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Notes and Examples on Intuitionistic Fuzzy Metric Space

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Abstract

Park introduced and discussed in [11] a notion of intuitionistic fuzzy metric space which is based both on the idea of intuitionistic fuzzy set due to Atanassov [1], and the concept of a fuzzy metric space given by George and Veeramani in [5] and [9]. We show an application and some examples of intuintionistic fuzzy metric spaces.

Keywords: Fuzzy metric; Compact subset; Intuitionistic fuzzy metric spaces; Hausdorff fuzzy metric.

1. Introduction and Preliminaries

In [8] the topology $\tau_{(M,N)}$ generated by an intuitionistic fuzzy metric space $(X, M, N,*, ♦)$ coincides with the topology τ_M generated by the fuzzy metric space $(X, M,*)$, and thus, the results obtained in [8] are immediate consequences of the corresponding and well-known results for fuzzy metric spaces. In this paper we show an applied of intuintionistic fuzzy metric spaces and Some illustrative examples are given.

Definition 1.1 Definition 1.1. A binary operation $*$: [0, 1] × [0, 1] ⟶ [0, 1] is continuous t-norm if $*$ is satisfying the following conditions:

1) $*$ is associative and commutative;

2) $*$ is continuous;

3) $a * 1 = a$ for all $a \in [0, 1]$;

4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.

Definition 1.2 A binary operation ♦ : [0, 1] × [0, 1] ⟶ [0, 1] is continuous t-conorm if ♦ is satisfying the following conditions:

1) ♦ is associative and commutative;

2) ♦ is continuous;

3) a♦0 = a for all $a \in [0, 1]$;

4) a♦b ≤ c♦d whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.

Definition 1.3: A fuzzy metric space is a triple (X, M, $*$) such that X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$, and $s, t \geq 0$:

1) $M(x, y, t) > 0$;

2) $M(x, y, t) = 1$ if and only if $x = y$;

3) $M(x, y, t) = M(y, x, t)$;

4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$;

5) $M(x, y, -): (0, \infty) \to [0, 1]$ is continuous.

Remark: If (X, M, $*$) is a fuzzy metric space, we will say that (M, $*$) is a fuzzy metric on X .

Our basic reference for general topology is [4]. George and Veeramani proved in [7] that every fuzzy metric $(M,*)$ on X generates a Hausdorff first topology τ_M on X which has as a base the family of open sets of the form $\tau_M = \{B_M(x,r,t):\ x \in X, r \in (0,1), t > 0\}$, where,

$$B_M(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\} for\ all\ x \in X, r \in (0,1) and t > 0\}.$$

Theorem 1.4: Let $(X, M,*)$ be a fuzzy metric space.Then (X, τ_M) is a metrizable topological space.

proof: [6, 7, 10].

2. Main results

Definition 2.1: A 5-tuple $(X, M, N,*, \blacklozenge)$ is said to be an intuitionistic fuzzy metric space if X is an nonempty set, $*$ is a continuous t-norm, $\blacklozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions:

1) $M(x,y,t) + N(x,y,t) \leq 1$;

2) $M(x,y,t) > 0$;

3) $M(x,y,t) = 1$ for all $t > 0$ if and only if $x = y$;

4) $M(x,y,t) = M(y,x,t)$;

5) $M(x,z,t+s) \geq M(x,y,t) * M(y,z,s)$ for all $x,y,z \in X, s,t > 0$;

6) $M(x,y,-) : (0,\infty) \to [0,1]$ is continuous;

7) $N(x,y,t) > 0$;

8) $N(x,y,t) = 0$ for all $t > 0$ if and only if $x = y$;

9) $N(x,y,t) = N(y,x,t)$;

10) $N(x,z,t+s) \leq N(x,y,t) \blacklozenge N(y,z,s)$ for all $x,y,z \in X, s,t > 0$;

11) $N(x,y,-) : (0,\infty) \to [0,1]$ is continuous;

Then (M, N) is called an intuitionistic fuzzy metric on X.

The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of nonnearness between x and y with respect to t, respectively.

Given a fuzzy metric space $(X, M, N,*, \blacklozenge)$, we shall denote the set of all nonempty compact subsets of $(X, \tau_{(M,N)})$, by $K_0(X)$.

Park proved in [11], among other results, that each intuitionistic fuzzy metric (M, N) on X generates a Hausdorff first countable topology $\tau_{(M,N)}$ on X which has as a base the family of open sets of the form $\{B(x,r,t) : x \in X, r \in (0,1), t > 0\}$, where, $B(x,r,t) = \{y \in X : M(x,y,t) > 1 - r, N(x,y,t) < r\}$ for all $x \in X, r \in (0,1)$ and $t > 0$.

Theorem 2.2. Let $(X, M, N,*, \blacklozenge)$ be an intuitionistic fuzzy metric space. Then, for each $x \in X, r \in (0,1), t > 0$, we have $B(x,r,t) = B_M(x,r,t)$.

Proof: It is clear that $B(x,r,t) \subseteq B_M(x,r,t)$. Now suppose that $y \in B_M(x,r,t)$.

Then $M(x,y,t) > 1 - r$, so, by condition (1) of Definition 2.1, we have:

$1 \geq M(x,y,t) + N(x,y,t) > 1 - r + N(x,y,t)$.

Hence N (x, y, t) < r, and consequently, y ∈ B(x, r, t).

Corollary2.3. Let $(X, M, N, *, \blacklozenge)$ be an intuitionistic fuzzy metric space. Then the topologies $\tau_{(M,N)}$ and τ_M coincide on X.

Proof: It will be deduce From theorem 2.2.

Corollary 2.4. Let $(X, M, N, *, \blacklozenge)$ be an intuitionistic fuzzy metric space. Then $(X, \tau_{(M,N)})$ is a metrizable topological space.

Proof: By Theorems 1.5 and 2.2.

Theorem 2.5. Let $(X, M, N, *, \blacklozenge)$ be an intuitionistic fuzzy metric space. Then the pair $(M_N, \square)$ is a fuzzy metric on X, where M_N is defined on $X^2 \times (0,\infty)$ by $M_N(x,y,t) = 1 - N(x,y,t)$ and $\square$ is the continuous t-norm defined by $a \square b = 1 - ((1-a) \blacklozenge (1-b))$.

proof :see[9].

Remark. Let $(X, M, N, *, \blacklozenge)$ is an intuitionistic fuzzy metric space and let $(M_N, \square)$ be the fuzzy metric constructed in theorem 2.5. Then $\tau_{MN} \subseteq \tau_M$, because for each x ∈ X, r ∈ (0, 1) and t > 0 we have, by theorem2.2, that

$$B_M(x,r,t) \subseteq \{y \in X : N(x,y,t) < r\} = \{y \in X : 1 - N(x,y,t) > 1 - r\} = B_{MN}(x,r,t).$$

Definition 2.6. Let $(X, M, N, *, \blacklozenge)$ be a fuzzy metric space. We define functions H_M and H_M on $K_0(X) \times K_0(X) \times (0,\infty)$ by,

$H_M(A,B,t) = min\{inf_{a\in A} M(a,B,t), inf_{b\in B} M(A,b,t)\}$,

and

$H_N(A,B,t) = max\{sup_{a\in A} N(a,B,t), sup_{b\in B} N(A,b,t)\}$,

for all $A, B \in K_0(X)$ and t > 0. where $M(a,B,t) = sup\{M(a,b,t) : b \in B\}$ and $N(a,B,t) = inf\,\{N(a,b,t) : b \in B\}$.

Remark. It was proved in [13] that $(H_M, \square)$ is a fuzzy metric on $\mathcal{K}_.(X)$ called the Hausdorff fuzzy metric of $(M, \square)$.

Now suppose that $((X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space. In was proved in [10] that $(H_M, H_N, *, \diamond)$ is a fuzzy metric on $\mathcal{K}_.(X)$ called the Hausdorff intuitionistic metric of $(M, N, *, \diamond)$.

Lemma 2.7 [12]

Let (X, d) be a metric space. Then, the Hausdorff fuzzy metric $(H_{M_d}, .)$ of the standard fuzzy metric $(M_d, .)$ coincides with the standard fuzzy metric $(M_{H_d}, .)$ of the Hausdorff metric H_d on $\mathcal{K}_.(X)$.

Theorem 2.8. Let (X, d) be a metric space. Then, the Hausdorff fuzzy metric $(H_{M_d}, H_{N_d}, \cdot)$ of the standard fuzzy metric $(M_{d,} N_d, \cdot)$ coincides with the standard fuzzy metric $(M_{H_d}, N_{H_d}, \cdot)$ of the Hausdorff metric H_d on $K_0(X)$.

proof .We consider the Hausdorff fuzzy metric $(H_{MN}, \square)$ of metric space defined by theorem2.5 . By theorem 2.7 we have $H_{M_d}(A, B, t) = M_{H_d}(A, B, t)$ (1). It is enough we show that $H_{N_d}(A, B, t) = N_{H_d}(A, B, t)$. The standard fuzzy metric $(N_d, .)$ for every $a \in A, b \in B$ is $N_d(a, b, t) = (d(a, b))/(t + d(a, b)$. Let $A, B \in K_0(X)$and $t > 0$ and for every $a \in A$ difined $N_d(a, B, t) = (d(a, B))/(t + d(a, B)$. By theorem2.5 we have;

$M_N(x, y, t) = 1 - N(x, y, t)$ (2)

and by (1) we see;

$H_{M_{N_d}}(A, B, t) = M_{N_{H_d}}(A, B, t)$ (3)

and,

$H_{N_d}(A, B, t) = max\{sup_{a \in A} N_d(a, B, t), sup_{b \in B} N_d(A, b, t)\}=$

$max\left\{sup_{a \in A}\left(1 - M_{N_d}(a, B, t)\right), sup_{b \in B}\left(1 - M_{N_d}(A, b, t)\right)\right\} =$

$max\{1 - inf_{a \in A} M_{N_d}(a, B, t), 1 - inf_{b \in B} M_{N_d}(A, b, t)\}=$

$1 - min\{inf_{a \in A} M_{N_d}(a, B, t), inf_{b \in B} M_{N_d}(A, b, t)\} = 1 - H_{M_{N_d}}(A, B, t),$

Therefore$H_{N_d}(A, B, t) = 1 - H_{M_{N_d}}(A, B, t)$. By (3) we have $H_{N_d}(A, B, t) = 1 - M_{N_{H_d}}$. By (2) we have $H_{N_d}(A, B, t) = N_{H_d}(A, B, t)$.

Example2.9. Let d be the Euclidean metric on R, and let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be two compact intervals. Then $H_d(A, B) = max\{|a_1 - b_1|, |a_2 - b_2||\}$; so, by Theorem 2.9, $H_{N_d}(A, B, t) = N_{H_d}(A, B, t)$ then $H_{M_d}(A, B, t) = t/(t + max\{|a_1 - b1, a2 - b2|\})$ and$HNd(A,B,t) = NHd(A,B,t)$ thus $HNd(A,B,t) = max\{|a_1 - b_1|, |a_2 - b_2||\}/(t + max\{|a_1 - b_1|, |a_2 - b_2||\})$ for all $t > 0$.

Example 2.10. Let d be the discrete metric on a (nonempty) set X with $|X| \geq 2$. Let A and B be two nonempty finite subsets of X, with $A = B$. Then,

$H_d(A,B) = max\{sup_{a\in A}\, d(a,B), sup_{b\in B}\, d(A,b)\} = max\{1,1\} = 1$;

So, by Theorem 2.8,

$H_{M_d}(A,B,t) = t/(t+1)$ and $H_{N_d}(A,B,t) = 1/(t+1)$ for all $t > 0$.

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