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Zweier Ideal Convergent Sequence Spaces Defined By Orlicz Functions

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Abstract

An ideal *I* is a family of subsets of positive integers N which is closed under taking finite unions and subsets of its elements. In this article we introduce ideal convergent sequence spaces using Zweier transform and Orlicz function. We study some topological and algebraic properties. Further we prove some inclusion relations related to these new spaces.

Keywords: Ideal, *I*-convergence, Zweier sequence, Orlicz function.

1. Introduction

The concept of ideal convergence as a generalization of statistical convergence, and any concept involving ideal convergence plays a vital role not only in pure mathematics but also in other branches of science involving mathematics, especially in information theory, computer science, biological science, dynamical systems, geographic information systems, population modelling, and motion planning in robotics.

Kostyrko, et. al [16] was initially introduced the notion of I-convergence based on the structure of the admissible ideal I of subset of natural numbers N. Further details on ideal convergence, we refer to ([2-3], [7-13], [17], [21], [25-26], [29-33]), and many others.

Let X be a non-empty set. Then a family of set $I \subseteq 2^x$ (power sets of X) is said to be an *ideal* if I is *additive* i.e. $A, B \in I \Rightarrow A \cup B \in I$ and *hereditary* i.e. $A \in I, B \subseteq A \Rightarrow B \in I$.

A non-empty family of sets $F \subset 2^X$ is said to be a filter on N if and only if $\phi \notin F(I)$, for $A, B \in F(I)$ we have $A \cap B \in F(I)$ and for each $A \in F(I)$ and $B \supset A$ implies $B \in F(I)$.

An ideal $I \subseteq 2^X$ is called *non-trivial* if $I \neq 2^X$.

A non trivial ideal $I \subset 2^X$ is called *admissible* if $I \supset \{\{x\} : x \in X\}$.

A non trivial ideal I is *maximal* if there exist any non – trivial ideal $J \neq I$ containing I as a subset.

For each ideal I there is a filter $F(I)$ corresponding to I i.e. $F(I) = \{K \subseteq N : K^c \in I\}$, where $K^c = N - K$.

A subset A of N is said to have asymptotic density $\delta(A)$ if $\delta(A) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_A(k)$ exists, where χ_A is the characteristic function of A.

Remark 1. If we take $I = I_f = \{A \subseteq N$: A is a finite subset$\}$. Then I_f is a nontrivial admissible ideal of N and the corresponding convergence coincide with the usual convergence.
If we take $I = I_\delta = \{A \subseteq N: \delta(A) = 0\}$ where $\delta(A)$ denote the asymptotic density of the set A. Then I_δ is a non-trivial admissible ideal of N and the corresponding convergence coincide with the statistical convergence.

An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$, which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$.If the convexity of the regular function M is replaced by

$$M(x + y) \leq M(x) + M(y),$$

Then this function is called modulus function. The notion of modulus function was introduced by Nakano [22]. Ruckle [24] and Maddox [19] further investigated the modulus functions with application to sequence spaces.

Remark 2. If M is an Orlicz function, then $M(\lambda x) \le \lambda M(x)$ for all λ with $0 < \lambda < 1.$

An Orlicz function M is said to satisfy Δ_2-condition for all values of u if there exists a constant $K > 0$ such that $M(Lu) \le KLM(u)$ for all values of $L > 1$ (see [15]).

Lindenstrauss and Tzafriri [18] used the idea of Orlicz functions to construct the sequence space

$$l_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}.$$

The space l_M becomes a Banach space with the norm

$$\|x\| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \le 1 \right\},$$

which is called an Orlicz sequence space. The space l_M is closely related to the space l_p which is an Orlicz sequence space with $M(t) = t^p$ for $1 \le p < \infty.$

Later on Orlicz sequence spaces were investigated by Parashar and Chaudhary [23], Esi [5], Tripathy et al, [28], Bhardwaj and Singh [1], Et [4], Esi and Et [6] and many others.

The approach of constructing the new sequence spaces by means of the matrix domain of a particular limitation method have been recently employed by Malkowsky [20] and many others. Sengonul [27] defined the sequence $y = (y_i)$ which is frequently used as the Z^p-transformation of the sequence $x = (x_i)$ i.e.

$$y_i = px_i + (1-p)x_{i-1}$$

Where $x_{-1} = 0, p \ne 0, 1 < p < \infty$ and Z^p denotes the matrix $Z^p = (z_{ik})$ defined by

$$z_{ik} = \begin{cases} p, & (i = k) \\ 1-p, & (i-1 = k); (i,k \in N) \\ 0, & \text{otherwise.} \end{cases}$$

Sengonul [27] introduced the Zweier sequence spaces Z and Z_0 as follows

$$Z = \left\{ x = (x_k) \in w : Z^p x \in c \right\}$$

$$Z_0 = \left\{ x = (x_k) \in w : Z^p x \in c_0 \right\}$$

2. Definitions and Preliminaries

We assume throughout this paper that the symbols R and N as the set of real and natural numbers, respectively. Throughout the paper, we also denote I is an admissible ideal of subsets of N , unless otherwise stated.

A sequence $(x_k) \in w$ is said to be *I*-convergent to the number L if for every $\varepsilon > 0$, $\{k \in N : |x_k - L| \geq \varepsilon\} \in I$. In this case we write $I - \lim x_k = L.$

A sequence $(x_k) \in w$ is said to be *I-null* if $L = 0.$ In this case we write $I - \lim x_k = 0.$

Let *I* be an admissible ideal. A sequence $(x_k) \in w$ is said to be *I-Cauchy* if for every $\varepsilon > 0$ there exists a number $m = m(\varepsilon)$ such that $\{k \in N : |x_k - x_m| \geq \varepsilon\} \in I.$

A sequence $(x_k) \in w$ is said to be *I-bounded* if there exists $M > 0$ such that $\{k \in N : |x_k| > M\} \in I.$

A sequence space E is said to be *solid (or normal)* if $(\alpha_k x_k) \in E$ whenever $(x_k) \in E$ and for all sequence (α_k) of scalars with $|\alpha_k| \leq 1$ for all $k \in N$.

A sequence space E is said to be *symmetric* if $(x_k) \in E$ implies $(x_{\pi(k)}) \in E,$ where π is a permutation of N .

A sequence space E is said to be *sequence algebra* if $(x_k) * (y_k) = (x_k y_k) \in E$ whenever $(x_k), (y_k) \in E.$

A sequence space E is said to be *convergence free* if $(y_k) \in E$ whenever $(x_k) \in E$ and $x_k = 0$ implies $y_k = 0$.

Let $K = \{k_1 < k_2 < ...\} \subset N$ and let E be a sequence space. A K -step space of E is a sequence space $\lambda_K^E = \{(x_{k_n}) \in w : (x_k) \in E\}.$

A *canonical preimage* of a sequence $(x_{k_n}) \in \lambda_K^E$ is a sequence $(y_k) \in w$ defined by

$$y_k = \begin{cases} x_k, & \text{if } k \in K \\ 0, & \text{otherwise.} \end{cases}$$

A *canonical preimage* of a *step space* λ_K^E is a set of canonical preimages of all the elements in λ_K^E , i.e. y is in the canonical preimage of λ_K^E if and only if y is a *canonical preimage* of some $x \in \lambda_K^E$.

A sequence space E is said to be *monotone* if it contain the canonical preimages of its step spaces.

Throughout the article Z^I, Z_0^I, Z_∞^I, m_Z^I and $m_{Z_0}^I$ represents Zweier *I*-convergent, Zweier *I*-null, Zweier bounded *I*-convergent and Zweier bounded *I*-null sequence space, respectively.

The following results will be used for establishing some result of this article.

Lemma 1 [14]. The sequence space E is solid implies that E is monotone.

3. Main Results

In this article, we introduce the following classes of sequences:

$$Z^I(M) = \left\{ x = (x_k) : \left\{ k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n - L\right|}{\rho} \right) \geq \varepsilon \right\} \in I \right\} \text{for some } L \in C$$

$$Z_0^I(M) = \left\{ x = (x_k) : \left\{ k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho} \right) \geq \varepsilon \right\} \in I \right\}$$

$$Z_\infty^I(M) = \left\{ x = (x_k) : \left\{ k \in N : \exists K > 0, \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho} \right) \geq K \right\} \in I \right\}.$$

Also we write

$$m_Z^I(M) = Z^I(M) \cap Z_\infty^I(M), \qquad m_{Z_0}^I(M) = Z_0^I(M) \cap Z_\infty^I(M).$$

Theorem 1. For any Orlicz function M, the classes of sequence $Z^I(M), Z_0^I\left(M\right)$ and Z_∞^I are linear spaces.

Proof: We shall prove that result for the space $Z^I(M)$.

Let $(x_k), (y_k) \in Z^I(M)$ and let α, β be scalars. Then there exists positive numbers ρ_1 *and* ρ_2 such that

$$\left\{ k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n - L_1\right|}{\rho_1} \right) \geq \varepsilon \right\} \in I \quad \text{for some } L_1 \in C.$$

$$\left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p y\right)_n - L_2\right|}{\rho_2}\right) \geq \varepsilon\right\} \in I \text{ for some } L_2 \in C.$$

That is

$$A_1 = \left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n - L_1\right|}{\rho_1}\right) \geq \frac{\varepsilon}{2}\right\} \in I \quad (1)$$

$$A_2 = \left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p y\right)_n - L_2\right|}{\rho_2}\right) \geq \frac{\varepsilon}{2}\right\} \in I \quad (2)$$

Let $\rho_3 = \max\{2|\alpha|\rho_1, 2|\beta|\rho_2\}$. Since M is non-decreasing and convex function, we have

$$M\left(\frac{\left|\left(\alpha\left(Z^p x\right)_n + \beta\left(Z^p y\right)_n\right) - \left(\alpha L_1 + \beta L_2\right)\right|}{\rho_3}\right) \leq M\left(\frac{|\alpha|\left|\left(Z^p x\right)_n - L_1\right|}{\rho_3} + \frac{|\beta|\left|\left(Z^p y\right)_n - L_2\right|}{\rho_3}\right)$$

$$\leq M\left(\frac{\left|\left(Z^p x\right)_n - L_1\right|}{\rho_1}\right) + M\left(\frac{\left|\left(Z^p y\right)_n - L_2\right|}{\rho_2}\right).$$

Now, from (1) and (2), we have

$$\left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(\alpha\left(Z^p x\right)_n + \beta\left(Z^p y\right)_n\right) - \left(\alpha L_1 + \beta L_2\right)\right|}{\rho_3}\right) \geq \varepsilon\right\} \subset A_1 \cup A_2 \in I .$$

Therefore $\left(\alpha\left(Z^p x\right)_n + \beta\left(Z^p y\right)_n\right) \in Z^I(M)$.. Hence $Z^I(M)$ is a linear space.

Theorem 2. The space $Z_0^I(M)$ and $Z^I(M)$ are Banach spaces normed by

$$\left\|\left(Z^p x\right)_n\right\| = \inf\left\{\rho > 0 : \sup_k M\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right) \leq 1\right\}.$$

The proof of the theorem is easy, so omitted.

Theorem 3. Let M_1, M_2 be Orlicz functions that satisfy the Δ_2-condition. Then

$(i) W(M_2) \subseteq W(M_1.M_2),$
$(ii) W(M_1) \cap W(M_2) = W(M_1 + M_2)$ for $W = Z^I, Z_0^I, m_Z^I, m_{Z_0}^I.$

Proof. (i) Let $(x_k) \in Z_0^I(M_2)$. Then there exists $\rho > 0$ such that

$$\left\{ k \in N : \sum_{n=1}^{\infty} M_2\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho} \right) \geq \varepsilon \right\} \in I \tag{3}$$

Let $\varepsilon > 0$ and choose δ with $0 < \delta < 1$ such that $M_1(t) < \varepsilon$ for $0 \leq t \leq \delta.$

Write $\left(Z^p y\right)_n = M_2\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho} \right)$ and consider

$$\lim_{0 \leq \left(Z^p y\right)_n \leq \delta, k \in N} M_1(Z^p y)_n = \lim_{\left(Z^p y\right)_n \leq \delta, k \in N} M_1(Z^p y)_n + \lim_{\left(Z^p y\right)_n > \delta, k \in N} M_1(Z^p y)_n .$$

We have

$$\lim_{\left(Z^p y\right)_n \leq \delta, k \in N} M_1(Z^p y)_n = M_1(2) \lim_{\left(Z^p y\right)_n \leq \delta, k \in N} \left(Z^p y\right)_n . \tag{4}$$

For $Z^p y > \delta$, we have

$$\left(Z^p y\right)_n < \frac{\left(Z^p y\right)_n}{\delta} < 1 + \frac{\left(Z^p y\right)_n}{\delta}.$$

Since M_1 is non decreasing and convex, it follows that

$$M_1(Z^p y)_n < M_1\left(1 + \frac{\left(Z^p y\right)_n}{\delta} \right) < \frac{1}{2} M_1(2) + \frac{1}{2} M_1\left(\frac{2\left(Z^p y\right)_n}{\delta} \right).$$

Since M_1 satisfies the Δ_2-condition, we have

$$M_1(Z^p y)_n < \frac{1}{2} K \frac{\left(Z^p y\right)_n}{\delta} M_1(2) + \frac{1}{2} K \frac{\left(Z^p y\right)_n}{\delta} M_1(2) = K \frac{\left(Z^p y\right)_n}{\delta} M_1(2)$$

Hence

$$\lim_{\left(Z^p y\right)_n > \delta, k\in N} M_1\left(Z^p y\right)_n \le \max\left(1, K\delta^{-1}M_1(2)\right) \lim_{\left(Z^p y\right)_n > \delta, k\in N}\left(Z^p y\right)_n. \quad (5)$$

From (3), (4) and (5), we have

$$(x_k) \in Z_0^I(M_1.M_2).$$

Thus $Z_0^I(M_2) \subseteq Z_0^I(M_1.M_2)$. The other case can be proved similarly.

(ii) Let $(x_k) \in Z_0^I(M_1) \cap Z_0^I(M_2)$. Then there exists $\rho > 0$ such that

$$\left\{k \in N : \sum_{n=1}^{\infty} M_1\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right) \ge \varepsilon\right\} \in I$$

and
$$\left\{k \in N : \sum_{n=1}^{\infty} M_2\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right) \ge \varepsilon\right\} \in I \quad .$$

The rest of the proof follows the following equality

$$\lim_{k\in N}(M_1+M_2)\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right) = \lim_{k\in N} M_1\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right) + \lim_{k\in N} M_2\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right).$$

Taking $M_2(x) = x$ and $M_1(x) = M(x)$ for all $x \in [0,\infty)$ in Theorem 3(i). We have the following result.

Corollary 1. $W \subseteq W(M)$ for $W = Z^I, Z_0^I, m_Z^I, m_{Z_0}^I$.

Proposition 1. The space $Z_0^I(M)$ and $m_{Z_0}^I(M)$ are solid and monotone.

Proof: We shall prove the result for $Z_0^I(M)$. For $m_{Z_0}^I(M)$, the result can be proved similarly.

Let $(x_k) \in Z_0^I(M)$. then there exists $\rho > 0$ such that

$$\left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right) \ge \varepsilon\right\} \in I. \quad (6)$$

Let (α_k) be a sequence scalar with $|\alpha_k| \le 1$ for all $k \in N$. Then the result follows from (6) and the following inequality

$$M\left(\frac{\left|\alpha_k\left(Z^p x\right)_n\right|}{\rho}\right) \le \left|\alpha_k\right| M\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right) \le M\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho}\right) \text{for all } k \in N,$$

which follows from the remark.

That the space $Z_0^I(M)$ is monotone follows from the Lemma 1.

Proposition 2. The space $Z^I(M)$ and $m_Z^I(M)$ are neither monotone nor solid in general.

Proof: The proof of this result follows from the following example.

Example 1. Let $I = I_f$ and $M(x) = x$ for all $x \in [0, \infty)$. Consider the K-step space T_K of T defined as follows.

Let $(x_k) \in T$ and let $(y_k) \in T_K$ be such that $y_k = \begin{cases} x_k, \text{ if } k \text{ is } odd \\ 0, \text{ otherwise} \end{cases}$.

Consider the sequence (x_k) defined by $x_k = \frac{1}{2}$ for all $k \in N$. Then $(x_k) \in Z^I(M)$ but its K-step space preimage does not belongs to $Z^I(M)$. Thus $Z^I(M)$ is not monotone. Hence $Z^I(M)$ is not solid by Lemma 1.

Proposition 3. The space $Z^I(M)$ and $Z_0^I(M)$ are not convergence free in general.

Proof: The proof of this result follows from the following example.

Example 2. Let $I = I_f$ and $M(x) = x^2$ for all $x \in [0, \infty)$. Consider the sequence (x_k) and (y_k) defined by $x_k = \frac{1}{k^2}$ and $y_k = k^2$ for all $k \in N$.

Then (x_k) belongs to $Z^I(M)$ and $Z_0^I(M)$, but (y_k) does not belongs to both $Z^I(M)$ and $Z_0^I(M)$.

Hence the spaces are not convergence free.

Proposition 4. The spaces $Z^I(M)$ and $Z_0^I(M)$ are sequence algebra.

Proof: We proof that $Z_0^I(M)$ is sequence algebra. For the space $Z^I(M)$, the result can be proved similarly.

Let $(x_k), (y_k) \in Z_0^I(M)$. Then

$$\left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n\right|}{\rho_1}\right) \geq \varepsilon\right\} \in I \ \ for\ some\ \rho_1 > 0$$

and $$\left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p y\right)_n\right|}{\rho_2}\right) \geq \varepsilon\right\} \in I \ \ for\ some\ \rho_2 > 0.$$

Let $\rho = \rho_1 \rho_2 > 0$. Then we can show that

$$\left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n . \left(Z^p y\right)_n\right|}{\rho}\right) \geq \varepsilon\right\} \in I.$$

Thus $(x_k . y_k) \in Z_0^I(M)$. Hence $Z_0^I(M)$ is sequence algebra.

Theorem 4: Let M be a Orlicz function. Then $Z_0^I(M) \subset Z^I(M) \subset Z_\infty^I(M)$ and the inclusions are proper.

Proof: Let $(x_k) \in Z^I(M)$. Then there exist $L \in C$ and $\rho > 0$ such that

$$\left\{k \in N : \sum_{n=1}^{\infty} M\left(\frac{\left|\left(Z^p x\right)_n - L\right|}{\rho}\right) \geq \varepsilon\right\} \in I.$$

We have

$$M\left(\frac{\left|\left(Z^p x\right)_n\right|}{2\rho}\right) \leq \frac{1}{2} M\left(\frac{\left|\left(Z^p x\right)_n - L\right|}{\rho}\right) + M \frac{1}{2}\left(\frac{|L|}{\rho}\right).$$

Taking supremum over k on both sides we get $(x_k) \in Z_\infty^I(M)$. The inclusion $Z_0^I(M) \subset Z^I(M)$ is obvious.

That the inclusion is proper follows from the following example.

Example 3. Let $I = I_d, M(x) = x^2 \ \ for\ all\ x \in [0, \infty)$.

(**a**) Consider the sequence (x_k) defined by $x_k = 1$ for all $k \in N$. Then $(x_k) \in Z^I(M)$, but $(x_k) \notin Z_0^I(M)$.

(**b**) Consider the sequence (y_k) defined as

$$y_k = \begin{cases} 2, & if\ k\ is\ even \\ 0, & otherwise \end{cases}.$$

Then $(y_k) \in Z_\infty^I(M)$, but $(y_k) \notin Z^I(M)$.

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