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Fuzzy Mathematical modeling of Distribution Network through Location Allocation Model in a Three-level Supply Chain Design

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Abstract

Economic roles in all areas particularly in the steel industry have been grown dramatically. In this article, a new look to the field of mathematical modeling of distributed systems in terms of fuzzy location model and the theory of fuzzy has been allocated and an integer linear programming is used. The distribution system generally includes three levels so that the first level suppliers of iron ore, mining, steel and so on are placed. The second level involves locating distribution centers consider so that a limited number of distribution centers can serve as stations and the third level of local warehouses or factories in steel production are using the integer programming technique, a fuzzy mathematical model for distributed systems is presented. The second level of distribution center location selection techniques based on Fuzzy Analytical Hierarchy Process (FAHP) is proposed and its output as input in integer programming model is used. It's worth mentioning presented model is analyzed by software of maple 12.

Keywords: Mathematical Modeling, Integer Linear Programming, Distribution System, Fuzzy Analytical Hierarchy Process (FAHP).

1. Introduction

According to the growing variety of industries, including the steel industry, the traditional administration does not meet the required efficiency. The aim of this study is representing novel approach to modern management of all parts of steel industry. Thus, the supply chain management approach in the distributed network used to enable the integrated management of the distribution system. Some of the related factors are listed below:

- Definitive and fuzzy data in supply chain
- Waste in the supply chain
- Lead time in the supply chain network
- Efficiency and effectiveness of the supply chain
- Cost of the supply chain

In this study, the criteria of cost, efficiency, effectiveness, and lead time have been studied in a supply chain network to present the model in section 4. Needless to say, the network layout model has been used as a cover model.

First, Queuing Maximal Covering Location Problem (QMCLP) was presented by Marianov and Revelle in 2000 [1]. Second, Shavandi and Mahlooji in 2004 [2] proposed fuzzy queuing model of allocation problems. Then, fuzzy queuing location models on networks were presented by Perez & Vega [3]. Fuzzy AHP was developed by Zimmermann specifically for fuzzy numbers [4]. In another paper, Marianov and Serra [5] have proposed possible location allocation model with limited waiting time and queue length for the proposed systems. Moghadadi and et al. looked M/G/1 queue allocation model covering location problems. They also used Binary Quadratic Programming [6]. Similarly, Shavandi and Mahlooji proposed allocation hierarchy in 2006 [7]. Syam in 2008 presented allocation model with several service providers to design service systems that showed the breadth of research in this area [8].

Supply chain network design (SCND) decisions, as the most important strategic level decisions in supply chain management, concerned with complex interrelationships between various tiers, such as suppliers, plants, distribution centers and customer zones as well as determining the number, location and capacity of facilities to meet customer needs effectively. Supply chain management integrates interrelationships between various entities through creating alliance, such as, information-system integration and process integration, between entities to improve response to customers in various aspects such as, higher product variety and quality, lower costs and faster responses. One of the vital challenges for organizations in today's competitive markets is the need to respond to customer needs which are very volatile and can be occurred in volume and variety of customer needs Amiry 2011 [9]. Melo et al. (2009) [10] presented a general review on supply chain network design to identify basic features that such models must capture to support decision-making involved in strategic supply chain planning and support a variety of future research directions. Other interesting reviews in this field can be found in Dullaert et al. (2007) [11] and Snyder (2006) [12]. The most of the presented models in the literature focus on minimization of total costs and ignore other objectives such as responsiveness and flexibility which are effective in the success of supply chain management. Zanjirani Farahani et al. (2010) [13] give a comprehensive review on multi-criteria facility location problems in the SCND. Rajab ali pour Cheshmehgaz et al. (2011) [14] presented a multi-objective, multi-stage and flexible model to design logistics network with the aim of minimization response time and cost criteria. Amiri (2006) [15] presented a MILP model to coordinate production and distribution activities. Additionally, the presented model was able to determine the optimum number, location and capacity of facilities which should be opened. Altiparmak et al. (2006) [16] developed a SCND model in a practical case and then proposed a GA by using priority-based encoding to escape from infeasible solutions. Thanh et al. (2008) [17] proposed a dynamic MILP model for the facility location in the SCND. The proposed model includes strategic and tactical decisions. Pan and Nagi (2010) [18] developed a robust optimization approach to deal with demand uncertainty in supply chain network in agile manufacturing. The presented model was able to consider alliance costs, which is one of the important factors of agile supply chain, between opened facilities.

In section 3, Fuzzy AHP has been applied due to the different criteria in the real world. In reality, it enables us to elect distribution centers in different places. Distribution centers (Level II) must be selected according to a set of criteria provided by FAHP method. In section 4, case study of FAHP Model in the third section is considered. In section 5, Structure of model of allocation problem is presented in section 6, optimization mathematical model is presented in section 7, and presented model of section 6 is resolved. In section 8, we have concluded our analysis. And finally in section 9, references are listed.

2. Statement of the problem

In this study the modeling of distributed systems in terms of location models in a three-level supply chain is studied. The first level refers to suppliers of iron and ore. We have considered distribution centers for the second level and warehouses of steel factories for the third level. Although selection of distribution centers is made by FAHP method and output of FAHP model is considered for input of mixed integer programming model.

The assumptions are as follows:

- ✓ The model has been analyzed in three levels.
- ✓ Various costs such as the cost of construction of distribution centers, transportation, storage, and waiting in queue has been considered.
- ✓ Varying requirements are met at the third level (storage facility).
- ✓ Capacity of distribution centers is specified.
- ✓ Numbers of distribution centers are known and will be less than ideal options.
- ✓ Slack variables are used to revise the load centers.

3. Fuzzy Analytic Hierarchy Process

AHP is one of the well-known multi-criteria decision making techniques that was first proposed by Saaty [20]. The classical AHP takes into consideration the definite judgments of decision makers [19]. Although the classical AHP includes the opinions of experts and makes a multiple criteria evaluation, it is not capable of reflecting human's vague thoughts [21]. As the uncertainty of information and the vagueness of human feeling and recognition, it is difficult to provide exact numerical values for the criteria and to make evaluations which exactly convey the feeling and recognition of objects for decision makers. Therefore, most of the selection parameters cannot be given precisely. Thus experts may prefer intermediate judgments rather than certain judgments. So the fuzzy set theory makes the comparison process more flexible and capable to explain experts' preferences [22]. Different methods for the fuzzification of AHP have been proposed in the literature. AHP is firstly fuzzified by Laarhoven and Pedrycz [23] and in this study; fuzzy ratios which were defined by triangular membership functions were compared. Buckley used the comparison ratios based on trapezoidal membership functions [24]. Chang introduces a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons [25]. Kahraman, Ulukan, and Tolga proposed a fuzzy objective and subjective method based on fuzzy AHP [26]. Kulakand Kahraman made a selection among the transportation companies by using fuzzy axiomatic design and fuzzy AHP [27]. They developed fuzzy multi-attribute axiomatic design approach and compared it with fuzzy AHP.

In the following, Chang's extent analysis method is explained. Let X = {X1, X2, ... , X_n} be an object set, and U = {u1, u2, . . ., un} be a goal set. According to the method of extent analysis, each object is taken and extent analysis for each goal is performed, respectively.

Therefore, extent analysis values for each object can be obtained, with the following signs:M_{gi}^1, M_{gi}^2,..., M_{gi}^j where all the M_{gi}^j (i = 1, 2, . . ., n and j = 1, 2, . . ., m) are TFNs.

The steps of extent analysis can be given as in the following:

Step 1: The value of fuzzy synthetic extent with respect to the its object is defined as

$$\tilde{s}_i = \sum_{j=1}^{m} \tilde{M}_{gi}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{gi}^j\right]^{-1} \tag{1}$$

To obtain $\sum_{j=1}^{m} \tilde{M}_{gi}^j$ perform the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} \tilde{M}_{gi}^j = \left[\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum\nolimits_{j=1}^{m} u_j\right]^{-1} \tag{2}$$

And to obtain $\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{gi}^j$ the fuzzy addition operation $\tilde{M}_{gi}^j$ (j = 1, 2, . . ., m) values is performed such as

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{gi}^j = \left(\sum_{i=1}^{n}\sum_{i=1}^{n} m_i , \sum_{i=1}^{n} u_i\right) \tag{3}$$

And then the inverse of the above vector is computed in such as

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{gi}^j\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{l} u_i}\right) \tag{4}$$

Step 2: As $\widetilde{M}_2$ and $\widetilde{M}_2$are two triangular fuzzy numbers, the degree of possibility of $\widetilde{M}_2 \leq \widetilde{M}_1$is defined as

$$V(\widetilde{M}_2 \geq \widetilde{M}_1) = \sup_{y \geq x}\left[\min \mu_{\widetilde{M}_1}(x), \min \mu_{\widetilde{M}_2}(y)\right] \tag{5}$$

And can be equivalently expressed as follows:

$$V(\widetilde{M}_2 \geq \widetilde{M}_1) = \mu(d) = \begin{cases} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_2 \geq u_2 \\ \dfrac{l_2 - u_2}{(m_2 - u_2) - (m_1 - u_1)}, & \text{otherwise,} \end{cases} \tag{6}$$

Where d is the ordinate of the highest intersection point D between $u_{\widetilde{M}_2}$and$u_{\widetilde{M}_2}$as shown in Fig. 1. To compare $\widetilde{M}_2$ and$\widetilde{M}_1$, we need both values of $V(\widetilde{M}_2 \geq \widetilde{M}_2)$ and $V(\widetilde{M}_1 \geq \widetilde{M}_2)$

$$\acute{d}(A_i) = \min V(\tilde{S}_i \geq \tilde{S}_k).$$

Step 3: The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $\widetilde{M}_i$can be defined by $V(\widetilde{M} \geq \widetilde{M}_1, \widetilde{M}_2, \ldots, \widetilde{M}_k) = \min V(\widetilde{M} \geq \widetilde{M}_i)$, Where i = 1, . . ., k.

Assume that $\acute{d}(A_i) = \min V(\tilde{S}_i \geq \tilde{S}_k)$.

For k = 1, 2, . . .; k ≠ i. Then the weight vector is given by $W = \left(\acute{d}(A_1), \acute{d}(A_2), \ldots, \acute{d}(A_n)\right)^T$,

Where A_i(i = 1, 2, . . ., n) are $n$elements.

Step 4: Via normalization, the normalized weight vectors are $W = \left(\acute{d}(A_1), \acute{d}(A_2), \ldots, \acute{d}(A_n)\right)^T$, where W is a non-fuzzy number. Weight vector of risk factors can be obtained by either directly assigning or indirectly using pair-wise comparisons. Here, it is suggested that the decision makers use the linguistic variables in Table1 to evaluate the weight vector risk factors. After comparison is made, it is necessary to check the consistency ratio of the comparison. To do so, the graded mean integration approach is utilized for defuzzifying the matrix. According to the graded mean integration approach, a fuzzy number M= (m_1, m_2, m_3) be transformed into a crisp number by employing the below Eq. (7)

$$P(M) = M = \frac{m1 + 4m2 + m3}{6} \tag{7}$$

After the deffuzification of each value in the matrix, 'consistency ratio' (CR) of the matrix can easily be calculated and checked whether CR is smaller than .10 or not.

4. Case study of FAHP model

For Evaluating four criteria and five distribution center to be included that between five suggestion locations alone must be three selections.

Table 1. Weight of criteria

	Accessibility to roads	Create Cost	Level of General service	Environmental Condition
Accessibility to roads	(1.0,1.0,1.0)	(0.1,0.2,10)	(2.0,3.0,4.0)	(3.0,5.0,6.0)
Create Cost	(3.3,5.0,10)	(1.0,1.0,1.0)	(0.2,0.3,0.4)	(3.0,4.0,5.0)
Level of General service	(0.3,0.3,0.5)	(2.5,3.3,5.0)	(1.0,1.0,1.0)	(4.0,5.0,6.0)
Environmental Condition	(0.2,0.2,0.3)	(0.2,0.3,0.3)	(0.2,0.2,0.3)	(1.0,1.0,1.0)

Table 2. Weight of per criteria / per Alternative

Alternative/ Criteria	Accessibility to roads	Create Cost	Level of General service	Environmental Condition
A	0.61	0.63	0.11	0.72
B	0.37	0.35	0.27	0.30
C	0.45	0.58	0.24	0.36
D	0.13	0.12	0.78	0.12
E	0.97	0.51	0.29	0.42
Weight of Criteria	1.513	2.247	0.758	0.964

$$\begin{pmatrix} 0.61 & 0.63 & 0.11 & 0.72 \\ 0.37 & 0.35 & 0.27 & 0.3 \\ 0.45 & 0.58 & 0.24 & 0.36 \\ 0.13 & 0.12 & 0.78 & 0.12 \\ 0.97 & 0.51 & 0.29 & 0.42 \end{pmatrix} * \begin{matrix} 1.51 \\ 2.24 \\ 0.75 \\ 0.96 \end{matrix} = \begin{matrix} 3.106 \\ 1.833 \\ 2.504 \\ 1.165 \\ 3.227 \end{matrix}$$

The results are illustrated as follows in Tables 1 and 2 which the weight of each distribution center is:

$\mathrm{A} = 3.106$,$\mathrm{B} = 1.833$,$\mathrm{C} = 2.504$,$\mathrm{D} = 1.165$,$\mathrm{E} = 3.227$

Finally, distribution centers A, C and E will be selected.

5. Structure of model

5.1. Index

K: index of supplier (k=1, 2, 3,…, K)

j: index of Distribution Center (j=1,2,3,..., J)

K: index of Local Warehouse (l=1, 2, 3,..., L)

P: index of Type Product and Shipment (p=1, 2, 3,..., P)

5.2. Parameters

G_j: Maximum number of transfer shipment type of p to node j

G_l: Maximum number of transfer shipment type of p to node l

Q_j: Create cost of distribution center j

TC_{pkj}: transportation cost of per shipment type of p from node k to node j

TC_{pjl}: transportation cost of per shipment type of p from node j to node l

R_{pl}: sale price of per shipment type of p in factory warehouse l

S_l: is not used capacity quantity in factory warehouse l

S_j: is not used capacity quantity in factory warehouse j

5.3. Variables

F_j: if distribution center j to be created (1-0)

A_{kj}: Possibility of assigning node k to node j (1-0)

B_{jl}: Possibility of assigning node j to node l (1-0)

Z_{pKj}: number of transfer shipment type of p from supplier k to distribution center j

W_{pjl}: number of transfer shipment type of p from distribution center j to local warehouse l

D_l: Supply quantity in factory warehouse l

6. Optimization Model

$$Max\,\Pr ofit = \sum_p \sum_l \sum_j W_{pjl}.R_{pl}.B_{jl} - \sum_p \sum_k \sum_j Z_{pkj}.(H_{pj} + TC_{pkj}).A_{kj} - \sum_p \sum_l \sum_j W_{pjl}.(H_{pj} + TC_{pjl}).B_{jl} - \sum_j Q_j.F_j$$

$$\sum_p \sum_j B_{jl}.W_{pjl} + S_l = C_l \quad (8) \qquad \sum_p \sum_j A_{kj}.Z_{pkj} + S_j = G_j \quad (9)$$

$$A_{kj} \leq F_j (11) \sum_j F_j = e(10)$$

$$\sum_p \sum_k Z_{pkj} = \sum_p \sum_l W_{pjl} (12) \qquad \sum_p \sum_j W_{pjl} = D_l (13)$$

$$LowerLimit \leq D_l \leq UpperLimit(14)` \qquad A_{kj}, B_{jl}, F_j \in \{0,1\}, Z_{pkj}, W_{pjl} \geq 0(15)$$

7. Optimization model

Solution of optimization model accomplish by maple12 that result placed in table 3 until table 8.

Table 3. (p=1)

Z_{pkj}	j=1	j=2	j=3
K=1	8750	0	0
K=2	0	772500	613100
K=3	0	0	0

Table 4. (p=2)

Z_{pkj}	j=1	j=2	j=3
K=1	0	0	0
K=2	0	0	0
K=3	0	12500	14400

Table 5. (p=1)

W_{pjl}	j=1	j=2	j=3
K=1	0	87500	0
K=2	785000	0	0
K=3	15000	12500	600000

Table 6. (p=2)

W_{pjl}	L=1	L=2	L=3
j=1	0	0	**0**
j=2	0	0	**0**
j=3	0	0	**0**

Table 7.

A_{kj}	j=1	j=2	j=3
K=1	0	1	1
K=2	0	0	0
K=3	0	1	1

Table. 8

B_{jl}	L=1	L=2	L=3

j=1	0	0	0
j=2	0	1	0
j=3	1	1	1

Table. 9

$F_1=0$	$S_1=0$	$X_1=15000$	$D_1=100$	$Z_{optimum}=173125000$
$F_2=1$	$S_2=0$	$X_2=0$	$D_2=200$	
$F_3=1$	$S_3=14400$	$X_3=0$	$D_3=500$	

8. Conclusion

In this study, mathematical modeling of distributed systems has been used in terms of location models in the supply chain. First, the distribution centers have been selected among options proposed by Fuzzy AHP method. Furthermore, the model of the objective function is considered to maximize the total profit (the difference between total revenues and total cost) in a distributed system. The output of model represents optimized number of distribution centers and the centers with assigned priority levels, respectively according to the factors such as waiting time, service rate and input rate of shipment. Need for new approaches in the area of decision-making within the steel industry are essential. In order to cause competitive factors affecting the market, industry decision-makers should be considered. Moreover, the utilization of mathematical models and simulations in distributed systems can be one of these approaches which result in achieving desired goals. Finally, presented model via software Maple 12 has been studied which shows low required time of calculation and performance of model.

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