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Common Fixed Point Results for R - Weakly Commuting Mappings in Generalized Fuzzy Metric Spaces

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Abstract

In this paper, we prove two common fixed point theorems involving R-*weakly commuting mappings in the context of* $\mathcal{M}$*-fuzzy metric spaces. Our results generalizes the earlier results of Pant* [8], *Vasuki* [15] *and Som* [13,14] *in fuzzy metric spaces.*

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1. Introduction

Zadeh introduced fuzzy sets in his seminal work in 1965[18] and after a decade in 1975 Kramosil and Michalek in [6] introduced fuzzy metric spaces by generalizing the definition of probabilistic metric spaces. Afterwards, fuzzy metric spaces were introduced by fuzzify metric spaces from different angles. Some of these definitions are obtainable in [3], [4] and [5]. Fixed point results in fuzzy metric spaces in the sense of Kramosil and Michalek [6] have been obtained in works. The main purpose of this modification is to introduce some desirable topological properties such as Hausdroff property. For more detail, one can refers to papers [1], [2], [7], [12] and [17]. Fixed point results were discussed in modified $\mathcal{M}$-fuzzy metric spaces defined in the sense of Sedghi and Shobe [10]. Presently we take up some issues of fixed point theory involving R-commuting mappings is such spaces. .In this paper, we obtain some common fixed point results on $\mathcal{M}$-fuzzy metric spaces generalizing the earlier results of Pant [8], Vasuki [15] and Som [13,14] in fuzzy metric spaces.

Definition 1.1:[9] A binary operation *: [0,1] × [0,1] →[0,1] is a continuous t-norm if it satisfies the following conditions

(i) * is associative and commutative
(ii) * is continuous
(iii) a*1=a for all a ∈ [0,1].
(iv) a*b ≤ c*d whenever a ≤ c and b ≤ d, for each a, b, c, d ∈ [0,1].

Examples for continuous t-norm are a*b = ab and a*b = min {a,b}

Definition 1.2:[11] Let X be a non empty set. A generalized metric (or D* - metric) on X is a function D*: $X^3 \to [0,\propto)$ that satisfies the following conditions for each x, y, z, a ∈ X.

1. D* (x, y, z) ≥ 0

2. D* (x, y, z) = 0 if and only if x = y = z

3. D* (x, y, z) = D* (ρ{x, y, z}) where ρ is permutation.

4. D* (x, y, z) ≤ D* (x, y, a) + D* (a, z, z).

The pair (X, D*) is called generalized metric (or D* - metric) space.

Definition 1.3: [**10]** A 3-tuple (X, $\mathcal{M}$,*) is called $\mathcal{M}$-fuzzy metric space if X is an arbitrary non-empty set, * is a continuous t-norm, and $\mathcal{M}$ is a fuzzy set on X^3 x (0,∞), satisfying the following conditions for each x,y,z,a∈X and t, s > 0

(FM-1) $\mathcal{M}$(x,y,z,t) > 0

(FM-2) $\mathcal{M}$(x,y,z,t) = 1 iff x = y = z

(FM-3) $\mathcal{M}$(x,y,z,t) = $\mathcal{M}$(p{x,y,z},t), where p is a permutation function.

(FM-4) $\mathcal{M}$(x,y,a,t)* $\mathcal{M}$(a,z,z,s) ≤ $\mathcal{M}$(x,y,z,t + s)

(FM-5) $\mathcal{M}(x,y,z,\cdot) : (0,\infty) \to [0,1]$ is continuous

(FM-6) $\lim_{t\to\infty} \mathcal{M}(x, y, z, t) = 1$.

Examples 1.4: [11] Let X be a nonempty set, D * is the D* metric on X. Denote a*b = ab for all $a,b\in[0,1]$. For each $t\in (0,\infty)$, define $\mathcal{M}(x, y, z, t) = \frac{t}{t+D*(x,y,z)}$ for all $x,y,z\in X$, then $(X, \mathcal{M},*)$ is a $\mathcal{M}$-fuzzy metric space. We call this $\mathcal{M}$-fuzzy metric induced by D* -metric space. Thus every D* - metric induces a $\mathcal{M}$-fuzzy metric.

Lemma 1. 5: [10] Let $(X, \mathcal{M},*)$ be a $\mathcal{M}$-fuzzy metric space. Then for every $t > 0$ and for every $x,y\in X$. We have $\mathcal{M}(x,x,y,t) = \mathcal{M}(x,y,y,t)$.

Definition 1. 6: **[16]** Let $(X, \mathcal{M},*)$ be a $\mathcal{M}$-fuzzy metric space and $\{x_n\}$be a sequence in X then

(a) $\{x_n\}$ is said to be converges to a point $x\in X$ if $\lim_{n\to\infty} \mathcal{M}(x, x, x_n, t) = 1$ for all $t > 0$.
(b) $\{x_n\}$ is called Cauchy sequence if $\lim_{n\to\infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t) = 1$ for all $t > 0$ and $p > 0$.
(c) a $\mathcal{M}$-fuzzy metric space $(X, \mathcal{M},*)$ is said to be complete in which every Cauchy sequence is convergent.

Definition 1.7: The mappings f and g of $\mathcal{M}$-fuzzy metric space $(X, \mathcal{M},*)$ into itself is said to be weakly commuting $\mathcal{M}(fgx, gfx, gfx, t) \geq \mathcal{M}(fx, gx, gx, t)$ for all $x\in X$, $t > 0$.

Definition 1.8: Let f and g be two self mapping of a $\mathcal{M}$-fuzzy metric space $(X, \mathcal{M},*)$. Then the mapping are said to be R-weakly commuting provided there exists some positive real number R such that $\mathcal{M}(fgx, gfx, gfx, t) \geq \mathcal{M}(fx, gx, gx, t/R)$ for all $x\in X$, $t > 0$ and $R > 0$.

2. Fixed point theorem for three self maps

Theorem 2.1: Let S and T be two continuous self mappings of a complete $\mathcal{M}$-fuzzy metric space $(X, \mathcal{M},*)$, where * is a continuous t-norm. Let A be a self mappings in X such that:

(2.1) {A,S} and {A,T} are R-weakly commuting;

(2.2) $A(X) \subseteq S(X) \cap T(X)$

(2.3) $\left(\frac{1}{\mathcal{M}(Ax,Ay,Az,t)} - 1\right) \leq \varphi \left[\max\left\{\left(\frac{1}{\mathcal{M}(Sx,Ay,Ty,t)} - 1\right), \left(\frac{1}{\mathcal{M}(Sx,Ax,Ay,t)} - 1\right), \left(\frac{1}{\mathcal{M}(Ty,Ay,Az,t)} - 1\right.\right.\right.$

where $\varphi: [0,\infty) \to [0, \infty)$ is a continuous function such that $\varphi(t) < t$ for all $t > 0$ and $\varphi(0) = 0$;
(2.4) let $x_n \to x$, $y_n \to y$ and $z_n \to z$, $t > 0$ implies $\mathcal{M}(x_n, y_n, z_n, t) \to \mathcal{M}(x, y, z, t)$

Then A, S and T have a unique common fixed point in X.

Proof : Let $x_0 \in X$ be an arbitrary point. As $A(X) \subseteq S(X)$. So, there exists a point $x_1 \in X$ such that $Ax_0 = Sx_1$.

Again $A(X) \subseteq T(X)$, hence there exists another point $x_2 \in X$ such that $Ax_1 = Tx_2$. In general,

we get points x_{2n+1} and x_{2n+2} in X, such that $Sx_{2n+1} = Ax_{2n}$, $Tx_{2n+2} = Ax_{2n+1}$ for n = 0, 1…

Let for t > 0,

$$H_n(t) = \left(\frac{1}{\mathcal{M}(Ax_n, Ax_{n+1}, Ax_{n+1}, t)} - 1\right)$$

Therefore $H_{2n}(t) = \left(\frac{1}{\mathcal{M}(Ax_{2n}, Ax_{2n+1}, Ax_{2n+1}, t)} - 1\right) = \left(\frac{1}{\mathcal{M}(Ax_{2n+1}, Ax_{2n}, Ax_{2n}, t)} - 1\right)$

$$\leq \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(Sx_{2n+1}, Ax_{2n}, Tx_{2n}, t)} - 1\right), (-1), \left(\frac{1}{\mathcal{M}(Tx_{2n}, Ax_{2n}, Ax_{2n}, t)} - 1\right)\right\}\right]$$

$$= \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(Ax_{2n}, Ax_{2n}, Ax_{2n-1}, t)} - 1\right), (-1), \left(\frac{1}{\mathcal{M}(Ax_{2n-1}, Ax_{2n}, Ax_{2n}, t)} - 1\right)\right\}\right]$$

$$= \varphi[\max\{H_{2n-1}(t), H_{2n}(t), H_{2n-1}(t)\}]$$

$$= \varphi[\max\{H_{2n-1}(t), H_{2n}(t)\}] \tag{2.5}$$

We claim that $H_{2n}(t) < H_{2n-1}(t)$. If not, then $\max\{H_{2n-1}(t), H_{2n}(t)\} = H_{2n}(t)$

Therefore, by (2.5), $H_{2n}(t) \leq \varphi[H_{2n}(t)] < H_{2n}(t)$, which is a contradiction.

Hence, $H_{2n}(t) < H_{2n-1}(t)$. This gives, by (2.5),

$$H_{2n}(t) \leq \varphi[H_{2n-1}(t)] < H_{2n-1}(t) \tag{2.6}$$

Now, $H_{2n+1}(t) =$

$$\left(\frac{1}{\mathcal{M}(Ax_{2n+1}, Ax_{2n+2}, Ax_{2n+2}, t)} - 1\right) = \left(\frac{1}{\mathcal{M}(Ax_{2n+2}, Ax_{2n+1}, Ax_{2n+1}, t)} - 1\right)$$

$$\leq \varphi\left[\max\left\{\begin{array}{c}\left(\frac{1}{\mathcal{M}(Sx_{2n+2}, Ax_{2n+1}, Tx_{2n+1}, t)} - 1\right), \left(\frac{1}{\mathcal{M}(Sx_{2n+2}, Ax_{2n+2}, Ax_{2n+1}, t)} - 1\right), \\ \left(\frac{1}{\mathcal{M}(Tx_{2n+1}, Ax_{2n+1}, Ax_{2n+1}, t)} - 1\right)\end{array}\right\}\right]$$

$$= \varphi\left[\max\left\{\begin{array}{c}\left(\frac{1}{\mathcal{M}(Ax_{2n+1}, Ax_{2n+1}, Ax_{2n}, t)} - 1\right), \left(\frac{1}{\mathcal{M}(Ax_{2n+1}, Ax_{2n+2}, Ax_{2n+1}, t)} - 1\right), \\ \left(\frac{1}{\mathcal{M}(Ax_{2n}, Ax_{2n+1}, Ax_{2n+1}, t)} - 1\right)\end{array}\right\}\right]$$

$$= \varphi[\max\{H_{2n}(t), H_{2n+1}(t), H_{2n}(t)\}] = \varphi[\max\{H_{2n}(t), H_{2n+1}(t)\}]$$

As done above, we can easily prove that $H_{2n+1}(t) < H_{2n}(t)$.

Therefore, $H_{2n+1}(t) < H_{2n}(t) < H_{2n-1}(t)$.

Hence, $\{H_n(t)\}$ is a decreasing sequence of positive real numbers, hence it tends to a limit $L \geq 0$.

We claim that L= 0. If not, taking limit as $n \to \infty$, as φ is a continuous function, (2.6) gives $L \leq \varphi(L) < L$, a contradiction. Hence L= 0, that is, for t > 0

$$\lim_{n\to\infty} \mathcal{M}(Ax_n, Ax_{n+1}, Ax_{n+1}, t) = 1 \tag{2.7}$$

Now, for any positive integer p, we have

$$\mathcal{M}(Ax_n, Ax_{n+p}, Ax_{n+p}, t) \geq \mathcal{M}(Ax_n, Ax_{n+1}, Ax_{n+1}, t/p) * \mathcal{M}(Ax_{n+1}, Ax_{n+2}, Ax_{n+2}, t/p) * \ldots * \mathcal{M}(Ax_{n+p-1}, Ax_{n+p}, Ax_{n+p}, t/p).$$

Taking limit as $n \to \infty$, in the above inequality and using (2.7) and continuity of *, we have

$$\lim_{n\to\infty} \mathcal{M}(Ax_n, Ax_{n+p}, Ax_{n+p}, t) \geq \lim_{n\to\infty} \mathcal{M}(Ax_n, Ax_{n+1}, Ax_{n+1}, t/p) * \mathcal{M}(Ax_{n+1}, Ax_{n+2}, Ax_{n+2}, t/p) * \ldots * \mathcal{M}(Ax_{n+p-1}, Ax_{n+p}, Ax_{n+p}, t/p)$$

$$\geq 1*1* \underbrace{\quad}_{\text{p times}} *1 = 1$$

In other words, $\lim_{n\to\infty} \mathcal{M}(Ax_n, Ax_{n+p}, Ax_{n+p}, t) = 1$ for all t > 0 and positive integer p.

Thus $\{Ax_n\}$ is a Cauchy sequence and by completeness of X, we have $\{Ax_n\}$ converges to a point $z \in X$.

Obviously, the subsequences $\{Sx_{2n+1}\}$ and $\{Tx_{2n}\}$ of $\{Ax_n\}$ also converges to the same limit.

$$\text{Thus } Ax_n \to z,\ Sx_{2n+1} \to z \text{ and } Tx_{2n} \to z \text{ as } n \to \infty \tag{2.8}$$

Since pair {A, S} is R-weakly commuting, we get

$$\left(\frac{1}{\mathcal{M}(ASx_{2n+1}, SAx_{2n+1}, SAx_{2n+1}, t)} - 1\right) \leq \left(\frac{1}{\mathcal{M}(Ax_{2n+1}, Sx_{2n+1}, Sx_{2n+1}, t/R)} - 1\right)$$

$$= \left(\frac{1}{\mathcal{M}(Ax_{2n+1}, Ax_{2n}, Ax_{2n}, t/R)} - 1\right)$$

Taking limit as $n \to \infty$ we have by virtue of (2.5) and the continuity of S gives,

$$\lim_{n\to\infty}\left(\frac{1}{\mathcal{M}(ASx_{2n+1}, SAx_{2n+1}, SAx_{2n+1}, t)} - 1\right) \leq \lim_{n\to\infty}\left(\frac{1}{\mathcal{M}(Ax_{2n+1}, Ax_{2n}, Ax_{2n}, t/R)} - 1\right)$$

$$= \left(\frac{1}{\mathcal{M}(z, z, z, t/R)} - 1\right) = 0$$

Again $Ax_n \to z$, therefore by continuity of S, $SAx_n \to Sz$.

Therefore, by the above inequality, $\lim_{n\to\infty} ASx_{2n+1} = \lim_{n\to\infty} SAx_{2n+1} = Sz$ (2.9)

Similarly, as pair {A,T} is R-weakly commuting and using continuity of T, we can easily prove

$\lim_{n\to\infty} ATx_{2n+1} = \lim_{n\to\infty} TAx_{2n+1} = Tz$ (2.10)

Now, we prove that Sz = z. Suppose otherwise. Then there exists t > 0, such that

$\left(\frac{1}{\mathcal{M}(Sz,z,z,t)} - 1\right) > 0.$ By (2.3) we have,

$$\left(\frac{1}{\mathcal{M}(ASx_{2n+1},Ax_{2n},Ax_{2n},t)} - 1\right)$$

$$\leq \varphi\left[\max\left\{\begin{matrix}\left(\frac{1}{\mathcal{M}(S^2x_{2n+1},Ax_{2n},Tx_{2n},t)} - 1\right), \left(\frac{1}{\mathcal{M}(S^2x_{2n+1},ASx_{2n+1},Ax_{2n},t)} - 1\right), \\ \left(\frac{1}{\mathcal{M}(Tx_{2n},Ax_{2n},Ax_{2n},t)} - 1\right)\end{matrix}\right\}\right]$$

Taking limit as $n \to \infty$ we have by virtue of (2.5) , (2.8) ,(2.9) and the continuity of S,

$$\left(\frac{1}{\mathcal{M}(Sz,z,z,t)} - 1\right) \leq \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(Sz,z,Sz,t)} - 1\right), \left(\frac{1}{\mathcal{M}(Sz,Sz,z,t)} - 1\right), \left(\frac{1}{\mathcal{M}(Sz,z,z,t)} - 1\right)\right\}\right]$$

$$= \varphi\left(\frac{1}{\mathcal{M}(Sz,z,z,t)} - 1\right) < \left(\frac{1}{\mathcal{M}(Sz,z,z,t)} - 1\right), \text{ which is a contradiction}$$

Thus z is a fixed point of S.

Next, we can show that z is a fixed point of A.

If otherwise, z is not a fixed point of A, then $\left(\frac{1}{\mathcal{M}(Az,z,z,t)} - 1\right) > 0$. By (2.3) we have,

$$\left(\frac{1}{\mathcal{M}(Az,Ax_{2n},Ax_{2n},t)} - 1\right) \leq \varphi\left[\max\left\{\begin{matrix}\left(\frac{1}{\mathcal{M}(Sz,Ax_{2n},Tx_{2n},t)} - 1\right) \\ \left(\frac{1}{\mathcal{M}(Sz,Az,Ax_{2n},t)} - 1\right), \left(\frac{1}{\mathcal{M}(Tx_{2n},Ax_{2n},Ax_{2n},t)} - 1\right)\end{matrix}\right\}\right]$$

As $n \to \infty$, we have,

$$\left(\frac{1}{\mathcal{M}(Az,z,z,t)} - 1\right) \leq \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(Sz,z,z,t)} - 1\right), \left(\frac{1}{\mathcal{M}(Sz,z,z,t)} - 1\right), \left(\frac{1}{\mathcal{M}(z,z,z,t)} - 1\right)\right\}\right]$$

$$= \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(z,z,z,t)} - 1\right), \left(\frac{1}{\mathcal{M}(z,z,z,t)} - 1\right), \left(\frac{1}{\mathcal{M}(z,z,z,t)} - 1\right)\right\}\right]$$

$$= \varphi\,[\max\,(0,0,0)]$$

$$= \varphi(0) = 0$$

which is a contradiction to the fact that $\left(\frac{1}{\mathcal{M}(Az,z,z,t)} - 1\right) > 0$. Hence z is a fixed point of A.

Now we claim that z is also a fixed point of T. Suppose not, then $\left(\frac{1}{\mathcal{M}(\mathrm{Tz,z,z,t})}-1\right)>0$.

By (2.3) we have,

$$\left(\frac{1}{\mathcal{M}(\mathrm{Az,ATx_{2n},ATx_{2n},t})}-1\right) \leq \varphi\left[\max\left\{\begin{array}{c}\left(\frac{1}{\mathcal{M}(\mathrm{Sz,ATx_{2n},T^2x_{2n},t})}-1\right),\\ \left(\frac{1}{\mathcal{M}(\mathrm{Sz,Az,ATx_{2n},t})}-1\right),\left(\frac{1}{\mathcal{M}(\mathrm{T^2x_{2n},ATx_{2n},ATx_{2n},t})}-1\right)\end{array}\right\}\right]$$

Taking limit as n $\to\infty$, it gives

$$\left(\frac{1}{\mathcal{M}(\mathrm{z,Tz,Tz,t})}-1\right) \leq \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(\mathrm{z,Tz,Tz,t})}-1\right),\left(\frac{1}{\mathcal{M}(\mathrm{z,z,Tz,t})}-1\right),\left(\frac{1}{\mathcal{M}(\mathrm{Tz,Tz,Tz,t})}-1\right)\right\}\right]$$

$$= \varphi\left[\left(\frac{1}{\mathcal{M}(\mathrm{z,Tz,Tz,t})}-1\right)\right]$$

$$< \left(\frac{1}{\mathcal{M}(\mathrm{Tz,z,z,t})}-1\right)$$

which is a contradiction. Hence z is a fixed point of T.

Uniqueness: Let v be another fixed point of A, S and T . That is Av = Sv = Tv

and $\left(\frac{1}{\mathcal{M}(\mathrm{z},v,v,\mathrm{t})}-1\right)>0$. That is $\left(\frac{1}{\mathcal{M}(\mathrm{Az,A}v,\mathrm{A}v,\mathrm{t})}-1\right)>0$.

Now , by (2.3) we have,

$$\left(\frac{1}{\mathcal{M}(\mathrm{Az,A}v,\mathrm{A}v,\mathrm{t})}-1\right) \leq \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(\mathrm{Sz,A}v,\mathrm{T}v,\mathrm{t})}-1\right),\left(\frac{1}{\mathcal{M}(\mathrm{Sz,Az,A}v,\mathrm{t})}-1\right),\left(\frac{1}{\mathcal{M}(\mathrm{T}v,\mathrm{A}v,\mathrm{A}v,\mathrm{t})}-1\right)\right\}\right]$$

$$= \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(\mathrm{z},v,v,\mathrm{t})}-1\right),\left(\frac{1}{\mathcal{M}(\mathrm{z,z},v,\mathrm{t})}-1\right),\left(\frac{1}{\mathcal{M}(v,v,v,\mathrm{t})}-1\right)\right\}\right]$$

$$= \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(\mathrm{z},v,v,\mathrm{t})}-1\right),\left(\frac{1}{\mathcal{M}(\mathrm{z,z},v,\mathrm{t})}-1\right),0\right\}\right]$$

$$= \varphi\left[\left(\frac{1}{\mathcal{M}(\mathrm{z},v,v,\mathrm{t})}-1\right)\right]$$

$$< \left(\frac{1}{\mathcal{M}(\mathrm{z},v,v,\mathrm{t})}-1\right)$$

which is a contradiction. Hence z is a fixed point of A, S and T. This completes the proof.

Taking T = S in the above theorem we get the following corollary unifying Vasuki's Theorem 1.9 [11], which in turn also generalizes the result of Pant [5].

Corollary 2.2: Let S be a continuous self mappings of a complete $\mathcal{M}$-fuzzy metric space (X, $\mathcal{M}$,*), where * is a continuous t-norm. Let A be another self mappings of X such that:

(2.11) {A, S} is R-weakly commuting;

(2.12) $A(X) \subseteq S(X)$;

(2.13)
$$\left(\frac{1}{\mathcal{M}(Ax,Ay,Az,t)} - 1\right) \leq \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(Sx,Ay,Sy,t)} - 1\right), \left(\frac{1}{\mathcal{M}(Sx,Ax,Ay,t)} - 1\right), \left(\frac{1}{\mathcal{M}(Sy,Ay,Az,t)} - 1\right)\right\}\right]$$

where $\varphi: [0,\infty) \to [0, \infty)$ is a continuous function such that $\varphi(t) < t$ for all $t > 0$ and $\varphi(0) = 0$;

(2.14) let $x_n \to x$, $y_n \to y$ and $z_n \to z$, $t > 0$ implies $\mathcal{M}(x_n, y_n, z_n, t) \to \mathcal{M}(x, y, z, t)$.

Remark 2.3: The conclusions of Theorem 2.1 remain true if we replace condition (2.3) by any of the following conditions:

(2.15) $$\left(\frac{1}{\mathcal{M}(Ax,Ay,Az,t)} - 1\right) \leq \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(Sx,Ay,Ty,t)} - 1\right) * \left(\frac{1}{\mathcal{M}(Sx,Ax,Ay,t)} - 1\right)\right\}\right]$$

where $\varphi: [0,\infty) \to [0, \infty)$ is a continuous function such that $\varphi(t) < t$ for all $t > 0$ and $\varphi(0) = 0$;

(2.16) $\mathcal{M}(Ax, Ay, Az, t) \geq r\,[\min\{\mathcal{M}(Sx, Ay, Ty, t), \mathcal{M}(Sx, Ax, Ay, t), \mathcal{M}(Ty, Ay, Az, t)\}]$

where $r : [0, 1] \to [0, 1]$ is a continuous function such that $r(t) > t$ for all $t < 1$ and $r(1) = 1$.

Remark 2.4: The conclusions of Corollary 2.2 remain true if we replace condition (2.13) by any of the following conditions:

(2.17) $$\left(\frac{1}{\mathcal{M}(Ax,Ay,Az,t)} - 1\right) \leq \varphi\left[\max\left\{\left(\frac{1}{\mathcal{M}(Sx,Ay,Sy,t)} - 1\right) * \left(\frac{1}{\mathcal{M}(Sx,Ax,Ay,t)} - 1\right)\right\}\right]$$

where $\varphi: [0,\infty) \to [0, \infty)$ is a continuous function such that $\varphi(t) < t$ for all $t > 0$ and $\varphi(0) = 0$;

(2.18) $\mathcal{M}(Ax, Ay, Az, t) \geq r\,[\min\{\mathcal{M}(Sx, Ay, Sy, t), \mathcal{M}(Sx, Ax, Ay, t)\}]$

where $r : [0, 1] \to [0, 1]$ is a continuous function such that $r(t) > t$ for all $t < 1$ and $r(1) = 1$.

Remark 2.5: From Theorem 2.1 and Remark 2.3, it is clear that above results generalizing the earlier results of Pant [8], Vasuki [15] and Som [13,14] in fuzzy metric spaces.

3. Fixed point theorem for four self maps:

We prove a common fixed point theorem for four maps:

Theorem 3. 1: Let S and T be two continuous self-mappings of a complete $\mathcal{M}$-fuzzy metric space $(X, \mathcal{M}, *)$, when * is a continuous t-norm. Let A and B be two self mapping of X satisfying:

(3.1) $A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$

(3.2) {A,T} and{B, S} are R-weakly commuting pairs

(3.3) $$a\left(\frac{1}{\mathcal{M}(Tx,Sy,Ax,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tx,Ax,Sz,t)}-1\right)+c\left(\frac{1}{\mathcal{M}(Sy,Bz,By,t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(Ax,Sz,Sy,t)}-1\right),\left(\frac{1}{\mathcal{M}(By,Tx,Sy,t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(Ax,Sz,By,t)}-1\right)$$

for all x, y, z$\in$X , where a, b, c $\in$[0,1], q >2 with q > a + b + c. Then, A, B, S and T have a unique common fixed point.

Proof: Let $x_0 \in X$ be any arbitrary point. Since $A(X) \subseteq S(X)$, there is a point $x_1 \in X$ such that $Ax_0 = Sx_1$. Again since $B(X) \subseteq T(X)$, for this x_1, there is an $x_2 \in X$ such that $Bx_1=Tx_2$ and so on. Inductively, we get a sequence $\{y_n\}$ in X such that $y_{2n} = Ax_{2n} = Sx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Tx_{2n+2}$, n = 0, 1, 2. . .

Let for t > 0, $H_n(t) = \left(\frac{1}{\mathcal{M}(y_n,y_{n+1},y_{n+1},t)}-1\right) \geq 0$ for all n.

Putting $x = x_{2n}$, $y = x_{2n+1}$, and $z = x_{2n+1}$ in (3.3), we get,

$$a\left(\frac{1}{\mathcal{M}(Tx_{2n},Sx_{2n+1},Ax_{2n},t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tx_{2n},Ax_{2n},Sx_{2n+1},t)}-1\right)+\ c\left(\frac{1}{\mathcal{M}(Sx_{2n+1},Bx_{2n+1},Bx_{2n+1},t)}-1\right)$$

$$+\min\left\{\left(\frac{1}{\mathcal{M}(Ax_{2n},Sx_{2n+1},Sx_{2n+1},t)}-1\right),\left(\frac{1}{\mathcal{M}(Bx_{2n+1},Tx_{2n},Sx_{2n+1},t)}-1\right)\right\}$$

$$\geq q\left(\frac{1}{\mathcal{M}(Ax_{2n},Sx_{2n+1},Bx_{2n+1},t)}-1\right)$$

$$a\left(\frac{1}{\mathcal{M}(y_{2n-1},y_{2n},y_{2n},t)}-1\right)+b\left(\frac{1}{\mathcal{M}(y_{2n-1},y_{2n},y_{2n},t)}-1\right)+c\left(\frac{1}{\mathcal{M}(y_{2n},y_{2n+1},y_{2n+1},t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(y_{2n},y_{2n},y_{2n},t)}-1\right),\left(\frac{1}{\mathcal{M}(y_{2n+1},y_{2n-1},y_{2n},t)}-1\right)\right\}\ \geq q\left(\frac{1}{\mathcal{M}(y_{2n},y_{2n},y_{2n+1},t)}-1\right)$$

$\Rightarrow$ $aH_{2n-1}(t) + bH_{2n-1}(t) + cH_{2n}(t) + 0 \geq qH_{2n}(t)$.

$\Rightarrow$ $(a + b) H_{2n-1}(t) \geq (q - c)H_{2n}(t)$.

$\Rightarrow$ $H_{2n}(t) \leq \frac{a+b}{q-c} H_{2n-1}(t) < pH_{2n-1}(t) < H_{2n-1}(t)$, where $p = \frac{a+b}{q-c} < 1$. (3.4)

Again putting $x = x_{2n+1}$, $y = x_{2n+2}$, $z = x_{2n+2}$, in (3.3) we have

$$a\left(\frac{1}{\mathcal{M}(Tx_{2n+1},Sx_{2n+2},Ax_{2n+1},t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tx_{2n+1},Ax_{2n+1},Sx_{n+2},t)}-1\right)+c\left(\frac{1}{\mathcal{M}(Sx_{2n+2},Bx_{2n+2},Bx_{2n+2},t)}-\right.$$
$$\left.1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(Ax_{2n+1},Sx_{2n+2},Sx_{2n+2},t)}-1\right),\left(\frac{1}{\mathcal{M}(Bx_{2n+2},Tx_{2n+1},Sx_{2n+2},t)}-1\right)\right\}$$
$$\geq q\left(\frac{1}{\mathcal{M}(Ax_{2n+1},Sx_{2n+2},Bx_{2n+2},t)}-1\right)$$

$$a\left(\frac{1}{\mathcal{M}(y_{2n},y_{2n+1},y_{2n+1},t)}-1\right)+b\left(\frac{1}{\mathcal{M}(y_{2n},y_{2n+1},y_{2n+1},t)}-1\right)+c\left(\frac{1}{\mathcal{M}(y_{2n+1},y_{2n+2},y_{2n+2},t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(y_{2n+1},y_{2n+1},y_{2n+1},t)}-1\right),\left(\frac{1}{\mathcal{M}(y_{2n+2},y_{2n},y_{2n+1},t)}-1\right)\right\}$$
$$\geq q\left\{\frac{1}{\mathcal{M}(y_{2n+1},y_{2n+1},y_{2n+2},t)}-1\right\}$$

$\Rightarrow$ $aH_{2n}(t) + bH_{2n}(t) + cH_{2n+1}(t) + 0 \geq q\, H_{2n+1}(t)$.

$\Rightarrow$ $(a + b)\, H_{2n}(t) \geq (q - c)\, H_{2n+1}(t)$.

$\Rightarrow$ $H_{2n+1}(t) \leq \frac{a+b}{q-c} H_{2n}(t)$.

$\Rightarrow$ $H_{2n+1}(t) \leq \frac{a+b}{q-c} H_{2n}(t) \leq p\, H_{2n}(t) < H_{2n-1}(t)$ where $p = \frac{a+b}{q-c} < 1$. (3.5)

Hence for $t > 0$, $\{H_n(t)\}$ is a decreasing sequence of positive real numbers and therefore tends to a limit $L \geq 0$. If $L > 0$, taking limit as $n \to \infty$ on (3.4), we get $L< L$. which is a contradiction. Hence $L = 0$, $\lim_{n\to\infty} \mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) = 1$ for all $t > 0$. Now for any positive integer p, we have

$$\mathcal{M}(y_n, y_{n+p}, y_{n+p}, t) \geq \mathcal{M}(y_n, y_{n+1}, y_{n+1}, t/p) * \mathcal{M}(y_{n+1}, y_{n+2}, y_{n+2}, t/p)$$
$$* \ldots * \mathcal{M}(y_{n+p-1}, y_{n+p}, y_{n+p}, t/p) \geq 1*1*\ldots*1 = 1$$

$$\Rightarrow \lim_{n\to\infty} \mathcal{M}(y_n, y_{n+p}, y_{n+p}, t) \geq \lim_{n\to\infty} \mathcal{M}(y_n, y_{n+1}, y_{n+1}, t/p) * \mathcal{M}(y_{n+1}, y_{n+2}, y_{n+2}, t/p) * \ldots *$$
$$\mathcal{M}(y_{n+p-1}, y_{n+p}, y_{n+p}, t/p)$$
$$\geq 1*1* \underline{\text{p times}} *1 = 1$$

Thus $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, there is a point $u \in X$, such that $y_n \to u$ and this implies that $\{Ax_{2n}\}$ and $\{Bx_{2n+1}\}$ converges to u such that the sequence $\{Sx_{2n+1}\}$ and $\{Tx_{2n+2}\}$ also converges to u, that is $Sx_{2n+1} \to u$ and $Tx_{2n+1} \to u$ as $n \to \infty$. We show that u is a common fixed point of A, B, S and T

Since the pair {A, T} is R-weakly commuting, therefore

$$\left(\frac{1}{\mathcal{M}(ATx_{2n}, TAx_{2n}, TAx_{2n}, t)} - 1\right) \leq \left(\frac{1}{\mathcal{M}(Tx_{2n}, Ax_{2n}, Ax_{2n}, t/R)} - 1\right) \to 0 \text{ as } n \to \infty.$$

Therefore, $\lim_{n\to\infty} ATx_{2n} = \lim_{n\to\infty} TAx_{2n} = Tu$ (as T is continuous).

Now, we claim that u is fixed point of T. Suppose not, then for any $t > 0$, we get $x = Tx_{2n}$, $y = x_{2n+1}$, $z = x_{2n+1}$ from (3.3),

$$a\left(\frac{1}{\mathcal{M}(T^2x_{2n},Sx_{2n+1},ATx_{2n},t)}-1\right)+b\left(\frac{1}{\mathcal{M}(T^2x_{2n},ATx_{2n},Sx_{2n+1},t)}-1\right)+$$

$$c\left(\frac{1}{\mathcal{M}(Sx_{2n+1},Bx_{2n+1},Bx_{2n+1},t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(ATx_{2n},Sx_{2n+1},Sx_{2n+1},t)}-1\right)+\left(\frac{1}{\mathcal{M}(Bx_{2n+1},T^2x_{2n+1},Sx_{2n+1},t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(ATx_{2n},Sx_{2n+1},Bx_{2n+1},t)}-1\right)$$

As $n\to\infty$, we have

$$a\left(\frac{1}{\mathcal{M}(Tu,u,Tu,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tu,Tu,u,t)}-1\right)+c\left(\frac{1}{\mathcal{M}(u,u,u,t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(Tu,u,u,t)}-1\right),\left(\frac{1}{\mathcal{M}(u,Tu,u,t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(Tu,u,u,t)}-1\right)$$

$$\Rightarrow\qquad (a+b+1)\left(\frac{1}{\mathcal{M}(Tu,u,u,t)}-1\right)\geq q\left(\frac{1}{\mathcal{M}(Tu,u,u,t)}-1\right)$$

$$\Rightarrow\qquad (a+b+1)\geq q.$$

$\Rightarrow\qquad (a+b)\geq(q-1)$, which is a contradiction. Thus u is a fixed point of T.

Next, we claim that u is a fixed point of A. Suppose that $Au\neq u$, then for any $t>0$, $\left(\frac{1}{\mathcal{M}(Au,u,u,t)}-1\right)>0$.

Again using (3.3), we get $x = u$, $y = x_{2n+1}$, $z = x_{2n+1}$.

$$a\left(\frac{1}{\mathcal{M}(Tu,Sx_{2n+1},Au,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tu,Au,Sx_{2n+1},t)}-1\right)+c\left(\frac{1}{\mathcal{M}(Sx_{2n+1},Bx_{2n+1},Bx_{2n+1},t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(Au,Sx_{2n+1},Sx_{2n+1},t)}-1\right),\left(\frac{1}{\mathcal{M}(Bx_{2n+1},Tu,Sx_{2n+1},t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(Au,Sx_{2n+1},Bx_{2n+1},t)}-1\right)$$

As $n\to\infty$, we have

$$a\left(\frac{1}{\mathcal{M}(u,u,Au,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(u,Au,u,t)}-1\right)+c\left(\frac{1}{\mathcal{M}(u,u,u,t)}-1\right)$$

$$+\min\left\{\left(\frac{1}{\mathcal{M}(Au,u,u,t)}-1\right),\left(\frac{1}{\mathcal{M}(u,u,u,t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(Au,u,u,t)}-1\right)$$

$$\Rightarrow\qquad (a+b)\left\{\frac{1}{\mathcal{M}(u,u,Au,t)}-1\right\}\geq q\left(\frac{1}{\mathcal{M}(Au,u,u,t)}-1\right)$$

$\Rightarrow\qquad (a+b)\geq q$, which is a contradiction

Therefore Au = u, Similarly, using R-weakly commutatively of pair {B, S} and from (3.3),

we can easily get Su = u and Bu = u. Thus u is a common fixed point of A, B, S and T.

Uniqueness: Let u and v be two fixed points. Putting x = u and y = v and z = v in (3.3), we have

$$a\left(\frac{1}{\mathcal{M}(Tu,Sv,Au,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tu,Au,Sv,t)}-1\right)+c\left(\frac{1}{\mathcal{M}(Sv,Bv,Bv,t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(Au,Sv,Sv,t)}-1\right),\left(\frac{1}{\mathcal{M}(Bv,Tu,Sv,t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(Au,Sv,Bv,t)}-1\right)$$

$$\Rightarrow a\left(\frac{1}{\mathcal{M}(u,v,u,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(u,u,v,t)}-1\right)+c\left(\frac{1}{\mathcal{M}(v,v,v,t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(u,v,v,t)}-1\right),\left(\frac{1}{\mathcal{M}(v,u,v,t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(u,v,v,t)}-1\right)$$

$$\Rightarrow \quad (a+b+1)\left(\frac{1}{\mathcal{M}(u,v,u,t)}-1\right)\geq q\left(\frac{1}{\mathcal{M}(u,v,v,t)}-1\right)$$

$$\Rightarrow \quad (a+b+1)\geq q.$$

$\Rightarrow$ $(a + b) \geq q - 1$, which is a contradiction .

Hence A, B, S and T have a unique common fixed point.

Taking A = B, we get the following corollary.

Corollary 3.2: Let S and T be two continuous self-mappings of a complete $\mathcal{M}$-fuzzy metric space (X, $\mathcal{M}$,*) , when * is a continuous t-norm. Let A be a self mapping on X satisfying:

(3.6) $A(X) \subseteq S(X) \cap T(X)$;

(3.7) {A,T} and{B,S} are R-weakly commuting pairs ;

(3.8) $$a\left(\frac{1}{\mathcal{M}(Tx,Sy,Ax,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tx,Ax,Sz,t)}-1\right)+c\left(\frac{1}{\mathcal{M}(Sy,Az,Ay,t)}-1\right)+$$

$$\min\left\{\left(\frac{1}{\mathcal{M}(Ax,Sz,Sy,t)}-1\right),\left(\frac{1}{\mathcal{M}(Ay,Tx,Sy,t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(Ax,Sz,Ay,t)}-1\right)$$

for all x, y, z$\in$X , where a, b, c $\in$[0,1], q >2 with q > a + b + c. Then, A, S and T have a unique common fixed point.

Taking T = S we can get another corollary of Theorem 3.1 as mentioned below.

Corollary 3.3: Let T be continuous self-mappings of a complete $\mathcal{M}$-fuzzy metric space (X, $\mathcal{M}$,*) , when * is a continuous t-norm. Let A and B be two self mapping of X satisfying:

(3.9) $A(X) \subseteq T(X)$ and $B(X) \subseteq T(X)$

(3.10) {A, T} and{B, T} are R-weakly commuting pairs

(3.11) $a\left(\frac{1}{\mathcal{M}(Tx,Ty,Ax,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tx,Ax,Tz,t)}-1\right)+c\left(\frac{1}{\mathcal{M}(Ty,Az,Ay,t)}-1\right)+$

$$\min\left\{\left(\frac{1}{\mathcal{M}(Ax,Tz,Ty,t)}-1\right),\left(\frac{1}{\mathcal{M}(Ay,Tx,Ty,t)}-1\right)\right\}\geq q\left(\frac{1}{\mathcal{M}(Ax,Tz,Ay,t)}-1\right)$$

for all x, y, z∈X , where a, b, c ∈[0,1], q > 2 with q > a + b + c. Then, A, B and S have a unique common fixed point.

Remark 3.4: The conclusions of Theorem 3.1 remain true if we replace condition (3.3) by any of the following conditions:

(3.12) $a\left(\frac{1}{\mathcal{M}(Tx,Sy,Ax,t)}-1\right)+b\left(\frac{1}{\mathcal{M}(Tx,Ax,Sz,t)}-1\right)+c\left(\frac{1}{\mathcal{M}(Sy,Bz,By,t)}-1\right)$

$\geq q\left(\frac{1}{\mathcal{M}(Ax,Sz,By,t)}-1\right)$ for all x, y, z∈X , where a, b, c ∈[0,1], q >2 with q > a + b + c;

(3.13) $a\mathcal{M}(Tx,Ty,Ax,t)+b\mathcal{M}(Tx,Ax,Tz,t)+c\mathcal{M}(Ty,Az,Ay,t)+$

$$\min\{\mathcal{M}(Ax,Tz,Ty,t),\mathcal{M}(Ay,Tx,Ty,t)\}\leq q\mathcal{M}(Ax,Tz,Ay,t)$$

for all x, y, z∈X , where a, b, c ∈[0,1], q > 0 with q < a + b + c + 1.

Remark 3.5: From Theorem 3.1 and Remark 3.4, it is clear that above results generalizing the earlier results of Som [14] in fuzzy metric spaces.

Example 3.3: Let $X=[0,\infty)$ and D* is the standard D* -metric on X. Denote a $*$b = a.b for all a, b ∈ [0, 1]. For each t ∈ (0,∞), define $\mathcal{M}$ (x, y, z, t) $=\frac{t}{t+D*(x,y,z)}$ for all x, y, z in X. Let A, B, S and T be self maps on X defined as:

Ax = Bx = $\frac{3x}{4}$ and Sx = Tx = 2x for all x in X. Clearly,

(i) S and T be two continuous self-mappings on X;
(ii) X be a complete $\mathcal{M}$-fuzzy metric space (X, $\mathcal{M}$,*) , when * is a continuous t-norm;
(iii) A(X) ⊆ S(X) and B(X) ⊆ T(X) ;
(iv) {A, T} and{B, S} are R-weakly commuting pairs as both pairs commute at coincidence points;
(v) {A, T} and {B, S} satisfies inequality (3.3) for all x, y, z∈X ,
where $a=\frac{1}{4},b=\frac{1}{4},c=\frac{1}{2}$, q > 2 with q > a + b + c.

Hence, all conditions of Theorem 3.1 are satisfied and x = 0 is a unique common fixed point of A, B, S and T.

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