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A quick method to calculate the super-efficient point in multi-objective assignment problems

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Abstract

The present study has presented a method to obtain the best non-dominated point (the point having the least distance to the ideal point) for the multi-objective assignment problems which is more efficient and is so quick while simple, compared with other similar methods in other studies. This method does not need any parameters or point (even the ideal point) to solve the problem and effectively turns solving a multi-objective assignment problem into solving the single-objective assignment problem. Moreover, it gives the best non-dominated point as the solution. Finally, a numeral example has been brought to compare this method with proposed methods in other studies.

Keywords: Multi-objective optimization, Assignment problems, Integer programming.

1. Introduction

Assignment problem is among the familiar problems in the real world. In the meantime, multi-objective assignment problems are specifically important, because the real problems are encountered with the multi-objective assignment problems and single-objective assignment problems are not as many as the aforementioned ones. Multi-objective assignment problems are the problems of multi-objective integer programming, thus their solving through the regular methods is not possible because of the solution region being non-convex. So far a great number of methods have been proposed to solve these types of problems [1, 3, 4] which need much more time and complicated calculations to

reach the best non-dominated solution (an solution which has the least distance to the ideal point). A simple method is proposed for the single-objective assignment problems like the Hungarian method.
Simplicity of A method, like the Hungarian method to solve the single-objective assignment problem , and the importance of the non-dominated points in multi-objective problems, on the other hand, has led to looking for a method which without calculating all of the non-dominated points while being simple, specifies the best non-dominated point for the decision-maker.
This paper introduces a method to solve multi-objective assignment problems. It is clear that ultimately the decision-maker will choose one among all of the non-dominated points and this point is usually a point has the least distance from the ideal point. To choose this best non-dominated point two following methods can be followed:
i) The regular method which has been mentioned so far, finding all of the non-dominated points and then choosing the best non-dominated point among them. This method is not efficient because of these reasons:
1) some of the methods have been proposed for the multi-objective assignment problems cannot calculate all of the non-dominated points,
2) calculating all of the non-dominated points through the methods have been explained so far is so time-consuming and onerous.
ii) The method presented in this study gives only a non-dominated point to the decision-maker which this point is the one having the least distance to the ideal point. This method seems to be useful according to these three reasons:
1) this method transform a multi-objective assignment problem into a single-objective assignment problem, because in this method there is even no need to calculate the ideal point,
2) calculations and the required time in this method is exactly matched to solve a single-objective assignment problem, and
3) while being simple, this method makes it possible for the decision-maker to make the best decision at the least amount of time.
The outline of this paper is as follows: In the second part, some of the definitions and symbols are being used in this study will be presented. The third part is dedicated to the method presentation. The fourth part included a numerical example to illustrate this method and comparing it with the other methods [1]. Finally, the fifth part is about the conclusion.

2. Definitions and symbols

A multi objective assignment problem is formulated generally as follows:

$$\min z_k(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^k x_{ij},\ k = 1,2,\dots,p$$

$$s.t. \begin{cases} \sum_{j=1}^{n} x_{ij} = 1,\ i = 1,2,\dots,n \\ \sum_{j=1}^{n} x_{ij} = 1,\ j = 1,2,\dots,n \\ x_{ij} \in \{0,1\},\ i,\ j = 1,2,\dots,n \end{cases} \tag{1}$$

Where all objective function coefficients c_{ij}^k are positive integers. Henceforth, the feasible region of the above problem, to avoid repeating with S, and the dependent cost matrix to the target function of

k th is shown by C^k and the set of $Z(x)=(z_1(x),z_2(x),...,z_p(x))$ where $x=(x_{11},...,x_{nn})\in S$ is shown by Y in the target space.

Definition 1: *The $x^* \in S$ is called efficient when there is no $x \in S$ so that*

$$\forall i \in \{1,2,..,p\};\ z_i(x) \le z_i(x^*),$$
$$\exists j \in \{1,2,..,p\};\ z_j(x) < z_j(x^*).$$

Thus, $Z(x^*)$ is called the non-dominated point.

Definition 2: *Assume $x^{i*} \in S$ for each $i \in \{1,...,p\}$ is the optimized point of $z_i(\mathrm{x})$ on the S, then the $(z_1(x^{1*}),z_2(x^{2*}),...,z_p(x^{p*}))$ point in the target space is called the ideal point and shown by I.*

Definition 3: *The efficient point of $x^* \in S$ is called super-efficient when $Z(x^*)$ has the least distance to the ideal point, so in this case $Z(x^*)$ is called the best non-dominated point.*

3. Method presentation

In multi-objective problems, the decision-maker usually looks for a super-efficient point. Thus, it is attempted in this paper to present a method to reach the super-efficient point consuming the least time and doing the least possible calculations. In fact, a type of $x \in S$ to minimize the distance function of $d(Z(x),I)$. To calculate the distance of $d(Z(x),I)$ different norms are used. Here, to keep the linearity of the target function the L_1 norm is used.

Theorem 1: *x^* is the optimized problem's solution*

$$P_1 : \min |z_1(x)-I_1|+...+|z_p(x)-I_p|$$
$$s.t.\quad x \in S$$

if and only if it is the optimized solution of the following problem:

$$P_2 : \min z_1(x)+...+z_p(x)$$
$$s.t.\ \ x \in S$$

Proof: Since according to the ideal point definition I_i is the optimized solution of $\begin{cases}\min\ z_i(x)\\ s.t.\ \ x \in S\end{cases}$ for each $i \in \{1,...,p\}$, then $z_i(x)-I_i$ is always a positive amount and all of the absolute values can be deleted from the P_1 problem. Also, the fixed values of I_i do not affect the optimization.

Theorem 2: *Any optimized point of P_1 problem is an efficient point of problem (1).*

Proof: Assume x^* as the optimized point of P_1, it is intended to show that x^* is an efficient point for problem (1). It is assumed inversely that this is not true, then there is a $y \in S$ which for each i, $z_i(y) \le z_i(x^*)$ and for at least one j, $z_j(y) \le z_j(x^*)$, and this is contrast with the x^* being optimized for problem P_2. Therefore, x^* is an efficient point for problem (1).

Now, using the (1) and (2) theorems it is shown that obtaining the super-efficient point of problem (1) is equal to obtaining the optimized point of problem P_2.

Theorem 3: x^* *is the optimized point of problem* P_2 *if and only if it is the super-efficient point of problem (1).*

Proof: According to the super-efficient point definition and theorem (2) it can be simply proved that x^* is the super-efficient point of problem (1) if and only if it is the optimized point of problem P_1. Then, based on theorem (1) this theorem is proved.

Regarding the above explanations and theorems finding the super-efficient solution of the multi-objective assignment of problem (1) is equal to finding the optimized point of the single-objective assignment problem of P_2 with the cost matrix of $C = C^1 + C^2 + ... + C^p$. Accordingly, using one of the single-objective assignment methods such as Hungarian method the solution of super-efficient for the problem (1) can be easily obtained, i.e. practically a multi-objective assignment problem has been turned into a single-objective one.
In the next part a numerical example has been applied for better understanding on the presented method and investigating the efficiency and comparing the operating speed with other proposed methods.

4. Investigating and comparing the method using the numerical example

To investigate the operating speed and efficiency of the presented method, and reaching the super-efficient solution compared with other methods only an example mentioned in source [1] is considered:

Example: *Consider the three-objective assignment problem with the three-cost following matrices*

$$C^1 = \begin{pmatrix} 2 & 5 & 4 & 7 \\ 3 & 3 & 5 & 7 \\ 3 & 8 & 4 & 2 \\ 6 & 8 & 4 & 2 \end{pmatrix}, C^2 = \begin{pmatrix} 3 & 3 & 6 & 2 \\ 5 & 3 & 7 & 3 \\ 5 & 2 & 7 & 4 \\ 4 & 6 & 3 & 5 \end{pmatrix}, C^3 = \begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 6 & 4 & 7 & 3 \end{pmatrix}$$

which has been solved in [1] using a two-phase method for multi-objective problems with integers and applying several algorithm consuming a lot of time. Here, the super-efficient point and the best non-dominated point of the problem are being solved according to the presented method in this study and it needs only a single-objective assignment problem. To this aim the cost-matrix of C *is obtained as following:*

$$C = C^1 + C^2 + C^3 = \begin{pmatrix} 9 & 10 & 15 & 12 \\ 13 & 12 & 16 & 13 \\ 12 & 13 & 16 & 8 \\ 16 & 15 & 12 & 13 \end{pmatrix}$$

Now, using the Hungarian method the single-objective assignment problem with the cost-matrix C is solved which its optimized point is

$$x^* = \begin{cases} x_{11} = x_{22} = x_{34} = x_{43} = 1 \\ x_{ij} = 0 \qquad o.w \end{cases}$$

and the optimized value of the target function is $Z(x^*) = (9,13,16)$. *According to what has been mentioned and proved in the third part of this paper* x^* *is the super-efficient point and* $Z(x^*)$ *is the best non-dominated solution of the tri-objective assignment problem, and its solving is easily finished.*

As it was seen in the above example, the best non-dominated solution was calculated solving only a single-objective assignment using the method in this paper consuming the minimum time and also calculations. The best non-dominated now is calculated with the regular method for comparison. To this aim, first of all the ideal point which needs the solving of three single-objective assignment problems should be calculated. By these calculations the ideal point $I = (9,11,13)$ is obtained. In the next step all of the non-dominated points should be calculated for making it possible to calculate the point among them which has the least distance from the ideal point. In [1] the set of all non-dominated points was calculated spending a lot of time and doing very complicated calculations as following:

$$\begin{aligned} &y^1 = (9,13,16), d^1 = 5, \quad y^2 = (19,11,17), d^2 = 14, \\ &y^3 = (14,18,15), d^3 = 14, \ \ y^4 = (20,17,14), d^4 = 18, \\ &y^5 = (14,20,14), d^5 = 15, \ \ y^6 = (18,18,14), d^6 = 17, \\ &y^7 = (18,20,13), d^7 = 18, \end{aligned}$$

where d^j is the distance of the non-dominated point of j th, i.e. y_j from the ideal point with the norm L_1 and from here the best non-dominated point is $y^1 = Z(x^*)$, the same solution reached to using the presented method without calculating all of the non-dominated points, and even the ideal point easily spending the least possible time.

5. Conclusion

As it was observed through the presented numerical example, using the method which mentioned in this paper it is possible to obtain the best non-dominated solution without calculating all of the non-dominated points for each multi-objective assignment problem only with solving a single-objective problem, while in this method there is even no need to calculate the ideal point either.
Among the things can be done to continue this paper is investigating the mentioned method with other norms such as $L_\infty, L_2, \ldots$.

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