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λ-FUZZY FIXED POINTS IN FUZZY MRTRIC SPACES

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Abstract

In this paper fuzzy metric space and λ-fuzzy fixed points for maps in fuzzy metric spaces are redefined. Instead of fuzzy numbers or real numbers are used to define fuzzy metric.

Key words and Phrases : Fuzzy metric space, λ-Fuzzy fixed point, Fuzzy Cauchy sequence, fuzzy Complete metric space.

1 Introduction

The concept of fuzzy sets was introduced by Zadeh [11]. It was developed extensively by many authors and used in various fields. To use this concept in topology and analysis, several researchers have defined fuzzy metric spaces in various ways. Many authors have studied fixed point theory in fuzzy metric spaces for fuzzy mappings (see [1-10]).

In this paper, we give an implicit relation on fuzzy metric spaces and present some fixed point theorems. This theorems are generalization of some previous fixed point theorems given by some authors. Now, we begin with some defnitions, and we consider λ-fuzzy fixed point in fuzzy metric spaces.

2 FUZZY METRIC SPACE

For the convenience of reading, some basic concepts of fuzzy points and denotations are presented below. Fuzzy point are the fuzzy sets being of the following from in the sense of Pu [6],

$$x_\lambda(y) = \begin{cases} \lambda & y = x \\ 0 & y \neq x \end{cases}$$

where X is a nonempty set and $\lambda \in [0,1]$ In this paper, fuzzy points are usually denoted by (x,λ) and the set of all the fuzzy points on X is denoted by $P_F(X)$. Particularly, when $X = R$, is denoted by $S_F(R)$.

Definition 2.1 . *Suppose (x,λ) and (y,γ) are two fuzzy points. A series of definitions contains the following onse;*

(1) We say $(x,\lambda) \pm (y,\gamma)$ if $x > y$ or $(x,\lambda) = (y,\gamma)$.

(2) (x,λ) is said to be no less than (y,γ) if $x \geq y$, denoted by $(x,\lambda) \succ (y,\gamma)$ (3) (x,λ) is said to be nonnegative if $x \geq 0$.

The set of all the nonnegative fuzzy scalars denoted by $S_F^+(R)$.

We now present the definition of ordinary fuzzy metric spaces except that $\leq$ is replaced by $\prec$ in the triangle inequality. This is because that there exist no reasonable complete order in $S_F^+(R)$.

Definition 2.2. *Suppose X is a nonempty set and*

$$\rho_F : P_F(X) \times P_F(X) \to S_F^+(R)$$

is a mapping. $(P_F(X), \rho_F)$ is said to be a ordinary fuzzy metric space if for any (x,λ), (y,γ), (z,ρ) in $P_F(X)$, ρ_F satisfies the following three conditions,

(1) $\rho_F((x,\lambda),(y,\gamma)) = 0$ iff $x = y$ and $\lambda = \gamma = 1$, therefore $(x,\lambda) = (y,\gamma)$;

(2) $\rho_F((x,\lambda),(y,\gamma)) = \rho_F((y,\gamma),(x,\lambda))$;

(3) $\rho_F((x,\lambda),(z,\rho)) \prec \rho_F((x,\lambda),(y,\gamma)) + \rho_F((y,\gamma),(z,\rho))$. ρ_F is called a fuzzy metric defined in $P_F(X)$ and $\rho_F((x,\lambda),(y,\gamma))$ is called a fuzzy distance between the two fuzzy points

Example 2.1 *Suppose $(P_F(X), \rho_F)$ is an ordinary fuzzy metric space. The distance of any two fuzzy points (x,λ) , (y,γ) in $P_F(X)$ is defined by*

$$\rho_F((x,\lambda),(y,\gamma)) = (d((x,y), min\{\lambda,\gamma\}),$$

where $d(x, y)$ is the distance between x and y defined in metric space (X, d) . Then $(P_F(X), \rho_F)$ is a ordinary fuzzy metric space.

Proof. It suffices to prove that ρ_F satisfies the three conditions in Definition 2.2

(1). Suppose (x, λ) and (y, γ) are two fuzzy points in $P_F(X)$. Since $\rho_F(x, y)$ is a distance between x and y , one has $\rho_F(x, y) \geq 0$.

It follows that $\rho_F((x, \lambda), (y, \gamma)) = (d(x, y), min\{\lambda, \gamma\}) = 0$ which is equal to that $x = y$ and $\lambda = \gamma = 1$.

(2). For any $\{(x, \lambda), (y, \gamma)\} \subset P_F(X)$, one has

$$\rho_F((x, \lambda), (y, \gamma)) = (d(x, y), min\{\lambda, \gamma\})$$

$$= (d(y, x), min\{\gamma, \lambda\})$$

$$= \rho_F((y, \gamma), (x, \lambda)).$$

(3). For any $\{(x, \lambda), (y, \gamma), (z, \rho)\} \subset P_F(X)$,

$$\rho_F((x, \lambda), (z, \rho)) = (d(x, z), min\{\lambda, \rho\})$$

$$\prec (d(x, y) + d(y, z), min\{\lambda, \rho, \gamma\})$$

$$= (d(x, y), min\{\lambda, \gamma\}) + (d(y, z), min\{\gamma, \rho\}).$$

$$= d((x, \lambda), (y, \gamma)) + d((y, \gamma), (z, \rho)).$$

The example given above show that a ordinary fuzzy metric space can be constructed by a metric space in the usual sense, called an induced metric space of it and the metric of the space is an induced metric of the original one.

Definition 2.3. *A λ -fuzzy set A can be regarded as a subset of fuzzy points in $P_F(X)$, i.e.,*

$$\{(x, \lambda) : A(x) \geq \lambda\}$$

The set of all the $\lambda -$ fuzzy sets on X is denoted by $P_F^\lambda(X)$;

$$P_F^\lambda(X) = \{(B, \lambda) : B \; is \; \lambda - \; fuzzy \; set\}.$$

Definition 2.4 . *Let $\{(x_n, \lambda_n)\}$ be a sequence of fuzzy scalar. We say that the sequence $\{(x_n, \lambda_n)\}$ is convergent to (x, λ) , $\lambda \neq 0$, and write $\lim_{n\to\infty}(x_n, \lambda_n) = (x, \lambda)$ if $\lim_{n\to\infty} x_n = x$ and*

$\{\lambda_i \mid \lambda_i < \lambda, i \in N\}$ *is a finite set and there exits a convergence subsequence of* λ_i *, denoted by* λ_{i_k} *, such that* $\lim_{k\to\infty} \lambda_{i_k} = \lambda$.

Definition 2.5. *Let X be a fuzzy metric space,* $A, B \in P_F^{\lambda}(X)$ *and* $T:(A,\lambda)\bigcup(B,\lambda) \to (A,\lambda)\bigcup(B,\lambda)$ *be a map such that* $T((A,\lambda)) \subseteq (B,\lambda), T((B,\lambda)) \subseteq (A,\lambda)$. *The point* $(x,\lambda) \in (A,\lambda)\bigcup(B,\lambda)$ *is fuzzy fixed point of T, if* $T(x,\lambda) = (x,\lambda)$.

Definition 2.6. *A sequence of fuzzy points* $(x_n, \lambda_n) \in (P_F(X), \rho_f)$ *is said to be a fuzzy Cauchy sequence if there exists a* $\lambda \in (0,1]$ *such that*

$$\lim_{n\to\infty} \rho_F((x_{m+n}, \lambda_{m+n}), (x_n, \lambda_n)) = 0 \quad \forall m \in N.$$

Definition 2.7. *An induced fuzzy metric space is said to be complete if any fuzzy Cauchy sequence in it has a unique limit in the space.*

3. $\lambda-$Fuzzy fixed point

In this section we present $\lambda-$fuzzy fixed point theorems;

Theorem 3.1. *Let* X *be a fuzzy complete metric space,* $A, B \in P_F^{\lambda}(X)$ *and* $T:(A,\lambda)\bigcup(B,\lambda) \to (A,\lambda)\bigcup(B,\lambda)$ *be a map. There is a continuous map* $\varphi : S_F^{+}(R) \to [0;1)$, $\varphi(\rho_F((x,\lambda);(y,\lambda))) < b$, $\varphi(0) = 0$, $b < 1$, $\alpha + 2\beta < 1$, $(x,\lambda),(y,\lambda) \in P_F(X)$ *and one of the following statement is satisfy;*

(i) $$\rho_F(T(x,\lambda),T(y,\lambda)) \prec \phi(\rho_F(x,\lambda),(y,\lambda))(\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta(\rho_F((x,\lambda),T(x,\lambda)) + \rho_F((y,\lambda),T(y,\lambda))),$$

(ii) $\rho_F(T(x,\lambda),T(y,\lambda)) \prec \varphi(\rho_F(x,\lambda),(y,\lambda))(\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta\rho_F(T(x,\lambda),(y,\lambda))),$

(iii) $\rho_F(T(x,\lambda),T(y,\lambda)) \prec \varphi(\rho_F(x,\lambda),(y,\lambda))\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta\rho_F(T(x,\lambda),(y,\lambda)).$

then T has an unique fuzzy fixed point in $(A,\lambda)\bigcup(B,\lambda)$.

Proof. Choose $(x,\lambda) \in P_F(X)$, suppose (i) is true, then

$$\rho_F(T(x,\lambda),T^2(x,\lambda) \prec \varphi(\rho_F((x,\lambda),T(x,\lambda))(\alpha\rho_F((x,\lambda),T(x,\lambda))$$

$$+\beta(\rho_F((x,\lambda),T(x,\lambda)) + \rho_F(T(x,\lambda),T^2(x,\lambda)))$$

$$\prec b(\alpha\rho_F((x,\lambda),T(x,\lambda))$$

$$+\beta(\rho_F((x,\lambda),T(x,\lambda))+\rho_F(T(x,\lambda)),T^2(x,\lambda)))$$

Therefore

$$\rho_F(T(x,\lambda),T^2(x,\lambda)) \prec \frac{\alpha b+\beta b}{1-\beta b}\rho_F((x,\lambda),T(x,\lambda))$$

Now if $k=\frac{\alpha b+\beta b}{1-\beta b}$, then $k \prec 1$ and

$$\rho_F(T(x,\lambda),T^2(x,\lambda) \prec k(\rho_F((x,\lambda),T(x,\lambda))$$

Also $\rho_F(T^2(x,\lambda),T^3(x,\lambda) \prec k^2(\rho_F((x,\lambda),T(x,\lambda))$, therefore $\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda)) \prec k^n(\rho_F((x,\lambda),T(x,\lambda))$, for all $n \geq 1$.

Suppose (ii) is true then

$$\rho_F(T(x,\lambda),T^2(x,\lambda)) \prec \varphi(\rho_F((x,\lambda),T(x,\lambda))(\alpha\rho_F((x,\lambda),T(x,\lambda))$$

$$+\beta\rho_F(T^2(x,\lambda),T(x,\lambda))$$

$$\prec b(\alpha\rho_F((x,\lambda),T(x,\lambda))+\beta\rho_F(T(x,\lambda),T^2(x,\lambda))$$

Therefore

$$\rho_F(T(x,\lambda),T^2(x,\lambda)) \prec \frac{\alpha b}{1-\beta b}(\rho_F((x,\lambda),T(x,\lambda))).$$

Now if $k=\frac{\alpha b}{1-\beta b}$, then $k \prec 1$ and

$$\rho_F(T(x,\lambda),T^2(x,\lambda) \prec k(\rho_F((x,\lambda),T(x,\lambda))).$$

Therefore

$$\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda) \prec k^n(\rho_F((x,\lambda),T(x,\lambda)))\ \textit{for all}\ n \geq 1.$$

Suppose (iii) is true then

$$\rho_F(T(x,\lambda),T^2(x,\lambda)) \prec \varphi(\rho_F((x,\lambda),T(x,\lambda)))\alpha\rho_F((x,\lambda),T(x,\lambda))+\beta(\rho_F(T(x,\lambda),T^2(x,\lambda))$$

$$\prec b\alpha\rho_F((x,\lambda),T(x,\lambda))+\beta\rho_F(T(x,\lambda),T^2(x,\lambda)).$$

Therefore

$$\rho_F(T(x,\lambda),T^2(x,\lambda) \prec \frac{\alpha b}{1-\beta b}\rho_F((x,\lambda),T(x,\lambda).$$

Now if $k = \dfrac{\alpha b}{1-\beta}$, then $k \prec 1$ and

$$\rho_F(T(x,\lambda),T^2(x,\lambda)) \prec k\rho_F((x,\lambda),T(x,\lambda)).$$

Also $\rho_F(T^2(x,\lambda),T^3(x,\lambda) \prec k^2\rho_F((x,\lambda),T(x,\lambda)),$

therefore $\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda)) \prec k^n\rho_F((x,\lambda),T(x,\lambda)),$ for all $n \geq 1.$

In each case

$$\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda)) \to 0$$

This implies $\{T^n(x,\lambda)\}$ is a fuzzy Cauchy sequence.

By fuzzy completeness of X, there is a $(x^*,\lambda) \in X$ such that $T^n(x,\lambda) \to (x^*,\lambda).$

If (i) is true, then

$$\rho_F(T(x^*,\lambda),(x^*,\lambda)) \prec \rho_F(T^{n+1}(x,\lambda),T(x^*,\lambda)) + \rho_F(T^{n+1}(x,\lambda),(x^*,\lambda))$$

$$\prec \varphi(\rho_F(T^n(x,\lambda)),(x^*,\lambda))(\alpha\rho_F(T^n(x,\lambda)),(x^*,\lambda)$$

$$+\beta\rho_F(T^n(x,\lambda)),T^{n+1}(x^*,\lambda)) + \rho_F((x^*,\lambda),T(x^*,\lambda))$$

$$+\rho_F(T^{n+1}(x,\lambda)),(x^*,\lambda).$$

If (ii) is true, then

$$\rho_F(T(x^*,\lambda),(x^*,\lambda)) \prec \rho_F(T^{n+1}(x,\lambda),T(x^*,\lambda)) + \rho_F(T^{n+1}(x,\lambda),(x^*,\lambda))$$

$$\prec \varphi(\rho_F(T^n(x,\lambda)),(x^*,\lambda))(\alpha\rho_F(T^n(x,\lambda)),(x^*,\lambda)$$

$$+\beta\rho_F(T^{n+1}(x,\lambda)),(x^*,\lambda)) + \rho_F(T^{n+1}(x,\lambda)),(x^*,\lambda).$$

If (iii) is true, then

$$\rho_F(T(x^*,\lambda),(x^*,\lambda)) \prec \rho_F(T^{n+1}(x,\lambda),T(x^*,\lambda)) + \rho_F(T^{n+1}(x,\lambda),(x^*,\lambda))$$

$$\prec \varphi(\rho_F(T^n(x,\lambda)),(x^*,\lambda)).$$

Since $T^n(x,\lambda) \to (x^*,\lambda)$ and $\varphi(0)=0$, and φ is continuous, therefore

$\rho_F(T(x^*,\lambda),(x^*,\lambda)) = 0,$ and so $T(x^*,\lambda) = (x^*,\lambda).$

If (y^*,λ) is an another $\lambda-$fuzzy fixed point of T and (i) is true, then

$$\rho_F((x^*,\lambda),(y^*,\lambda)) = \rho_F(T(x^*,\lambda),T(y^*,\lambda))$$

$$\prec \rho_F((x^*,\lambda),(y^*,\lambda))(\alpha\rho_F((x^*,\lambda),(y^*,\lambda))$$

$$+\beta(\rho_F(T(x^*,\lambda),(x^*,\lambda))+\rho_F(T(y^*,\lambda),(y^*,\lambda))).$$

Hence $\rho_F((x^*,\lambda),(y^*,\lambda)) \prec \alpha b\rho_F((x^*,\lambda),(y^*,\lambda)),$ since $\alpha b<1$ and so

$$\rho_F((x^*,\lambda),(y^*,\lambda)) = 0.$$

Therefore,

$$(x^*,\lambda) = (y^*,\lambda)$$

and the $\lambda-$fuzzy fixed point of T is unique.

If (y^*,λ) is an another fixed point of T and (ii) is true, then

$$\rho_F((x^*,\lambda),(y^*,\lambda)) = \rho_F(T(x^*,\lambda),T(y^*,\lambda))$$

$$\prec \rho_F((x^*,\lambda),(y^*,\lambda))(\alpha\rho_F((x^*,\lambda),(y^*,\lambda))+\beta\rho_F(T(x^*,\lambda),(y^*,\lambda))).$$

Hence $\rho_F((x^*,\lambda),(y^*,\lambda)) \prec b(\alpha\rho_F((x^*,\lambda),(y^*,\lambda))+\beta\rho_F((x^*,\lambda),(y^*,\lambda)),$

since $\alpha+\beta<1, b<1$ and $(1-\alpha b-\beta b)\rho_F((x^*,\lambda),(y^*,\lambda)) \prec 0$ and so $\rho_F((x^*,\lambda),(y^*,\lambda)) = 0.$ Therefore, $(x^*,\lambda) = (y^*,\lambda),$ and the $\lambda-$fuzzy fixed point of T is unique.

If (y^*,λ) is an another fixed point of T and (iii) is true, then

$$\rho_F((x^*,\lambda),(y^*,\lambda)) = \rho_F(T(x^*,\lambda),T(y^*,\lambda))$$

$$\prec \varphi\rho_F((x^*,\lambda),(y^*,\lambda))\alpha\rho_F((x^*,\lambda),(y^*,\lambda)+\beta\rho_F((y^*,\lambda),(x^*,\lambda)).$$

Hence $\rho_F((x^*,\lambda),(y^*,\lambda)) \prec b\alpha\rho_F((x^*,\lambda),(y^*,\lambda)) + \beta\rho_F((x^*,\lambda),(y^*,\lambda)),$ since $\alpha b + \beta < \alpha + \beta < 1,$ $b < 1$ and $(1-\alpha b-\beta)\rho_F((x^*,\lambda),(y^*,\lambda)) \prec 0$ and so $\rho_F((x^*,\lambda),(y^*,\lambda)) = 0.$ Therefore, $(x^*,\lambda) = (y^*,\lambda),$ and the $\lambda-$fuzzy fixed point of T is unique.

Definition 3.1. *Let* X *be a fuzzy metric space and* $(A,\lambda),(B,\lambda) \in P_F^{\lambda}(X)$. *We define*

$$\rho_F((A,\lambda),(B,\lambda)) = \inf\{\rho_F((x,\lambda),(y,\lambda)) : (x,\lambda) \in (A,\lambda), (y,\lambda) \in (B,\lambda)\}.$$

Let $T : A\bigcup B \to (A,\lambda)\bigcup(B,\lambda)$ be a map. We say that the point $(x,\lambda) \in A\bigcup B$ is a $\lambda-$fuzzy approximate best proximity point of the pair $((A,\lambda),(B,\lambda))$, if $\rho_F((x,\lambda),T(x,\lambda)) \prec \rho_F((A,\lambda),(B,\lambda)) + \varepsilon$, for some $\varepsilon > 0$.

Proposition 3.1. *Let* X *be a fuzzy metric space,* $T : A\bigcup B \to (A,\lambda)\bigcup(B,\lambda)$ *a map and* $(A,\lambda),(B,\lambda) \in P_F^{\lambda}(X)$. *If*

$$\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda)) \to \rho_F((A,\lambda),(B,\lambda)), \; as\; n \to \infty.$$

then the pair $((A,\lambda),(B,\lambda))$ has an $\lambda-$fuzzy approximate best proximity pair.

Proof. For any $\varepsilon > 0$. There exists $n_0 \in N$ such that

$$\rho_F(T^{n_0}(x,\lambda),T(T^{n_0}(x,\lambda)) = \rho_F(T^{n_0}(x,\lambda),T^{n_0+1}(x,\lambda)) \prec \rho_F((A,\lambda),(B,\lambda)) + \varepsilon.$$

Therefore $T_0^n(x,\lambda)$ is a $\lambda-$fuzzy approximate best proximity point of the pair $((A,\lambda),(B,\lambda))$.

Theorem 3.2. *Let* X *be a fuzzy complete metric space,* $A,B \in P_F^{\lambda}(X)$ *and* $T : (A,\lambda)\bigcup(B,\lambda) \to (A,\lambda)\bigcup(B,\lambda)$ *be a map. There is a continuous map* $\varphi : S_F^{+}(R) \to [0;1)$, $\varphi(\rho_F((x,\lambda);(y,\lambda))) < b$, $\varphi(0) = 0$, $b < 1$ *and* $\alpha + 2\beta + \gamma < 1$ *such that for* $(x,\lambda),(y,\lambda) \in P_F(X)$ *and one of the following statement is satisfy;*

(i)
$$\rho_F(T(x,\lambda),T(y,\lambda)) \prec \varphi(\rho_F(x,\lambda),(y,\lambda))(\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta(\rho_F((x,\lambda),T(x,\lambda)) + \rho_F((y,\lambda),T(y,\lambda)) + \gamma\rho_F((A$$

(ii)
$$\rho_F(T(x,\lambda),T(y,\lambda)) \prec \varphi(\rho_F(x,\lambda),(y,\lambda))(\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta\rho_F(T(x,\lambda),(y,\lambda)) + \gamma\rho_F((A,\lambda),(B,\lambda))),$$

(iii)
$\rho_F(T(x,\lambda),T(y,\lambda)) \prec \varphi(\rho_F(x,\lambda),(y,\lambda))\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta\rho_F((Tx,\lambda),(y,\lambda) + \gamma\rho_F((A,\lambda),(B,\lambda)).$

then T has a $\lambda-$fuzzy approximate best proximity point of the pair $((A,\lambda),(B,\lambda))$.

Proof. Let $x \in X$, (i) is true, then

$$\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda)) \prec k^n\rho_F((x,\lambda),T(x,\lambda)) + (1-k^n)\rho_F((A,\lambda),(B,\lambda)),\ \textit{for all } n \geq 1,$$

where $k = \dfrac{\alpha b + \beta b}{1-\beta b}$. If (ii) is true and $k = \dfrac{\alpha b}{1-\beta b}$, then $k \prec 1$ and

$$\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda)) \prec k^n\rho_F((x,\lambda),T(x,\lambda)) + (1-k^n)\rho_F((A,\lambda),(B,\lambda)),\ \textit{for all } n \geq 1$$

If (iii) is true and $k = \dfrac{\alpha b}{1-\beta}$, then $k \prec 1$ and

$$\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda)) \prec k^n\rho_F((x,\lambda),T(x,\lambda)) + (1-k^n)\rho_F((A,\lambda),(B,\lambda)),\ \textit{for all } n \geq 1.$$

Therefore

$$\rho_F(T^n(x,\lambda),T^{n+1}(x,\lambda)) \to \rho_F((A,\lambda),(B,\lambda)).$$

By Proposition 3.1, the pair $((A,\lambda),(B,\lambda))$ has a $\lambda-$fuzzy approximate best proximity pair. +

Theorem 3.3. *Let* X *be a fuzzy complete metric space,* $A,B \in P_F^\lambda(X)$ *and* $T:(A,\lambda)\bigcup(B,\lambda) \to (A,\lambda)\bigcup(B,\lambda)$ *be a map. There is a continuous map* $\varphi: S_F^+(R) \to [0;1)$, $\varphi(\rho_F((x,\lambda);(y,\lambda))) < b$, $\varphi(0)=0$, $b<1$ *and* $\alpha + 2\beta + \gamma < 1$ *such that for* $(x,\lambda),(y,\lambda) \in P_F(X)$ *and one of the following statement is satisfy;*

(i)
$\rho_F(T(x,\lambda),T(y,\lambda)) \prec \phi(\rho_F(x,\lambda),(y,\lambda))(\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta(\rho_F((x,\lambda),T(x,\lambda)) + \rho_F((y,\lambda),T(y,\lambda)))$
$+\gamma\rho_F((A,\lambda),(B,\lambda)),$
(ii)
$\rho_F(T(x,\lambda),T(y,\lambda)) \prec \varphi(\rho_F(x,\lambda),(y,\lambda))(\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta\rho_F(T(x,\lambda),(y,\lambda))) + \gamma\rho_F((A,\lambda),(B,\lambda)),$

(iii)
$\rho_F(T(x,\lambda),T(y,\lambda)) \prec \varphi(\rho_F(x,\lambda),(y,\lambda))\alpha\rho_F((x,\lambda),(y,\lambda)) + \beta\rho_F((Tx,\lambda),(y,\lambda) + \gamma\rho_F((A,\lambda),(B,\lambda)).$
Also let (x_n,λ) be as follows by

$$(x_{n+1},\lambda) = T(x_n,\lambda)\ \ \textit{for some}\ \ (x_1,\lambda) \in (A,\lambda)\bigcup(B,\lambda),\ n \in N.$$

If (x_n,λ) has a convergent sequence in A, then there exists a $(x_0,\lambda)\in A$ such that $\rho_F((x_0,\lambda),T(x_0,\lambda))=\rho_F((A,\lambda),(B,\lambda)))$.

Proof. For $(x_1,\lambda)\in X$, by Theorem 3.1, for every three case we have

$$\rho_F(T^n(x_1,\lambda),T^{n+1}(x_1,\lambda_)) \to \rho_F((A,\lambda),(B,\lambda))),$$

that is, $\rho_F((x_n,\lambda),T(x_n,\lambda))\to\rho_F((A,\lambda),(B,\lambda)))$. Since (x_n,λ) has a subsequence (x_{n_k},λ) that is convergent to $(x_0,\lambda)\in(A,\lambda)\bigcup(B,\lambda)$ and T is continuous. Therefore $\rho_F((x_0,\lambda),(Tx_0,))=\rho_F((A,\lambda),(B,\lambda)))$.

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