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Numerical solution of singular Fredholm integro–differential equations of the second kind via Petrov–Galerkin method by using Legendre multiwavelet

S. Akhavan

Department of Mathematics

Faculty of technical and engineering

Islamic Azad University, Khomeinishahr branch

Khomeinishahr, Iran

akhavan@iaukhsh.ac.ir

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Abstract

It is of interest to know whether we can solve a singular Fredholm integro-differential equation of the second kind with an infinite or semi-infinite range of integrate via Petrov–Galerkin method by using Legendre multiwavelet. For this purpose, we directly deal with infinite range of integrate. We introduce some change of variables for mapping infinite interval into a finite interval. After that, we use Petrov–Galerkin method with Legendre multiwavelet basis that yields linear system. Numerical results of our example will demonstrate accuracy and efficiency of the proposed method.

Keywords: : Fredholm integro–differential equations, Petrov–Galerkin method, singularity, Legendre multiwavelets.

1. Introduction

Many mathematical formulation of physical phenomena often contain singular integro–differential equation. These equations arise in the Dirichlet problem, potential problem, radiative equilibrium, elastic contact problems, and many others. The term "singularity" is usually used for any lack of analytically in the problem. The following features make a singular problem:

1. An infinite or semi–infinite range of integrate.
2. A discontinuous derivative in the kernel or driving term like "Green‘s function".
3. An infinite or non–exiting derivative of some finite order like $f(t) = \sqrt{1 - t^2}$.

Singular integral and integro–differential equations can not be analytically solved easily so it is required to obtain the approximate solution. In this work, our problem belongs to first group due to belongs to first group due to the range of integral is $[0,\infty]$. In fact, our problem consist of a singular Fredholm integro–differential equation of the second kind with two boundary conditions. The general form of the equations is:

$$\begin{aligned} &\sum_{i=0}^{2} a_i(t)x^{(i)}(t) - \int_0^\infty k(s,t)x(s)ds = f(t) \quad , \quad 0 \le t \le \infty \\ &x(0) = \alpha \\ &x(\infty) = \beta \end{aligned} \tag{1}$$

where the function $f(t)$, the kernel $k(s,t)$ and $a_i(t)$ for each $i = 0,1,\dots,m$ are known and $x(t)$ the exact solution is an unknown. Some methods ignore this singularity but dealing directly with that is not difficult. For example, we can use a Nystrom method that a quadrature rule constructed for this range (such as Gauss-Laguerre) or we can use an expansion method. For this method, we should choose a set of expansion function defined on the infinite interval. Two choices for interval $[0,\infty]$ are

$$\begin{aligned} &h_n(t) = e^{-\alpha t} L_n(t) \\ &h_n(t) = (1+z)^q T_n(z) \quad , \quad z = (\tfrac{2\alpha}{t+\alpha}) - 1 \end{aligned}$$

where α, q are parameters [3]. An alternative to deal directly with infinite range is mapping onto a finite interval and then solve the finite interval equation. In this paper, this rule is used.

In [1], Alpert constructed a class of like-wavelet basis for $L^2[0,1]$ and applied them for approximating the solution of the Fredholm integral equation of the second kind. The numerical method employed in [1] was the Galerkin method. In [4,5], the wavelet Petrov–Galerkin schemes based on discontinuous orthogonal multiwavelets were described. In this paper we use Alpert‘s multiwavelets by using Petrov–Galerkin method.

But numerical methods includes quadrature, collocation and Galerkin methods for Eq. (1) are used ago that their analysis may be found in [5,6,7,8]. On the other hand, the Petrovâ€“-Galerkin method for Fredholm integral equations has been studied in [2]. We can see from [2] that one of the advantages of the Petrov-â€“Galerkin method is allowing us choose two different spaces for the trial space and the test space while the order of convergence be similar to the Galerkin method. We want to use this approach in process of solving problem.

This paper is organized as follows: at first, we give a brief summary of construction of Legendre multiwavelet. In Section 3, we review The Petrov–Galerkin method and its convergence. To facilitate access to the individual topics, the 2 and 3 chapters are rendered as self-contained as possible. Section 4 exhibits a numerical method for transferring a singular integro–differential equation to a linear system by Petrov–Galerkin method. Section 5 illustrates some numerical examples to show the accuracy and advantages of method presented. Finally Section 6 concludes the paper.

2. Legendre multiwavelet

In this section ,we want to construct one base for $L^2[0,1]$ that is comprised of dilates and translates of a finite set of functions $h_1, h_2, \dots, h_k$. In particular, this base consists of orthonormal system

$$h_{j.m}^n(x) = 2^{m/2} h_j(2^m x - n) \quad , \quad j = 1,\dots,k; m,n \in Z \tag{2}$$

where the functions $h_1, h_2, \dots, h_k$ are piecewise polynomial with the following properties:

1. to vanish outside interval [0,1]
2. being orthogonal to low–order polynomials (have vanishing moments)

$$\int_0^1 h_j(x)x^i dx = 0 \quad , \quad i = 0,1,\dots,k-1 \quad , \quad j = 1,2,\dots,k$$

If we employ the multi–resolution analysis, we will composite S_m^k. At first, suppose that $k \in N$ and $m = 0,1,2,\dots$, we define a space S_m^k of piecewise polynomial functions;

$$S_m^k = \left\{ f : f(x) = \begin{cases} a polynomial\ of\ degree < k & \frac{n}{2^m} \le x \le \frac{n+1}{2^m} \\ 0 & otherwise \end{cases} \right\}$$

where $n = 0,1,\dots,2^m - 1$. You can see that $\dim S_m^k = 2^m k$ and

$$S_0^k \subset S_1^k \subset \cdots \subset S_m^k \subset \cdots$$

By this assumption, the $2^m k$ dimensional space R_m^k can define as to be the orthogonal complement of S_m^k in S_{m+1}^k,

$$S_m^k \oplus R_m^k = S_{m+1}^k \quad , \quad R_m^k \perp S_m^k$$

The following decomposition can immediately be obtained:

$$S_m^k = S_0^k \oplus R_0^k \oplus R_1^k \oplus \cdots \oplus R_{m-1}^k$$

If the functions $h_1, h_2, \dots, h_k : R \to R$ form an orthogonal basis for R_0^k, the k functions $f_1, f_2, \dots, f_k : R \to R$ supported on the interval $[-1,1]$ can be constructed by the following form:

$$f_i(x) = \begin{cases} p_{k-1}(x) & 0 \le x \le 1 \\ (-1)^{i+k-1} p_{k-1}(-x) & -1 \le x \le 0 \end{cases} \quad , \quad i = 1,2,\dots,k$$

where $p_{k-1}(x)$ is a polynomial of degree $k-1$ with indeterminate coefficients. These functions have the following properties:

1. The functions $f_1, f_2, \dots, f_k$ satisfy the following orthogonality and normality conditions:

$$\int_{-1}^1 f_i(x) f_j(x) dx \equiv \langle f_i, f_j \rangle = \delta_{ij} \quad , \quad i,j = 1,\dots,k$$

2. The function f_j has vanishing moments

$$\int_{-1}^1 f_i(x) x^i dx = 0 \quad , \quad i = 0,1,\dots,j+k-2$$

We can now define $h_1, h_2, \dots, h_k$ by the following formula

$$h_i(x) = \sqrt{2} f_i(2x-1) \quad , \quad i = 1,\dots,k$$

and obtain the equality

$$R_0^k = Linearspan\{h_i(x): i = 1, \dots, k\}$$

and, more generally,

$$R_m^k = Linearspan\{h_{j,m}^n : h_{j,m}^n(x) = 2^{m/2} h_j(2^m x - n), j = 1, \dots, k; n = 0, \dots, 2^m - 1\}$$

Now, for each positive integer k, we let S_0^k the trial space, be the space of polynomials of degree less than k on the interval [0,1] and them vanish elsewhere. In this case, we suppose

$$S_0^k = Linearspan\{L_0(x), L_1(x), \dots, L_{k-1}(x)\}$$

where $L_i(x)$ are orthonormal Legendre polynomials.

For making R_0^k the test space, we have to derive all $f_i(x)$ for each k. For example, suppose $k = 3$ then

$$f_1(x) = \begin{cases} ax^2 + bx + c & 0 \le x \le 1 \\ -ax^2 + bx - c & -1 \le x \le 0 \\ 0 & otherwise \end{cases}$$

$$f_2(x) = \begin{cases} dx^2 + ex + f & 0 \le x \le 1 \\ dx^2 - ex + f & -1 \le x \le 0 \\ 0 & otherwise \end{cases}$$

$$f_3(x) = \begin{cases} gx^2 + hx + i & 0 \le x \le 1 \\ -gx^2 + hx - i & -1 \le x \le 0 \\ 0 & otherwise \end{cases}$$

Under two above properties, we can make a linear system such that all unknown coefficients derive from that. Although this system do not have unique solution, you can uniquely see all $f_i(x)$ for each k in [1]. Then $h_1(x), h_2(x), h_3(x)$ will derive. After that, we can form a basis for R_m^k with each m, k.

3. The Petrov–Galerkin method and its convergence

In this section, we present a brief review of the Petrov-Galerkin method and conditions of its convergence. We follow the notations of [1]. If X is a Banach space with the norm $\|.\|$ and X^* is its dual space, then two different sequences of finite dimensional subspaces $X_n \subseteq \mathbf{X}$ and $Y_n \subseteq X^*$ can be chosen such that satisfying the condition (H):

(H) : For each $x \in \mathbf{X}$ and $y \in X^*$, there exist $x_n \in X_n$ and $y_n \in Y_n$ such that

- $\|x_n - x\| \to 0$ and $\|y_n - y\| \to 0$ as $n \to \infty$
- $\dim X_n = \dim Y_n \quad n = 1,2, \dots$

In Petrov-Galerkin method, that is a numerical method, we seek $x_n \in X_n$ so as each $y_n \in Y_n$ be orthogonal on both sides of Eq. (1).

$$\left\langle (\sum_{i=0}^{m} a_i(t)D^{(i)} - K)x_n, y_n \right\rangle = \langle f, y_n \rangle \qquad forall \quad y_n \in Y_n \tag{3}$$

On the other hand, for $x \in X$, an element $p_n x \in X$ is called a generalized best approximation from X_n to x with respect to Y_n if it satisfies the equation

$$\langle x - p_n x, y_n \rangle = 0 \qquad forall \quad y_n \in Y_n \tag{4}$$

Thereupon, the Petrov-Galerkin method is a projection method with a generalized best approximation projection. For existence and uniqueness of the generalized best approximation, the following proposition exists:

For each $x \in X$, the generalized best approximation from X_n to x with respect to Y_n exists uniquely if and only if

$$Y_n \cap X_n^{\perp} = \{0\} \tag{5}$$

where $X_n^{\perp}$ denotes the annihilator of X_n in X^* that is the set of all functions satisfying a given set of conditions which is zero on every member of a given set and say that $X_n \perp Y_n$ if $Y_n \cap X_n^{\perp} \neq \{0\}$. By this condition p_n is a projection.

For the proof we refer the reader to [1].

But this condition is not sufficient for insurance every $x \in X$ has a unique Petrov–Galerkin approximation. Therefore, we have to introduce a new concept the regular pair. If there exists a linear operator $\Pi_n: X_n \to Y_n$ with $\Pi_n X_n = Y_n$ such that satisfying the condition

$$(H-1) \qquad \|x_n\| \leq c_1 \langle x_n, \Pi_n x_n \rangle^{1/2} \qquad forall \quad x_n \in X_n$$
$$(H-2) \qquad \|\Pi_n x_n\| \leq c_2 \|x_n\| \qquad forall \quad x_n \in X_n$$

where c_1 and c_2 are positive constants independent of n. The $\{X_n, Y_n\}$ is called a regular pair.

On the other hand, if X_n and Y_n satisfy the condition (H) and $\{X_n, Y_n\}$ be a regular pair, we have the following statements:

1. $\|P_n x - x\| \to 0 \quad$ as $\quad n \to \infty, \quad$ forall $\quad x \in X$.
2. $\| P_n x - x \| \leq C \| Q_n x - x \|$ for some constant $C > 0$ independent of n.

It means, for ensuring existence and uniqueness of approximation solution for every $x \in X$, we have to consider the condition (H), and the conditions $(H-1)$, $(H-2)$ for each construction separately.

If we choose S_m^k and $S_{m'}^{k'}$ such that $\dim S_m^k = \dim S_{m'}^{k'}$, the condition (H) will satisfy and by assumption linear operation $\Pi_n: S_m^k \to S_{m'}^{k'}$ as follow:

$$\Pi_n(x_n(t)) = \Pi_n \left(\sum_{j=1}^{2^m k} c_j b_j(t) \right) = \sum_{j=1}^{2^{m'} k'} \left(c_j d_j(t) \right) \tag{6}$$

where

$$S_m^k = \text{Linearspan}\{b_1(x), b_2(x), \dots, b_{2^m k}(x)\}$$
$$S_{m'}^{k'} = \text{Linearspan}\{d_1(x), d_2(x), \dots, d_{2^{m'} k'}(x)\} \quad (7)$$

the conditions $(H-1)$, $(H-2)$ will prove in two subsections.

3.1. Convergence of Legendre multiwavelet

The conditions $(H-1)$, $(H-2)$ must separately prove for each basis containing a regular pair.

By definition $\Pi_n X_n = Y_n$ and the norm $\|.\|$, we have

$$\langle x_n, \Pi_n x_n \rangle = \int_0^1 x_n(t)\Pi_n(x_n(t))dt$$
$$= \int_0^1 \left(\sum_{j=1}^{2^m k} c_j b_j(t)\right)\left(\sum_{j=1}^{2^{m'} k'} \left(c_j d_j(t)\right)\right) dt$$

By assumption $2^m k = \dim S_m^k = \dim S_{m'}^{k'} = 2^{m'} k'$, we can write

$$= \int_0^1 \left(\sum_{j=1}^{2^m k} c_j b_j(t)\right)\left(\sum_{j=1}^{2^m k} \left(c_j d_j(t)\right)\right) dt$$

If we rewrite this relation in matrices form, we will have

$$= \int_0^1 C^T \Phi(t) \Psi^T(t) C dt$$

where

$$\Phi(t) = [b_1(x), b_2(x), \dots, b_{2^m k}(x)]^T$$
$$\Psi(t) = \left[d_1(x), d_2(x), \dots, d_{2^{m'} k'}(x)\right]^T$$
$$C = [c_1, c_2, \dots, c_{2^m k}]^T$$

By definition $[B]_{i,j} = \int_0^1 b_i(t) d_j(t) dt$, we have

$$= C^T B C$$

where matrices B are diagonal with N positive integer as its diagonal entries that they are eigenvalues of B. Therefore

$$= c_1^2 b_{11} + c_2^2 b_{22} + \dots + c_N^2 b_{NN}$$

We can realize

$$\geq \lambda_{(\min B)}(c_1^2 + c_2^2 + \dots + c_N^2) = \lambda_{(\min B)} \|x_n\|$$

With choosing $c_1 = \frac{1}{\sqrt{\lambda_{(\min B)}}}$, this relation can be rewrite as follow

$$\|x_n\| \leq \frac{1}{\sqrt{\lambda_{(\min B)}}} \langle x_n, \Pi_n x_n \rangle^{1/2}$$

Clearly, we have

$$\|\Pi_n x_n\|_2^2 = \int_0^1 \left(\sum_{j=1}^{2^{m'}k'} \left(c_j d_j(t)\right)\right)^2 dt$$

By assumption $2^m k = \dim S_m^k = \dim S_{m'}^{k'} = 2^{m'}k'$, we can write

$$= \int_0^1 \left(\sum_{j=1}^{2^m k} \left(c_j d_j(t)\right)\right)^2 dt$$
$$= \int_0^1 \left(C^T\Psi(t)\right)^2 dt = \int_0^1 \left(C^T\Psi(t)\Psi^T(t)C\right)dt$$
$$= C^T\left\{\int_0^1 \left(\Psi(t)\Psi^T(t)\right)dt\right\}C$$

By orthonormality of basis

$$= C^T I C = \|x_n\|_2^2$$

This relation shows that the choice of an integer for $c_2 \geq 1$ yields $(H-2)$ condition.

4. Numerical method

In this section, we perform procedure with an algorithm consisting of three stages: the first stage "A" mapping the range of integrate $[0,\infty]$ to a finite interval $[0,R]$. The second stage "B" finds the infinite number of boundary condition $x(\infty)$, and the third stage "C" solves the Fredholm integro-differential equation of the second kind with singularity via Petrovâ€“Galerkin method by Legendre multiwavelet basis. The details of the new algorithm is as follows:

At first, to map the range of integrate $[0,\infty]$ into a finite interval $[0,R]$, we can introduce the change of variables;

$$\begin{cases} \hat{s} = \frac{R}{s+1} & ds = \frac{-R}{\hat{s}^2}d\hat{s} \\ \hat{t} = \frac{R}{t+1} & dt = \frac{-R}{\hat{t}^2}d\hat{s} \end{cases} \tag{8}$$

We find out ((1)) takes the form:

$$\begin{aligned} &\sum_{i=0}^{2} \hat{a}_i(\hat{t})\hat{x}^{(i)}(\hat{t}) \ - \int_0^R \frac{\hat{k}(\hat{s},\hat{t})\hat{x}(\hat{s})(-R)}{\hat{s}^2}d\hat{s} = \hat{f}(\hat{t}) \ \ , \ \ 0 \leq \hat{t} \leq R \\ &\hat{x}(R) = \alpha \\ &\hat{x}(0) = \beta \end{aligned} \tag{9}$$

We can now suppose $R = 1$ due to the range of Legendre multiwavelet. But other choices are $R \geq 2$ which make the change of variables, whereas the ranges of integrates are $[0,1]$. In fact, we truncate the interval $[0,R]$ to $[0,1]$. It is possible that $R > 2$ but we will not develop this point here.

We know $\hat{x}_n \in X_n$ and S_m^k forms a basis for the trial space X_n. Further, let $\hat{x}_n(\hat{t})$ be an approximation of exact solution $\hat{x}(\hat{t})$. We can write

$$\hat{x}_n(\hat{t}) = \sum_{i=1}^{N} c_i b_i(t)$$

If we substitute $\hat{x}_n(\hat{t})$ instead of $\hat{x}(\hat{t})$ in ((1)), we derive

$$\sum_{j=0}^{2} a_j(t)\sum_{q=1}^{N} c_q {b_q}^{(j)}(t) - \int_0^1 k(s,t)\left[\sum_{q=1}^{N} c_q b_q(s)\right]ds = f(t) \quad , \quad 0 \le t \le 1 \tag{10}$$

or

$$\sum_{j=0}^{2} a_j(t)C^T\Phi^{(j)}(t) - \int_0^1 k(s,t)C^T\Phi(s)ds = f(t)$$

This relation can be simplified as follow:

$$C^T WG - C^T K = f(t) \tag{11}$$

where

$$W = \begin{bmatrix} b_1(t) & {b_1}^{(1)}(t) & \cdots & {b_1}^{(m)}(t) \\ b_2(t) & {b_2}^{(1)}(t) & \cdots & {b_2}^{(m)}(t) \\ \vdots & \vdots & \vdots & \vdots \\ b_N(t) & {b_N}^{(1)}(t) & \cdots & {b_N}^{(m)}(t) \end{bmatrix}, G = \begin{bmatrix} a_0(t) \\ a_1(t) \\ \vdots \\ a_m(t) \end{bmatrix}$$

$$K = \begin{bmatrix} \int_0^1 k(s,t)b_1(s)\ ds \\ \int_0^1 k(s,t)b_2(s)\ ds \\ \vdots \\ \int_0^1 k(s,t)b_N(s)\ ds \end{bmatrix}$$

We now inner multiply both side in each element of Y_n basis, where $S_{m'}^{k'}$ forms a basis for Y_n (where $2^m k = \dim S_m^k = \dim S_{m'}^{k'} = 2^{m'} k'$ and $k \ge \begin{cases} m+1 & m = 2k_1 - 1 \\ m+2 & m = 2k_1 \end{cases}$.

$$C^T \int_0^1 WG\Psi^T(t)dt - C^T \int_0^1 K\Psi^T(t)dt = \int_0^1 f(t)\Psi(t)dt \tag{12}$$

where $\Psi(t) = (d_1(t), d_2(t), \ldots, d_N(t))^T$. The system (12) have the following matrix form

$$C^T[R - M] = F \tag{13}$$

or

$$[R - M]^T C = F \tag{14}$$

where

$$[R]_{i,j} = \int_0^1 [WG]_i d_j(t)dt \quad ,$$

$$[M]_{i,j} = \int_0^1 [K]_i d_j(t)dt = \int_0^1 \int_0^1 k(s,t)b_i(s)d_j(t)dsdt$$

In the ((14)) system, we could use two exact equations instead of some two row of approximation equations. These two additional equations derive from boundary conditions.

$$\begin{cases} \sum_{i=1}^{N} b_i(0) &= \alpha \\ \sum_{i=1}^{N} b_i(1) &= \beta \end{cases} \tag{15}$$

Solution of new system will derive the approximation solution.

5. Numerical results

In the following examples, we use Legendre multiwavelet basis for Petrov–Galerkin method with different values of k, n, $R = 1$. The computations associated with the examples were performed using Mathematica 8 software on a personal computer.

Example 5.1

$$\begin{gathered} (1+t)^2 x''(t) - (1+t)x'(t) + x(t) - \int_0^\infty \left(ts^2 - t\right) e^{-3s} x(s)\, \mathrm{d}s = \frac{4}{1+t} + \frac{2}{9}t \\ 0 \le t \le \infty \\ x(0) = 1 \\ x(\infty) = 0 \end{gathered} \tag{16}$$

with exact solution $x(t) = \frac{1}{1+t}$. After substituting the change of variables ((8)) takes the following form:

$$\frac{\hat{t}^4}{R^2\left(\frac{R}{\hat{t}}\right)^2 \widehat{x}''(\hat{t})} - \left(\frac{-\hat{t}^2}{R\left(\frac{R}{\hat{t}}\right)} + \frac{-2\hat{t}}{R\left(\frac{R}{\hat{t}}\right)^2}\right) \widehat{x}'(\hat{t}) + \hat{x}(\hat{t}) +$$
$$\int_0^R \frac{R}{s^2\left(\left(\frac{R-\hat{t}}{\hat{t}}\right)\left(\frac{R-\hat{s}}{\hat{s}}\right)^2 - \frac{R-\hat{t}}{\hat{t}}\right) e^{-3\left(\frac{R-\hat{s}}{\hat{s}}\right)} \hat{x}(\hat{s})} \mathrm{d}\hat{s} = \frac{4\hat{t}}{R} + \frac{2}{9\left(\frac{R-\hat{t}}{\hat{t}}\right)} \quad 0 \le \hat{t} \le R$$
$$\hat{x}(R) = 1$$
$$\hat{x}(0) = 0$$

with exact solution $\hat{x}(\hat{t}) = \frac{\hat{t}}{R}$. In Tables 1 the value of $\left\|\hat{x}_n(t_j) - \hat{x}(t_j)\right\|_\infty$ are computed where $t_j = \frac{j}{10}, 0 \le j \le 10$ with $R = 1$.

Example 5.2

$$x''(t) - 2x'(t) - 8x(t) - \int_0^\infty (ts^2 + t)x(s)ds = -\frac{3}{4}t \ , \ 0 \le t \le \infty \tag{17}$$

with exact solution $x(t) = e^{-2t}$. After substituting the change of variables ((8)) takes the following form

$$\frac{\hat{t}^4}{R^2}\hat{x}''(\hat{t}) + \left(-\frac{2\hat{t}}{R} + \frac{2\hat{t}^2}{R}\right)\hat{x}'(\hat{t}) - 8\hat{x}(\hat{t}) + \int_0^R \frac{R}{\hat{s}^2\left(\left(\frac{R-\hat{t}}{\hat{t}}\right)\left(\frac{R-\hat{s}}{\hat{s}}\right)^2 + \frac{R-\hat{t}}{\hat{t}}\right)\hat{x}(\hat{s})} d\hat{s}$$

$$= -\frac{3}{4\left(\frac{R-\hat{t}}{\hat{t}}\right)} \qquad 0 \le \hat{t} \le R$$

$$\hat{x}(R) = 1$$
$$\hat{x}(0) = 0$$

with exact solution $\hat{x}(\hat{t}) = e^{-2\left(\frac{R-\hat{t}}{\hat{t}}\right)}$. In Table 2 the values of $\left\|\hat{x}_n(t_j) - \hat{x}(t_j)\right\|_\infty$ are computed where $t_j = \frac{j}{10}, 1 \le j \le 10$ with $R = 1$.

Table 1. results of exp 1 with $R = 1$

X_n, Y_n	$\left\|\hat{x}_n(t_j) - \hat{x}(t_j)\right\|_\infty$
S_0^4, S_1^2	$6.56710*10^{-4}$
S_0^6, S_1^3	$7.12546*10^{-4}$
S_0^8, S_1^4	$7.19971*10^{-4}$
S_1^4, S_2^2	$2.850981*10^{-4}$

Table 2. results of exp 2 with $R = 1$

X_n, Y_n	$\left\|\hat{x}_n(t_j) - \hat{x}(t_j)\right\|_\infty$
S_0^4, S_1^2	$9.27298*10^{-2}$
S_0^6, S_1^3	$2.30143*10^{-2}$
S_0^8, S_1^4	$3.19595*10^{-3}$
S_1^4, S_2^2	$1.27511*10^{-1}$

6. Conclusion

In this paper, we solve the singular Fredholm integro–differential equations of the second kind which the interval of integrate is an infinite interval. Two change of variables are used to map $[0, \infty]$ into $[0,1]$. The choice of $[0,1]$ lies in the fact that support of Legendre multiwavelets is $[0,1]$. We use Petrov–Galerkin approach by using Legendre multiwavelet basis for discretization technique. Its accuracy and applicability were checked on some examples.

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