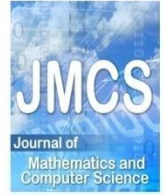


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Non-Archimedean Intuitionistic Fuzzy Continuity of Dectic Mappings

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Abstract

In this paper, we investigate the non-Archimedean intuitionistic fuzzy continuity through the existence of a certain solution of a fuzzy stability problem for the system of additive-quadratic-cubic-quartic functional equations.

Keywords: Non-Archimedean intuitionistic fuzzy normed spaces, Mixed type functional equation, Dectic mappings, Hyers-Ulam stability.

1. Introduction

The study of stability problems for functional equations is related to a question of Ulam [25] concerning the stability of group homomorphisms and affirmatively answered for Banach spaces by Hyers [12]. The results of Hyers were generalized by Aoki [2], Rassias [21] and Gavruta [10], respectively. We refer the reader to see, for instance, [1, 4, 5, 6, 7, 8, 9, 24] for more information on different aspects of stability of functional equations. The notion of intuitionistic fuzzy set introduced by Atanassov [3] has been used extensively in many areas of mathematics and sciences. The notion of intuitionistic fuzzy norm [22, 13, 20] is also useful one to deal with the inexactness and vagueness arising in modeling. There are many situations where the norm of a vector is not possible to find and the concept of intuitionistic fuzzy norm seems to be more suitable in such cases, that is, we can deal with such situations by modeling the inexactness through the intuitionistic fuzzy norm. The stability problem for Pexiderized quadratic functional equation, Jensen functional equation, cubic functional equation, functional equations associated with inner product spaces, and additive functional equation was considered in [14, 15, 16, 17, 18, 23], respectively, in the intuitionistic fuzzy normed spaces.

In this paper, we determine some stability results concerning the system of additive-quadratic-cubic-quartic functional equations

$$\begin{cases} f(ax_1 + bx_2, y, z, w) + f(ax_1 - bx_2, y, z, w) = 2af(x_1, y, z, w) \\ f(x, ay_1 + by_2, z, w) + f(x, ay_1 - by_2, z, w) = 2a^2 f(x, y_1, z, w) + 2b^2 f(x, y_2, z, w) \\ f(x, y, az_1 + bz_2, w) + f(x, y, az_1 - bz_2, w) = ab^2(f(x, y, z_1 + z_2, w) \\ \quad + f(x, y, z_1 - z_2, w)) + 2a(a^2 - b^2)f(x, y, z_1, w) \\ f(x, y, z, aw_1 + bw_2) + f(x, y, z, aw_1 - bw_2) = a^2b^2(f(x, y, z, w_1 + w_2) \\ \quad + f(x, y, z, w_1 - w_2)) + 2a^2(a^2 - b^2)f(x, y, z, w_1) - 2b^2(a^2 - b^2)f(x, y, z, w_2) \end{cases} \quad (1.1)$$

where $a, b \in \mathbb{Z} \setminus \{0\}$ with $a \neq \pm 1, \pm b$, in the setting of non-Archimedean intuitionistic fuzzy normed spaces. We also study the non-Archimedean intuitionistic fuzzy continuity through the existence of a certain solution of a fuzzy stability problem for the system of additive-quadratic-cubic-quartic functional equations. Note that, the function $f: \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x, y, z, w) = cxy^2z^3w^4$ is a solution of (1.1). In particular, putting $x = y = z = w$, we get a detric function $g: \mathbb{R} \rightarrow \mathbb{R}$ in one variable given by $g(x) := f(x, x, x, x) = cx^{10}$. The proof of the following proposition is evident, and we omit the details.

Proposition 1.1. Let X and Y be real linear spaces. If a mapping $f: X \times X \times X \times X \rightarrow Y$ satisfies system (1.1), then $f(\lambda x, \mu y, \eta z, \gamma w) = \lambda \mu^2 \eta^3 \gamma^4 f(x, y, z, w)$ for all $x, y, z, w \in X$, and all rational numbers $\lambda, \mu, \eta, \gamma$.

2. Non-Archimedean intuitionistic fuzzy normed spaces

In this section, we introduce the concept of non-Archimedean intuitionistic fuzzy normed space and further define the notions of convergence and Cauchy sequences in this new framework.

A valuation is a map $|\cdot|$ from a field K into $[0, \infty)$ such that 0 is the unique element having the 0 valuation, $|k_1 k_2| = |k_1| |k_2|$, and the triangle inequality holds, that is, $|k_1 + k_2| \leq |k_1| + |k_2|$, for all $k_1, k_2 \in K$. We say that a field K is valued if K carries a valuation. The usual absolute values of \mathbb{R} and \mathbb{C} are examples of valuations.

Let us consider a valuation which satisfies stronger condition than the triangle inequality. If the triangle inequality is replaced by $|k_1 + k_2| \leq \max \{|k_1|, |k_2|\}$ for all $k_1, k_2 \in K$ then, a map $|\cdot|$ is called non-Archimedean or ultrametric valuation, and field is called a non-Archimedean field. Clearly $|1| = |-1| = 1$ and $|n| \leq 1$, for all $n \in \mathbb{N}$. A trivial example of a non-Archimedean valuation is the map $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$.

Let X be a vector space over a field K with a non-Archimedean valuation $|\cdot|$. A non-Archimedean normed space is a pair $(X, \|\cdot\|)$, where $\|\cdot\|: X \rightarrow [0, \infty)$ is such that

- (i) $\|\cdot\| = 0$, if and only if $x = 0$;
- (ii) $\|\lambda x\| = \lambda \|x\|$, for all $\lambda \in K$ and
- (iii) the strong triangle inequality, $\|k_1 + k_2\| \leq \max \{\|k_1\|, \|k_2\|\}$ for all $x, y \in X$.

In [11], Hensel discovered the p -adic numbers as a number theoretical analogue of power series in complex analysis. The most interesting example of non-Archimedean spaces is p -adic numbers.

A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is said to be a continuous t -norm if it satisfies the following conditions:

(a) $*$ is associative and commutative, (b) $*$ is continuous, (c) $a * 1 = a$ for all $a \in [0,1]$ and (d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$.

A binary operation \diamond : $[0,1] \times [0,1] \rightarrow [0,1]$ is said to be a continuous t -conorm if it satisfies the following conditions:

(a') \diamond is associative and commutative, (b') \diamond is continuous, (c') $a \diamond 0 = a$ for all $a \in [0,1]$ and (d') $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$.

Definition 2.1. The five-tuple $(X, \mathcal{F}, \mathcal{P}, *, \diamond)$ is said to be a non-Archimedean intuitionistic fuzzy normed space (for short, non-Archimedean IFN space) if X is a vector space over a non-Archimedean field K , $*$ is a continuous t -norm, \diamond is a continuous t -conorm, and \mathcal{F}, \mathcal{P} are functions from $X \times \mathbb{R}$ to $[0,1]$ satisfying the following conditions. For every $x, y \in X$ and $s, t \in K$,

(i) $\mathcal{F}(x, t) + \mathcal{P}(x, t) \leq 1$, (ii) $\mathcal{F}(x, t) > 0$, (iii) $\mathcal{F}(x, t) = 1$ if and only if $x = 0$, (iv) $\mathcal{F}(ax, t) = \mathcal{F}\left(x, \frac{t}{|a|}\right)$ for each $a \neq 0$, (v) $\mathcal{F}(x, t) * \mathcal{F}(y, s) \leq \mathcal{F}(x + y, \max\{t, s\})$, (vi) $\mathcal{F}(x, .) : (0, \infty) \rightarrow [0,1]$ is continuous, (vii) $\lim_{t \rightarrow \infty} \mathcal{F}(x, t) = 1$ and $\lim_{t \rightarrow 0} \mathcal{F}(x, t) = 0$, (viii) $\mathcal{P}(x, t) < 1$, (ix) $\mathcal{P}(x, t) = 0$ if and only if $x = 0$, (x) $\mathcal{P}(ax, t) = \mathcal{P}\left(x, \frac{t}{|a|}\right)$ for each $a \neq 0$, (xi) $\mathcal{P}(x, t) \diamond \mathcal{P}(y, s) \geq \mathcal{P}(x + y, \max\{t, s\})$, (xii) $\mathcal{P}(x, .) : (0, \infty) \rightarrow [0,1]$ is continuous, (xiii) $\lim_{t \rightarrow \infty} \mathcal{P}(x, t) = 0$ and $\lim_{t \rightarrow 0} \mathcal{P}(x, t) = 1$.

In this case $(\mathcal{F}, \mathcal{P})$ is called a non-Archimedean intuitionistic fuzzy norm.

Example 2.2. Let $(X, \| \cdot \|)$ be a non-Archimedean normed space, $a * b = ab$ and $a \diamond b = \min\{a + b, 1\}$ for all $a, b \in [0,1]$. For all $x \in X$, all $t > 0$ and $k = 1, 2$, consider the following:

$$\mathcal{F}_k(x, t) = \begin{cases} \frac{t}{t+k\|x\|} & t > 0 \\ 0 & t \leq 0, \end{cases} \quad \text{and} \quad \mathcal{P}_k(x, t) = \begin{cases} \frac{k\|x\|}{t+k\|x\|} & t > 0 \\ 0 & t \leq 0. \end{cases}$$

Then $(X, \mathcal{F}_k, \mathcal{P}_k, *, \diamond)$ is a non-Archimedean intuitionistic fuzzy normed space.

Definition 2.3. Let $(X, \mathcal{F}, \mathcal{P}, *, \diamond)$ be a non-Archimedean intuitionistic fuzzy normed space. Then, a sequence $s = (s_n)$ is said to be

(i) convergent in $(X, \mathcal{F}, \mathcal{P}, *, \diamond)$ or simply $(\mathcal{F}, \mathcal{P})$ -convergent to $\lambda \in X$ if for every $\epsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{F}(s_n - \lambda, t) > 1 - \epsilon$ and $\mathcal{P}(s_n - \lambda, t) < \epsilon$ for all $n \geq n_0$. In this case we write $(\mathcal{F}, \mathcal{P})\text{-}\lim_{n \rightarrow \infty} s_n = \lambda$ and λ is called the $(\mathcal{F}, \mathcal{P})$ - limit of $s = (s_n)$.

(ii) Cauchy in $(X, \mathcal{F}, \mathcal{P}, *, \diamond)$ or simply $(\mathcal{F}, \mathcal{P})$ -Cauchy if for every $\epsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{F}(s_n - s_m, t) > 1 - \epsilon$ and $\mathcal{P}(s_n - s_m, t) < \epsilon$ for all $n \geq m \geq n_0$. A non-Archimedean IFN-space $(X, \mathcal{F}, \mathcal{P}, *, \diamond)$ is said to be complete if every $(\mathcal{F}, \mathcal{P})$ -Cauchy is $(\mathcal{F}, \mathcal{P})$ -convergent. In this case $(X, \mathcal{F}, \mathcal{P}, *, \diamond)$ is called non-Archimedean intuitionistic fuzzy Banach space.

Throughout this paper, we assume that $I = \{1, 2, 3, 4\}$.

3. Non-Archimedean intuitionistic fuzzy stability

In this section, we deal with the stability results concerning for system of functional equations (1.1) in non-Archimedean intuitionistic fuzzy normed spaces.

Theorem 3.1. Let X be a linear space over a non-Archimedean field K and let $(Z, \mathcal{F}', \mathcal{P}')$ be a non-Archimedean IFN space. Suppose that $\varphi_i: X \times X \times X \times X \times X \rightarrow Z$ for $i \in I$ be functions such that $|\alpha| < 1$ is far from zero and for some real number α with $|\alpha| > |\alpha|^{10}$,

$$\begin{cases} \mathcal{F}'(\varphi_i(\frac{x}{a}, \frac{y}{a}, \frac{z}{a}, \frac{w}{a}, \frac{v}{a}, t) \geq \mathcal{F}'(\varphi_i(x, y, z, w, v), |\alpha|t) \\ \mathcal{P}'(\varphi_i(\frac{x}{a}, \frac{y}{a}, \frac{z}{a}, \frac{w}{a}, \frac{v}{a}, t) \leq \mathcal{P}'(\varphi_i(x, y, z, w, v), |\alpha|t) \end{cases} \tag{3.1}$$

for all $x, y, z, w, v \in X, t > 0$ and $i \in I$. Let f be a mapping from $X \times X \times X \times X$ to a non-Archimedean intuitionistic fuzzy Banach space $(Y, \mathcal{F}, \mathcal{P})$ such that $f(x, 0, z, w) = 0, f(x, y, z, 0) = 0$ for all $x, y, z, w \in X$, and satisfying

$$\begin{cases} \mathcal{F}(f(ax_1 + bx_2, y, z, w) + f(ax_1 - bx_2, y, z, w) - 2af(x_1, y, z, w), t) \\ \geq \mathcal{F}'(\varphi_1(x_1, x_2, y, z, w), t), \\ \mathcal{P}(f(ax_1 + bx_2, y, z, w) + f(ax_1 - bx_2, y, z, w) - 2af(x_1, y, z, w), t) \\ \leq \mathcal{P}'(\varphi_1(x_1, x_2, y, z, w), t), \end{cases} \tag{3.2}$$

$$\begin{cases} \mathcal{F}(f(x, ay_1 + by_2, z, w) + f(x, ay_1 - by_2, z, w) - 2a^2 f(x, y_1, z, w) - 2b^2 f(x, y_2, z, w), t) \\ \geq \mathcal{F}'(\varphi_2(x, y_1, y_2, z, w), t), \\ \mathcal{P}(f(x, ay_1 + by_2, z, w) + f(x, ay_1 - by_2, z, w) - 2a^2 f(x, y_1, z, w) - 2b^2 f(x, y_2, z, w), t) \\ \leq \mathcal{P}'(\varphi_2(x, y_1, y_2, z, w), t), \end{cases} \tag{3.3}$$

$$\begin{cases} \mathcal{F}(f(x, y, az_1 + bz_2, w) + f(x, y, az_1 - bz_2, w) - ab^2(f(x, y, z_1 + z_2, w) + f(x, y, z_1 - z_2, w)) \\ - 2a(a^2 - b^2)f(x, y, z_1, w), t) \geq \mathcal{F}'(\varphi_3(x, y, z_1, z_2, w), t), \\ \mathcal{P}(f(x, y, az_1 + bz_2, w) + f(x, y, az_1 - bz_2, w) - ab^2(f(x, y, z_1 + z_2, w) + f(x, y, z_1 - z_2, w)) \\ - 2a(a^2 - b^2)f(x, y, z_1, w), t) \leq \mathcal{P}'(\varphi_3(x, y, z_1, z_2, w), t), \end{cases} \tag{3.4}$$

and

$$\begin{cases} \mathcal{F}(f(x, y, z, aw_1 + bw_2) + f(x, y, z, aw_1 - bw_2) - a^2b^2(f(x, y, z, w_1 + w_2) + f(x, y, z, w_1 - w_2)) \\ - 2a^2(a^2 - b^2)f(x, y, z, w_1) + 2b^2(a^2 - b^2)f(x, y, z, w_2), t) \geq \mathcal{F}'(\varphi_4(x, y, z, w_1, w_2), t), \\ \mathcal{P}(f(x, y, z, aw_1 + bw_2) + f(x, y, z, aw_1 - bw_2) - a^2b^2(f(x, y, z, w_1 + w_2) + f(x, y, z, w_1 - w_2)) \\ - 2a^2(a^2 - b^2)f(x, y, z, w_1) + 2b^2(a^2 - b^2)f(x, y, z, w_2), t) \leq \mathcal{P}'(\varphi_4(x, y, z, w_1, w_2), t), \end{cases} \tag{3.5}$$

for all $x, y, z, w, x_1, x_2, y_1, y_2, z_1, z_2, w_1, w_2 \in X$ and all $t > 0$. Then there exists a unique dectic mapping $D: X \times X \times X \times X \rightarrow Y$ such that

$$\begin{cases} \mathcal{F}(f(x, y, z, w) - D(x, y, z, w), t) \geq M(x, y, z, w, (|\alpha| - |\alpha|^{10})t) \\ \mathcal{P}(f(x, y, z, w) - D(x, y, z, w), t) \leq N(x, y, z, w, (|\alpha| - |\alpha|^{10})t) \end{cases} \tag{3.6}$$

for all $x, y, z, w \in X$ and $t > 0$, where

$$\begin{cases} M(x, y, z, w, t) = \mathcal{F}'(\varphi_1(x, 0, y, z, w), |2||a|^{-9}t) * \mathcal{F}'(\varphi_2(ax, y, 0, z, w), |2||a|^{-7}t) \\ \quad * \mathcal{F}'(\varphi_3(ax, ay, z, 0, w), |2||a|^{-4}t) * \mathcal{F}'(\varphi_4(ax, ay, az, w, 0), |2|t) \\ N(x, y, z, w, t) = \mathcal{P}'(\varphi_1(x, 0, y, z, w), |2||a|^{-9}t) \diamond \mathcal{P}'(\varphi_2(ax, y, 0, z, w), |2||a|^{-7}t) \\ \quad \diamond \mathcal{P}'(\varphi_3(ax, ay, z, 0, w), |2||a|^{-4}t) \diamond \mathcal{P}'(\varphi_4(ax, ay, az, w, 0), |2|t). \end{cases} \quad (3.7)$$

Proof. Put $x_2 = 0$ and replace x_1, y, z, w by $2x, 2y, 2z, 2w$ respectively, in (3.2) we get

$$\begin{cases} \mathcal{F}(f(2ax, 2y, 2z, 2w) - af(2x, 2y, 2z, 2w), t) \geq \mathcal{F}'(\varphi_1(2x, 0, 2y, 2z, 2w), |2|t) \\ \mathcal{P}(f(2ax, 2y, 2z, 2w) - af(2x, 2y, 2z, 2w), t) \leq \mathcal{P}'(\varphi_1(2x, 0, 2y, 2z, 2w), |2|t), \end{cases} \quad (3.8)$$

for all $x, y, z, w \in X$ and $t > 0$. Put $y_2 = 0$ and replace x, y_1, z, w by $2ax, 2y, 2z, 2w$ respectively, in (3.3) we get

$$\begin{cases} \mathcal{F}(f(2ax, 2ay, 2z, 2w) - a^2f(2ax, 2y, 2z, 2w), t) \geq \mathcal{F}'(\varphi_2(2ax, 2y, 0, 2z, 2w), |2|t) \\ \mathcal{P}(f(2ax, 2ay, 2z, 2w) - a^2f(2ax, 2y, 2z, 2w), t) \leq \mathcal{P}'(\varphi_2(2ax, 2y, 0, 2z, 2w), |2|t), \end{cases} \quad (3.9)$$

for all $x, y, z, w \in X$ and $t > 0$. Put $z_2 = 0$ and replace x, y, z_1, w by $2ax, 2ay, 2z, 2w$ respectively, in (3.4) we get

$$\begin{cases} \mathcal{F}(f(2ax, 2ay, 2az, 2w) - a^3f(2ax, 2ay, 2z, 2w), t) \geq \mathcal{F}'(\varphi_3(2ax, 2ay, 2z, 0, 2w), |2|t) \\ \mathcal{P}(f(2ax, 2ay, 2az, 2w) - a^3f(2ax, 2ay, 2z, 2w), t) \leq \mathcal{P}'(\varphi_3(2a2ax, 2ay, 2z, 0, 2w), |2|t), \end{cases} \quad (3.10)$$

for all $x, y, z, w \in X$ and $t > 0$. Put $w_2 = 0$ and replace x, y, z, w_1 by $2ax, 2ay, 2az, 2w$ respectively, in (3.5) we get

$$\begin{cases} \mathcal{F}(f(2ax, 2ay, 2az, 2aw) - a^4f(2ax, 2ay, 2az, 2w), t) \geq \mathcal{F}'(\varphi_4(2ax, 2ay, 2az, 2w, 0), |2|t) \\ \mathcal{P}(f(2ax, 2ay, 2az, 2aw) - a^4f(2ax, 2ay, 2az, 2w), t) \leq \mathcal{P}'(\varphi_4(2ax, 2ay, 2az, 2w, 0), |2|t), \end{cases} \quad (3.11)$$

for all $x, y, z, w \in X$ and $t > 0$. Thus for all $x, y, z, w \in X$ and $t > 0$, we have

$$\begin{cases} \mathcal{F}(f(2ax, 2ay, 2az, 2aw) - a^{10}f(2x, 2y, 2z, 2w), t) \\ \quad \geq \mathcal{F}'(\varphi_1(2x, 0, 2y, 2z, 2w), |2||a|^{-9}t) * \mathcal{F}'(\varphi_2(2ax, 2y, 0, 2z, 2w), |2||a|^{-7}t) \\ \quad * \mathcal{F}'(\varphi_3(2ax, 2ay, 2z, 0, 2w), |2||a|^{-4}t) * \mathcal{F}'(\varphi_4(2ax, 2ay, 2az, 2w, 0), |2|t) \\ \mathcal{P}(f(2ax, 2ay, 2az, 2aw) - a^{10}f(2x, 2y, 2z, 2w), t) \\ \quad \geq \mathcal{P}'(\varphi_1(2x, 0, 2y, 2z, 2w), |2||a|^{-9}t) \diamond \mathcal{P}'(\varphi_2(2ax, 2y, 0, 2z, 2w), |2||a|^{-7}t) \\ \quad \diamond \mathcal{P}'(\varphi_3(2ax, 2ay, 2z, 0, 2w), |2||a|^{-4}t) \diamond \mathcal{P}'(\varphi_4(2ax, 2ay, 2az, 2w, 0), |2|t). \end{cases} \quad (3.12)$$

Replace x, y, z, w by $\frac{x}{2}, \frac{y}{2}, \frac{z}{2}, \frac{w}{2}$ respectively, in (3.12) and using (3.1) we obtain

$$\begin{cases} \mathcal{F}(f(ax, ay, az, aw) - a^{10}f(x, y, z, w), t) \geq M(x, y, z, w, t) \\ \mathcal{P}(f(ax, ay, az, aw) - a^{10}f(x, y, z, w), t) \leq N(x, y, z, w, t) \end{cases} \quad (3.13)$$

for all $x, y, z, w \in X$ and $t > 0$. Replace x, y, z, w by $\frac{x}{a^{k+1}}, \frac{y}{a^{k+1}}, \frac{z}{a^{k+1}}, \frac{w}{a^{k+1}}$ respectively, in (3.12) and using (3.1) we obtain

$$\begin{cases} \mathcal{F}\left(a^{10(k+1)}f\left(\frac{x}{a^{k+1}}, \frac{y}{a^{k+1}}, \frac{z}{a^{k+1}}, \frac{w}{a^{k+1}}\right) - a^{10k}f\left(\frac{x}{a^k}, \frac{y}{a^k}, \frac{z}{a^k}, \frac{w}{a^k}\right), |a|^{10k}t\right) \geq M(x, y, z, w, |a|^{k+1}t) \\ \mathcal{P}\left(a^{10(k+1)}f\left(\frac{x}{a^{k+1}}, \frac{y}{a^{k+1}}, \frac{z}{a^{k+1}}, \frac{w}{a^{k+1}}\right) - a^{10k}f\left(\frac{x}{a^k}, \frac{y}{a^k}, \frac{z}{a^k}, \frac{w}{a^k}\right), |a|^{10k}t\right) \leq N(x, y, z, w, |a|^{k+1}t), \end{cases}$$

for all $x, y, z, w \in X, t > 0$ and an integer $k \geq 0$. By replacing t by $\frac{t}{|a|^{k+1}}$, we get

$$\begin{cases} \mathcal{F}\left(a^{10(k+1)}f\left(\frac{x}{a^{k+1}}, \frac{y}{a^{k+1}}, \frac{z}{a^{k+1}}, \frac{w}{a^{k+1}}\right) - a^{10k}f\left(\frac{x}{a^k}, \frac{y}{a^k}, \frac{z}{a^k}, \frac{w}{a^k}\right), \frac{|a|^{10k}}{|a|^{k+1}}t\right) \geq M(x, y, z, w, t) \\ \mathcal{P}\left(a^{10(k+1)}f\left(\frac{x}{a^{k+1}}, \frac{y}{a^{k+1}}, \frac{z}{a^{k+1}}, \frac{w}{a^{k+1}}\right) - a^{10k}f\left(\frac{x}{a^k}, \frac{y}{a^k}, \frac{z}{a^k}, \frac{w}{a^k}\right), \frac{|a|^{10k}}{|a|^{k+1}}t\right) \leq N(x, y, z, w, t). \end{cases} \quad (3.14)$$

It follows from

$$\begin{aligned} a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right) - f(x, y, z, w) \\ = \sum_{k=0}^{n-1} \left(a^{10(k+1)}f\left(\frac{x}{a^{k+1}}, \frac{y}{a^{k+1}}, \frac{z}{a^{k+1}}, \frac{w}{a^{k+1}}\right) - a^{10k}f\left(\frac{x}{a^k}, \frac{y}{a^k}, \frac{z}{a^k}, \frac{w}{a^k}\right) \right) \end{aligned}$$

and (3.14) that

$$\begin{aligned} \mathcal{F}\left(a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right) - f(x, y, z, w), \sum_{k=0}^{n-1} \frac{|a|^{10k}}{|a|^{k+1}}t\right) \\ \geq \prod_{k=0}^{n-1} \mathcal{F}\left(a^{10(k+1)}f\left(\frac{x}{a^{k+1}}, \frac{y}{a^{k+1}}, \frac{z}{a^{k+1}}, \frac{w}{a^{k+1}}\right) - a^{10k}f\left(\frac{x}{a^k}, \frac{y}{a^k}, \frac{z}{a^k}, \frac{w}{a^k}\right), \frac{|a|^{10k}}{|a|^{k+1}}t\right) \\ \geq M(x, y, z, w, t) \end{aligned}$$

$$\begin{aligned} \mathcal{P}\left(a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right) - f(x, y, z, w), \sum_{k=0}^{n-1} \frac{|a|^{10k}}{|a|^{k+1}}t\right) \\ \leq \prod_{k=0}^{n-1} \mathcal{P}\left(a^{10(k+1)}f\left(\frac{x}{a^{k+1}}, \frac{y}{a^{k+1}}, \frac{z}{a^{k+1}}, \frac{w}{a^{k+1}}\right) - a^{10k}f\left(\frac{x}{a^k}, \frac{y}{a^k}, \frac{z}{a^k}, \frac{w}{a^k}\right), \frac{|a|^{10k}}{|a|^{k+1}}t\right) \\ \leq N(x, y, z, w, t), \end{aligned}$$

for all $x, y, z, w \in X, t > 0$ and $n > 0$, where $\prod_{k=1}^n \varepsilon_k = \varepsilon_1 * \varepsilon_2 * \dots * \varepsilon_n$ and $\prod_{k=1}^n \varepsilon_k = \varepsilon_1 \diamond \varepsilon_2 \diamond \dots \diamond \varepsilon_n$. Replacing x, y, z, w by $\frac{x}{a^k}, \frac{y}{a^k}, \frac{z}{a^k}, \frac{w}{a^k}$ respectively in the last inequalities, we have

$$\begin{aligned} & \mathcal{F} \left(a^{10(n+p)} f \left(\frac{x}{a^{n+p}}, \frac{y}{a^{n+p}}, \frac{z}{a^{n+p}}, \frac{w}{a^{n+p}} \right) - a^{10p} f \left(\frac{x}{a^p}, \frac{y}{a^p}, \frac{z}{a^p}, \frac{w}{a^p} \right), \sum_{k=0}^{n-1} \frac{|a|^{10(k+p)}}{|\alpha|^{k+1}} t \right) \\ & \geq M(x, y, z, w, |\alpha|^p t) \\ & \mathcal{P} \left(a^{10(n+p)} f \left(\frac{x}{a^{n+p}}, \frac{y}{a^{n+p}}, \frac{z}{a^{n+p}}, \frac{w}{a^{n+p}} \right) - a^{10p} f \left(\frac{x}{a^p}, \frac{y}{a^p}, \frac{z}{a^p}, \frac{w}{a^p} \right), \sum_{k=0}^{n-1} \frac{|a|^{10(k+p)}}{|\alpha|^{k+1}} t \right) \\ & \leq N(x, y, z, w, |\alpha|^p t). \end{aligned}$$

Whence

$$\begin{aligned} & \mathcal{F} \left(a^{10(n+p)} f \left(\frac{x}{a^{n+p}}, \frac{y}{a^{n+p}}, \frac{z}{a^{n+p}}, \frac{w}{a^{n+p}} \right) - a^{10p} f \left(\frac{x}{a^p}, \frac{y}{a^p}, \frac{z}{a^p}, \frac{w}{a^p} \right), \sum_{k=0}^{n-1} \frac{|a|^{10(k+p)}}{|\alpha|^{k+p+1}} t \right) \\ & \geq M(x, y, z, w, t) \\ & \mathcal{P} \left(a^{10(n+p)} f \left(\frac{x}{a^{n+p}}, \frac{y}{a^{n+p}}, \frac{z}{a^{n+p}}, \frac{w}{a^{n+p}} \right) - a^{10p} f \left(\frac{x}{a^p}, \frac{y}{a^p}, \frac{z}{a^p}, \frac{w}{a^p} \right), \sum_{k=0}^{n-1} \frac{|a|^{10(k+p)}}{|\alpha|^{k+p+1}} t \right) \\ & \leq N(x, y, z, w, t) \end{aligned}$$

for all $x, y, z, w \in X, t > 0, n > 0$ and $p \geq 0$. Hence,

$$\left\{ \begin{aligned} & \mathcal{F} \left(a^{10(n+p)} f \left(\frac{x}{a^{n+p}}, \frac{y}{a^{n+p}}, \frac{z}{a^{n+p}}, \frac{w}{a^{n+p}} \right) - f \left(\frac{x}{a^p}, \frac{y}{a^p}, \frac{z}{a^p}, \frac{w}{a^p} \right), t \right) \\ & \geq M \left(x, y, z, w, \frac{t}{\sum_{k=p}^{n+p-1} \frac{|a|^{10k}}{|\alpha|^{k+1}}} \right) \\ & \mathcal{P} \left(a^{10(n+p)} f \left(\frac{x}{a^{n+p}}, \frac{y}{a^{n+p}}, \frac{z}{a^{n+p}}, \frac{w}{a^{n+p}} \right) - f \left(\frac{x}{a^p}, \frac{y}{a^p}, \frac{z}{a^p}, \frac{w}{a^p} \right), t \right) \\ & \leq N \left(x, y, z, w, \frac{t}{\sum_{k=p}^{n+p-1} \frac{|a|^{10k}}{|\alpha|^{k+1}}} \right). \end{aligned} \right. \tag{3.15}$$

Since $|\alpha| > |a|^{10}$, we have $\sum_{k=0}^{\infty} \frac{|a|^{10k}}{|\alpha|^{k+1}} < \infty$. This shows that $\left(a^{10n} f \left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n} \right) \right)$ is a Cauchy sequence in non-Archimedean intuitionistic fuzzy Banach space $(Y, \mathcal{F}, \mathcal{P})$ and so it converges to some point $D(x, y, z, w) \in Y$. Thus, we define a mapping $D: X \times X \times X \times X \rightarrow Y$ such that

$$D(x, y, z, w) = (\mathcal{F}, \mathcal{P}) - \lim_{n \rightarrow \infty} a^{10n} f \left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n} \right).$$

Hence for all $x, y, z, w \in X$ and $t > 0$, we have

$$\lim_{n \rightarrow \infty} \mathcal{F} \left(a^{10n} f \left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n} \right) - D(x, y, z, w) \right) = 1$$

$$\lim_{n \rightarrow \infty} \mathcal{P} \left(a^{10n} f \left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n} \right) - D(x, y, z, w) \right) = 0.$$

Moreover, if we put $p = 0$ in (3.15), we get

$$\mathcal{F}\left(a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right) - f(x, y, z, w), t\right) \geq M\left(x, y, z, w, \frac{t}{\sum_{k=0}^{n-1} \frac{|a|^{10k}}{|\alpha|^{k+1}}}\right)$$

$$\mathcal{P}\left(a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right) - f(x, y, z, w), t\right) \leq N\left(x, y, z, w, \frac{t}{\sum_{k=0}^{n-1} \frac{|a|^{10k}}{|\alpha|^{k+1}}}\right)$$

for all $x, y, z, w \in X, t > 0$ and $n > 0$. Therefore,

$$\mathcal{F}(D(x, y, z, w) - f(x, y, z, w), t) \geq \mathcal{F}\left(D(x, y, z, w) - a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right), t\right)$$

$$* \mathcal{F}\left(a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right) - f(x, y, z, w), t\right) \geq M\left(x, y, z, w, \frac{t}{\sum_{k=0}^{n-1} \frac{|a|^{10k}}{|\alpha|^{k+1}}}\right)$$

$$\mathcal{P}(D(x, y, z, w) - f(x, y, z, w), t) \geq \mathcal{P}\left(D(x, y, z, w) - a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right), t\right)$$

$$\diamond \mathcal{P}\left(a^{10n}f\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right) - f(x, y, z, w), t\right) \leq N\left(x, y, z, w, \frac{t}{\sum_{k=0}^{n-1} \frac{|a|^{10k}}{|\alpha|^{k+1}}}\right)$$

for all $x, y, z, w \in X, t > 0$ and $n > 0$. Letting $n \rightarrow \infty$ in the above inequalities, we

$$\mathcal{F}(D(x, y, z, w) - f(x, y, z, w), t) \geq M(x, y, z, w, (|\alpha| - |a|^{10})t)$$

$$\mathcal{P}(D(x, y, z, w) - f(x, y, z, w), t) \geq M(x, y, z, w, (|\alpha| - |a|^{10})t).$$

Hence, D satisfies (3.6). Let $x, y, z, w, x_1, x_2, y_1, y_2, z_1, z_2, w_1, w_2 \in X$, then

$$\left\{ \begin{array}{l} \mathcal{F}(D(ax_1 + bx_2, y, z, w) + D(ax_1 - bx_2, y, z, w) - 2aD(x_1, y, z, w), t) \\ \geq \mathcal{F}'\left(\varphi_1(x_1, x_2, y, z, w), \frac{|\alpha|^n}{|\alpha|^{10n}} t\right) \\ \mathcal{P}(D(ax_1 + bx_2, y, z, w) + D(ax_1 - bx_2, y, z, w) - 2aD(x_1, y, z, w), t) \\ \leq \mathcal{P}'\left(\varphi_1(x_1, x_2, y, z, w), \frac{|\alpha|^n}{|\alpha|^{10n}} t\right), \end{array} \right. \tag{3.16}$$

$$\left\{ \begin{array}{l} \mathcal{F}(D(x, ay_1 + by_2, z, w) + D(x, ay_1 - by_2, z, w) - 2a^2 D(x, y_1, z, w) - 2b^2 D(x, y_2, z, w), t) \\ \geq \mathcal{F}'\left(\varphi_2(x, y_1, y_2, z, w), \frac{|\alpha|^n}{|\alpha|^{10n}} t\right) \\ \mathcal{P}(D(x, ay_1 + by_2, z, w) + D(x, ay_1 - by_2, z, w) - 2a^2 D(x, y_1, z, w) - 2b^2 D(x, y_2, z, w), t) \\ \leq \mathcal{P}'\left(\varphi_2(x, y_1, y_2, z, w), \frac{|\alpha|^n}{|\alpha|^{10n}} t\right), \end{array} \right. \tag{3.17}$$

$$\begin{cases} \mathcal{F}(D(x, y, az_1 + bz_2, w) + D(x, y, az_1 - bz_2, w) - ab^2(D(x, y, z_1 + z_2, w) + D(x, y, z_1 - z_2, w)) \\ -2a(a^2 - b^2)D(x, y, z_1, w), t) \geq \mathcal{F}'\left(\varphi_3(x, y, z_1, z_2, w), \frac{|\alpha|^n}{|\alpha|^{10n}} t\right), \\ \mathcal{P}(D(x, y, az_1 + bz_2, w) + D(x, y, az_1 - bz_2, w) - ab^2(D(x, y, z_1 + z_2, w) + D(x, y, z_1 - z_2, w)) \\ -2a(a^2 - b^2)D(x, y, z_1, w), t) \leq \mathcal{P}'\left(\varphi_3(x, y, z_1, z_2, w), \frac{|\alpha|^n}{|\alpha|^{10n}} t\right), \end{cases} \quad (3.18)$$

and

$$\begin{cases} \mathcal{F}(D(x, y, z, aw_1 + bw_2) + D(x, y, z, aw_1 - bw_2) - a^2b^2(D(x, y, z, w_1 + w_2) + D(x, y, z, w_1 - w_2)) \\ -2a^2(a^2 - b^2)D(x, y, z, w_1) + 2b^2(a^2 - b^2)D(x, y, z, w_2), t) \geq \mathcal{F}'\left(\varphi_4(x, y, z, w_1, w_2), \frac{|\alpha|^n}{|\alpha|^{10n}} t\right), \\ \mathcal{P}(D(x, y, z, aw_1 + bw_2) + D(x, y, z, aw_1 - bw_2) - a^2b^2(D(x, y, z, w_1 + w_2) + D(x, y, z, w_1 - w_2)) \\ -2a^2(a^2 - b^2)D(x, y, z, w_1) + 2b^2(a^2 - b^2)D(x, y, z, w_2), t) \leq \mathcal{P}'\left(\varphi_4(x, y, z, w_1, w_2), \frac{|\alpha|^n}{|\alpha|^{10n}} t\right). \end{cases} \quad (3.19)$$

Letting $n \rightarrow \infty$ in (3.16), (3.17), (3.18) and (3.19), we get

$$\mathcal{F}(D(ax_1 + bx_2, y, z, w) + D(ax_1 - bx_2, y, z, w) - 2aD(x_1, y, z, w), t) = 1,$$

$$\mathcal{F}(D(x, ay_1 + by_2, z, w) + D(x, ay_1 - by_2, z, w) - 2a^2 D(x, y_1, z, w) - 2b^2 D(x, y_2, z, w), t) = 1,$$

$$\mathcal{F}(D(x, y, az_1 + bz_2, w) + D(x, y, az_1 - bz_2, w) - ab^2(D(x, y, z_1 + z_2, w) + D(x, y, z_1 - z_2, w)) \\ -2a(a^2 - b^2)D(x, y, z_1, w), t) = 1,$$

$$\mathcal{F}(D(x, y, z, aw_1 + bw_2) + D(x, y, z, aw_1 - bw_2) - a^2b^2(D(x, y, z, w_1 + w_2) + D(x, y, z, w_1 - w_2)) \\ -2a^2(a^2 - b^2)D(x, y, z, w_1) + 2b^2(a^2 - b^2)D(x, y, z, w_2), t) = 1,$$

and similarly, we obtain

$$\mathcal{P}(D(ax_1 + bx_2, y, z, w) + D(ax_1 - bx_2, y, z, w) - 2aD(x_1, y, z, w), t) = 0,$$

$$\mathcal{P}(D(x, ay_1 + by_2, z, w) + D(x, ay_1 - by_2, z, w) - 2a^2 D(x, y_1, z, w) - 2b^2 D(x, y_2, z, w), t) = 0,$$

$$\mathcal{P}(D(x, y, az_1 + bz_2, w) + D(x, y, az_1 - bz_2, w) - ab^2(D(x, y, z_1 + z_2, w) + D(x, y, z_1 - z_2, w)) \\ -2a(a^2 - b^2)D(x, y, z_1, w), t) = 0,$$

$$\mathcal{P}(D(x, y, z, aw_1 + bw_2) + D(x, y, z, aw_1 - bw_2) - a^2b^2(D(x, y, z, w_1 + w_2) + D(x, y, z, w_1 - w_2)) \\ -2a^2(a^2 - b^2)D(x, y, z, w_1) + 2b^2(a^2 - b^2)D(x, y, z, w_2), t) = 0,$$

for all $x, y, z, w, x_1, x_2, y_1, y_2, z_1, z_2, w_1, w_2 \in X$ and $t > 0$. This means that D satisfies (1.1) and so it is dectic mapping. To prove the uniqueness of D , assume that D' be another dectic mapping from $X \times X \times X \times X$ into Y , which satisfies (3.6). For all $x, y, z, w \in X$, all $n > 0$, clearly $D(x, y, z, w) = a^{10n} D\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right)$ and $D'(x, y, z, w) = a^{10n} D'\left(\frac{x}{a^n}, \frac{y}{a^n}, \frac{z}{a^n}, \frac{w}{a^n}\right)$. It follows from (3.6) that

$$\begin{cases} \mathcal{F}(D(x, y, z, w) - D'(x, y, z, w), t) \geq M\left(x, y, z, w, \frac{|\alpha|^n(|\alpha| - |\alpha|^{10})}{|\alpha|^{10n}} t\right) \\ \mathcal{P}(D(x, y, z, w) - D'(x, y, z, w), t) \leq N\left(x, y, z, w, \frac{|\alpha|^n(|\alpha| - |\alpha|^{10})}{|\alpha|^{10n}} t\right). \end{cases} \quad (3.20)$$

We see that the right-hand sides of (3.20) tending to 1 and 0, respectively, as $n \rightarrow \infty$. Therefore, $\mathcal{F}(D(x, y, z, w) - D'(x, y, z, w), t) = 1$ and $\mathcal{P}(D(x, y, z, w) - D'(x, y, z, w), t) = 0$, for all $x, y, z, w \in X$ and $t > 0$. Hence, $D(x, y, z, w) = D'(x, y, z, w)$. \square

Theorem 3.1. Let X be a linear space over a non-Archimedean field K and let $(Z, \mathcal{F}', \mathcal{P}')$ be a non-Archimedean IFN space. Suppose that $\varphi_i: X \times X \times X \times X \rightarrow Z$ for $i \in I$ be functions such that $|\alpha| < 1$ is far from zero and for some real number α with $0 < |\alpha| < |\alpha|^{10}$,

$$\begin{cases} \mathcal{F}'(\varphi_i(ax, ay, az, aw), t) \geq \mathcal{F}'(|\alpha|\varphi_i(x, y, z, w, v), t) \\ \mathcal{P}'(\varphi_i(ax, ay, az, aw), t) \leq \mathcal{P}'(|\alpha|\varphi_i(x, y, z, w, v), t) \end{cases}$$

for all $x, y, z, w, v \in X, t > 0$ and $i \in I$. Let f be a mapping from $X \times X \times X \times X$ to a non-Archimedean intuitionistic fuzzy Banach space $(Y, \mathcal{F}, \mathcal{P})$ such that $f(x, 0, z, w) = 0, f(x, y, z, 0) = 0$ for all $x, y, z, w \in X$, and satisfies (3.2), (3.3), (3.4) and (3.5). Then there exists a unique dectic mapping $D: X \times X \times X \times X \rightarrow Y$ such that

$$\begin{cases} \mathcal{F}(f(x, y, z, w) - D(x, y, z, w), t) \geq M(x, y, z, w, (|\alpha|^{10} - |\alpha|)t) \\ \mathcal{P}(f(x, y, z, w) - D(x, y, z, w), t) \leq N(x, y, z, w, (|\alpha|^{10} - |\alpha|)t) \end{cases}$$

for all $x, y, z, w \in X$ and $t > 0$, where $M(x, y, z, w, t)$ and $N(x, y, z, w, t)$ are defined as in Theorem 3.1.

Proof. The proof can be done by the same way as in Theorem 3.1. \square

4. Non-Archimedean intuitionistic fuzzy continuity

Recently, the intuitionistic fuzzy continuity is discussed in [19]. In this section, we establish non-Archimedean intuitionistic fuzzy continuity of continuous mappings satisfying (1.1) approximately.

Definition 4.1. Let $g: \mathbb{R} \rightarrow X$ be a mapping, where \mathbb{R} is endowed with the Euclidean topology and X is a non-Archimedean intuitionistic fuzzy normed space equipped with non-Archimedean intuitionistic fuzzy norm $(\mathcal{F}, \mathcal{P})$. Then, g is called non-Archimedean intuitionistic fuzzy continuous at a points $r_0 \in \mathbb{R}$ if for all $\epsilon > 0$ and $0 < \rho < 1$ there exists some $\delta > 0$ such that for each r with $0 < |r - r_0| < \delta$,

$$\mathcal{F}(g(rx) - g(r_0x), \epsilon) \geq \rho, \quad \mathcal{P}(g(rx) - g(r_0x), \epsilon) \leq 1 - \rho.$$

Theorem 4.2. Let X be a non-Archimedean space and let $(Z, \mathcal{F}', \mathcal{P}')$ be a non-Archimedean IFN space. Let $p > 10$ and $z_0 \in Z$. Let f be a mapping from $X \times X \times X \times X$ to a non-Archimedean intuitionistic fuzzy Banach space $(Y, \mathcal{F}, \mathcal{P})$ such that $f(x, 0, z, w) = 0, f(x, y, z, 0) = 0$ for all $x, y, z, w \in X$, and satisfying

$$\left\{ \begin{array}{l} \mathcal{F}(f(ax_1 + bx_2, y, z, w) + f(ax_1 - bx_2, y, z, w) - 2af(x_1, y, z, w), t) \\ \geq \mathcal{F}'((\|x_1\|^p + \|x_2\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, t), \\ \mathcal{P}(f(ax_1 + bx_2, y, z, w) + f(ax_1 - bx_2, y, z, w) - 2af(x_1, y, z, w), t) \\ \leq \mathcal{P}'((\|x_1\|^p + \|x_2\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, t), \end{array} \right. \quad (4.1)$$

$$\left\{ \begin{array}{l} \mathcal{F}(f(x, ay_1 + by_2, z, w) + f(x, ay_1 - by_2, z, w) - 2a^2 f(x, y_1, z, w) - 2b^2 f(x, y_2, z, w), t) \\ \geq \mathcal{F}'((\|x\|^p + \|y_1\|^p + \|y_2\|^p + \|z\|^p + \|w\|^p)z_0, t), \\ \mathcal{P}(f(x, ay_1 + by_2, z, w) + f(x, ay_1 - by_2, z, w) - 2a^2 f(x, y_1, z, w) - 2b^2 f(x, y_2, z, w), t) \\ \leq \mathcal{P}'((\|x\|^p + \|y_1\|^p + \|y_2\|^p + \|z\|^p + \|w\|^p)z_0, t), \end{array} \right. \quad (4.2)$$

$$\left\{ \begin{array}{l} \mathcal{F}(f(x, y, az_1 + bz_2, w) + f(x, y, az_1 - bz_2, w) - ab^2(f(x, y, z_1 + z_2, w) + f(x, y, z_1 - z_2, w)) \\ - 2a(a^2 - b^2)f(x, y, z_1, w), t) \geq \mathcal{F}'((\|x\|^p + \|y\|^p + \|z_1\|^p + \|z_2\|^p + \|w\|^p)z_0, t) \\ \mathcal{P}(f(x, y, az_1 + bz_2, w) + f(x, y, az_1 - bz_2, w) - ab^2(f(x, y, z_1 + z_2, w) + f(x, y, z_1 - z_2, w)) \\ - 2a(a^2 - b^2)f(x, y, z_1, w), t) \leq \mathcal{P}'((\|x\|^p + \|y\|^p + \|z_1\|^p + \|z_2\|^p + \|w\|^p)z_0, t), \end{array} \right. \quad (4.3)$$

and

$$\left\{ \begin{array}{l} \mathcal{F}(f(x, y, z, aw_1 + bw_2) + f(x, y, z, aw_1 - bw_2) - a^2b^2(f(x, y, z, w_1 + w_2) + f(x, y, z, w_1 - w_2)) \\ - 2a^2(a^2 - b^2)f(x, y, z, w_1) + 2b^2(a^2 - b^2)f(x, y, z, w_2), t) \\ \geq \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w_1\|^p + \|w_2\|^p)z_0, t) \\ \mathcal{P}(f(x, y, z, aw_1 + bw_2) + f(x, y, z, aw_1 - bw_2) - a^2b^2(f(x, y, z, w_1 + w_2) + f(x, y, z, w_1 - w_2)) \\ - 2a^2(a^2 - b^2)f(x, y, z, w_1) + 2b^2(a^2 - b^2)f(x, y, z, w_2), t) \\ \leq \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w_1\|^p + \|w_2\|^p)z_0, t) \end{array} \right. \quad (4.4)$$

for all $x, y, z, w, x_1, x_2, y_1, y_2, z_1, z_2, w_1, w_2 \in X$ and all $t > 0$. Then there exists a unique dectic mapping $D: X \times X \times X \times X \rightarrow Y$ such that

$$\left\{ \begin{array}{l} \mathcal{F}(f(x, y, z, w) - D(x, y, z, w), t) \\ \geq \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-9}(|a|^{10} - |a|^p)t) \\ * \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-7}(|a|^{10} - |a|^p)t) \\ * \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-4}(|a|^{10} - |a|^p)t) \\ * \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2|(|a|^{10} - |a|^p)t) \\ \mathcal{P}(f(x, y, z, w) - D(x, y, z, w), t) \\ \leq \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-9}(|a|^{10} - |a|^p)t) \\ \diamond \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-7}(|a|^{10} - |a|^p)t) \\ \diamond \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-4}(|a|^{10} - |a|^p)t) \\ \diamond \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2|(|a|^{10} - |a|^p)t) \end{array} \right. \quad (4.5)$$

for all $x, y, z, w \in X$ and all $t > 0$. Furthermore, if the mapping $g: \mathbb{R} \rightarrow Y$ defined by $g(r) := \frac{f(a^n rx, a^n ry, a^n rz, a^n rw)}{a^{10n}}$ is non-Archimedean intuitionistic fuzzy continuous for some $x, y, z, w \in X$ and all $n \in \mathbb{N}$ then the mapping $r \rightarrow D(rx, ry, rz, rw)$ from \mathbb{R} to Y is non-Archimedean intuitionistic fuzzy continuous.

Proof. Define $\varphi_i: X \times X \times X \times X \rightarrow Z$ by $\varphi_i(x, y, z, w) = (\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p + \|v\|^p)z_0$ for all $x, y, z, w, v \in X$ and all $i \in I$. Existence and uniqueness of the dectic mapping D satisfying (4.5) are deduced from Theorem 3.2. Note that, for all $x, y, z, w \in X$, all $t > 0$ and $n \in \mathbb{N}$ we have

$$\left\{ \begin{array}{l} \mathcal{F} \left(D(x, y, z, w) - \frac{f(a^n x, a^n y, a^n z, a^n w)}{a^{10n}}, t \right) \\ \geq \mathcal{F}' \left((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, \frac{|2||a|^{-9}|a|^{10n}(|a|^{10} - |a|^p)t}{|a|^{np}} \right) \\ * \mathcal{F}' \left((\|ax\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, \frac{|2||a|^{-7}|a|^{10n}(|a|^{10} - |a|^p)t}{|a|^{np}} \right) \\ * \mathcal{F}' \left((\|ax\|^p + \|ay\|^p + \|z\|^p + \|w\|^p)z_0, \frac{|2||a|^{-4}|a|^{10n}(|a|^{10} - |a|^p)t}{|a|^{np}} \right) \\ * \mathcal{F}' \left((\|ax\|^p + \|ay\|^p + \|az\|^p + \|w\|^p)z_0, \frac{|2||a|^{10n}(|a|^{10} - |a|^p)t}{|a|^{np}} \right) \\ \mathcal{P} \left(f(x, y, z, w) - \frac{f(a^n x, a^n y, a^n z, a^n w)}{a^{10n}}, t \right) \\ \leq \mathcal{P}' \left((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, \frac{|2||a|^{-9}|a|^{10n}(|a|^{10} - |a|^p)t}{|a|^{np}} \right) \\ \diamond \mathcal{P}' \left((\|ax\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, \frac{|2||a|^{-7}|a|^{10n}(|a|^{10} - |a|^p)t}{|a|^{np}} \right) \\ \diamond \mathcal{P}' \left((\|ax\|^p + \|ay\|^p + \|z\|^p + \|w\|^p)z_0, \frac{|2||a|^{-4}|a|^{10n}(|a|^{10} - |a|^p)t}{|a|^{np}} \right) \\ \diamond \mathcal{P}' \left((\|ax\|^p + \|ay\|^p + \|az\|^p + \|w\|^p)z_0, \frac{|2||a|^{10n}(|a|^{10} - |a|^p)t}{|a|^{np}} \right). \end{array} \right. \quad (4.6)$$

Fix $x, y, z, w, v \in X$ and $r_0 \in \mathbb{R}$. Given $\epsilon > 0$ and $0 < \rho < 1$. From (4.6) it is follows that

$$\left\{ \begin{aligned}
 & \mathcal{F} \left(D(rx, ry, rz, rw) - \frac{f(a^n rx, a^n ry, a^n rz, a^n rw)}{a^{10n}}, t \right) \\
 & \geq \mathcal{F}' \left((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p) z_0, \frac{|2||a|^{-9}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} \right) \\
 & * \mathcal{F}' \left((\|ax\|^p + \|y\|^p + \|z\|^p + \|w\|^p) z_0, \frac{|2||a|^{-7}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} \right) \\
 & * \mathcal{F}' \left((\|ax\|^p + \|ay\|^p + \|z\|^p + \|w\|^p) z_0, \frac{|2||a|^{-4}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} \right) \\
 & * \mathcal{F}' \left((\|ax\|^p + \|ay\|^p + \|az\|^p + \|w\|^p) z_0, \frac{|2||a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} \right) \\
 & \mathcal{P} \left(D(rx, ry, rz, rw) - \frac{f(a^n rx, a^n ry, a^n rz, a^n rw)}{a^{10n}}, t \right) \\
 & \leq \mathcal{P}' \left((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p) z_0, \frac{|2||a|^{-9}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} \right) \\
 & \diamond \mathcal{P}' \left((\|ax\|^p + \|y\|^p + \|z\|^p + \|w\|^p) z_0, \frac{|2||a|^{-7}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} \right) \\
 & \diamond \mathcal{P}' \left((\|ax\|^p + \|ay\|^p + \|z\|^p + \|w\|^p) z_0, \frac{|2||a|^{-4}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} \right) \\
 & \diamond \mathcal{P}' \left((\|ax\|^p + \|ay\|^p + \|az\|^p + \|w\|^p) z_0, \frac{|2||a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} \right).
 \end{aligned} \right. \tag{4.7}$$

For all $|r - r_0| < 1$ and $r \in \mathbb{R}$. Since

$$\lim_{n \rightarrow \infty} \frac{|2||a|^{-9}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} = \infty,$$

$$\lim_{n \rightarrow \infty} \frac{|2||a|^{-7}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} = \infty,$$

$$\lim_{n \rightarrow \infty} \frac{|2||a|^{-4}|a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} = \infty,$$

$$\lim_{n \rightarrow \infty} \frac{|2||a|^{10n}(|a|^{10} - |a|^p)t}{(1+r_0)^p|a|^{np}} = \infty.$$

Then there exists $n_0 \in \mathbb{N}$ such that

$$\left\{ \begin{aligned}
 & \mathcal{F} \left(D(rx, ry, rz, rw) - \frac{f(a^{n_0} rx, a^{n_0} ry, a^{n_0} rz, a^{n_0} rw)}{a^{10n_0}}, \epsilon \right) \geq \rho \\
 & \mathcal{P} \left(D(rx, ry, rz, rw) - \frac{f(a^{n_0} rx, a^{n_0} ry, a^{n_0} rz, a^{n_0} rw)}{a^{10n_0}}, \epsilon \right) \leq 1 - \rho
 \end{aligned} \right.$$

For all $|r - r_0| < 1$ and $r \in \mathbb{R}$. By the non-Archimedean intuitionistic fuzzy continuity of the mapping

$$r \rightarrow \frac{f(a^{n_0} rx, a^{n_0} ry, a^{n_0} rz, a^{n_0} rw)}{a^{10n_0}}, \text{ there exists } \delta < 1 \text{ such that for each } r \text{ with } |r - r_0| < \delta \text{ we have}$$

$$\begin{cases} \mathcal{F}\left(\frac{f(a^n rx, a^n ry, a^n rz, a^n rw)}{a^{10n}} - \frac{f(a^{n_0} r_0 x, a^{n_0} r_0 y, a^{n_0} r_0 z, a^{n_0} r_0 w)}{a^{10n_0}}, \epsilon\right) \geq \rho \\ \mathcal{P}\left(\frac{f(a^n rx, a^n ry, a^n rz, a^n rw)}{a^{10n}} - \frac{f(a^{n_0} r_0 x, a^{n_0} r_0 y, a^{n_0} r_0 z, a^{n_0} r_0 w)}{a^{10n_0}}, \epsilon\right) \leq 1 - \rho. \end{cases}$$

It follows that

$$\begin{cases} \mathcal{F}(D(rx, ry, rz, rw) - D(r_0x, r_0y, r_0z, r_0w), \epsilon) \geq \rho \\ \mathcal{P}(D(rx, ry, rz, rw) - D(r_0x, r_0y, r_0z, r_0w), \epsilon) \leq 1 - \rho \end{cases}$$

for each r with $|r - r_0| < \delta$. Hence, the mapping $r \rightarrow D(rx, ry, rz, rw)$ is non-Archimedean intuitionistic fuzzy continuous. \square

Theorem 4.3. Let X be a non-Archimedean space and let $(Z, \mathcal{F}', \mathcal{P}')$ be a non-Archimedean IFN space. Let $p < 10$ and $z_0 \in Z$. Let f be a mapping from $X \times X \times X \times X$ to a non-Archimedean intuitionistic fuzzy Banach space $(Y, \mathcal{F}, \mathcal{P})$ such that $f(x, 0, z, w) = 0, f(x, y, z, 0) = 0$ for all $x, y, z, w \in X$, and satisfying (4.1), (4.2), (4.3), (4.4). Then there exists a unique dectic mapping $D: X \times X \times X \times X \rightarrow Y$ such that

$$\begin{cases} \mathcal{F}(f(x, y, z, w) - D(x, y, z, w), t) \\ \geq \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-9}(|a|^p - |a|^{10})t) \\ * \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-7}(|a|^p - |a|^{10})t) \\ * \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-4}(|a|^p - |a|^{10})t) \\ * \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2|(|a|^p - |a|^{10})t) \\ \mathcal{P}(f(x, y, z, w) - D(x, y, z, w), t) \\ \leq \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-9}(|a|^p - |a|^{10})t) \\ \diamond \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-7}(|a|^p - |a|^{10})t) \\ \diamond \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2||a|^{-4}(|a|^p - |a|^{10})t) \\ \diamond \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)z_0, |2|(|a|^p - |a|^{10})t) \end{cases} \tag{4.8}$$

for all $x, y, z, w \in X$ and all $t > 0$. Furthermore, if the mapping $g: \mathbb{R} \rightarrow Y$ defined by $g(r) := a^{10n} f(\frac{rx}{a^n}, \frac{ry}{a^n}, \frac{rz}{a^n}, \frac{rw}{a^n})$ is non-Archimedean intuitionistic fuzzy continuous for some $x, y, z, w \in X$ and all $n \in \mathbb{N}$ then the mapping $r \rightarrow D(rx, ry, rz, rw)$ from \mathbb{R} to Y is non-Archimedean intuitionistic fuzzy continuous.

Proof. Define $\varphi_i: X \times X \times X \times X \rightarrow Z$ by $\varphi_i(x, y, z, w) = (\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p + \|v\|^p)z_0$ for all $x, y, z, w, v \in X$ and all $i \in I$. Existence and uniqueness of the dectic mapping D satisfying (4.5) are deduced from Theorem 3.2. Note that, for all $x, y, z, w \in X$, all $t > 0, n \in \mathbb{N}$ and all $i \in I$, we have

$$\mathcal{F}'\left(\varphi_i\left(\frac{x}{a}, \frac{y}{a}, \frac{z}{a}, \frac{w}{a}, \frac{v}{a}\right), t\right) = \mathcal{F}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p + \|v\|^p)z_0, |a|^p t),$$

$$\mathcal{P}'\left(\varphi_i\left(\frac{x}{a}, \frac{y}{a}, \frac{z}{a}, \frac{w}{a}, \frac{v}{a}\right), t\right) = \mathcal{P}'((\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p + \|v\|^p)z_0, |a|^p t).$$

Since $p < 10$, we have

$|a| = |a|^p > |a|^p$. By Theorem 3.1 there exists a unique dectic mapping D which satisfies (4.8). The rest of the proof can be done by the same way as in Theorem 4.1. \square

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