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Compact operators in Felbin's type fuzzy normed linear spaces

Ashraf Zohourmeskar

Department of basic science, Jundi-shapur university of Technology, Dezful, Iran, first University.

zohourmeskar@yahoo.com

Abstract

In this paper, we introduce fuzzy compact operators on Felbin's type fuzzy normed linear spaces. In particular, it is proved that an equivalent condition for the compactness of an operator in fuzzy normed linear spaces.

Key words and phrases: Fuzzy real number, Fuzzy norm linear space, Weakly fuzzy continuous, Fuzzy compact operator.

1 Introduction

In [4], Keleva and Seikkala introduced the concept of a fuzzy metric space and studied some of its properties. In [3], Felbin defined the fuzzy normed spaces and investigated some properties fuzzy normed space. With this definition of fuzzy normed linear space, it has been possible to introduce in this paper a notion of weakly fuzzy continuous operator over fuzzy normed linear spaces and to define "compact" for such an operator. The organization of this paper is as follows: section 2, comprises some useful definitions, notations and preliminary result. In section 3, the definition compact of the operator on fuzzy normed linear spaces is introduced and investing some important example of compact operator on fuzzy normed linear spaces.

2 Preliminaries

We denote the set of all real number by $\mathbb{R}$ and by $\mathbb{Z}^+$ the set of all positive integers.

In this paper, we consider the concept of fuzzy real number in the sense Xiao and Zhu [7] which is defined below:

A mapping $\eta(t): \mathbb{R} \longrightarrow [0,1]$, whose α-level set is denoted by $[\eta]\alpha = \{t: \eta(t) \geq \alpha\}$, is called a fuzzy real number if it satisfies two axioms:

(N1) There exists $t_0 \in \mathbb{R}$ such that $\eta(t_0) = 1$;

(N2) For each $\alpha \in (0,1]$; $[\eta]_\alpha = [\eta_\alpha^-, \eta_\alpha^+], where -\infty < \eta_\alpha^- \leq \eta_\alpha^+ < +\infty.$

The set of all fuzzy real number is denoted by $\mathbb{F}$. For each $r \in \mathbb{R}$, let $\bar{r} \in \mathbb{F}$ be defined by

$$\bar{r}(t) = \begin{cases} 1 & t = r, \\ 0 & t \neq r. \end{cases}$$

So $\bar{r}$ is a fuzzy real number and $\mathbb{R}$ can be embedded in $\mathbb{F}$. Let $\eta \in \mathbb{F}$, is called positive fuzzy real number if for all $t < 0, \eta(t) = 0$. The set of all positive fuzzy real numbers is denoted by $\mathbb{F}^+$.

A partial order "$\preccurlyeq$" in $\mathbb{F}$ is defined as follows, $\eta \preccurlyeq \delta$ if and only if for all $\alpha \in (0,1]$; $\eta_\alpha^- \leq \delta_\alpha^-$ and $\eta_\alpha^+ \leq \delta_\alpha^+$ where, $[\eta]_\alpha = [\eta_\alpha^-, \eta_\alpha^+]$, and $[\delta]_\alpha = [\delta_\alpha^-\ , \delta_\alpha^+]$.The strict inequality in $\mathbb{F}$ is defined by $\eta \prec \delta$ if and only if for all $\alpha \in (0,1]$; $\eta_\alpha^- < \delta_\alpha^-$and $\eta_\alpha^+ < \delta_\alpha^+$. According to Dubois and prade [2], the arithmetic operations $\oplus$,$\ominus$,$\odot$,$\oslash$ on $\mathbb{F} \times \mathbb{F}$ are defined by

$$(\eta \oplus \delta)(t) = sup_{t=r+s} min\{\eta(r), \delta(s)\}, \qquad t \in \mathbb{R}$$

$$(\eta \ominus \delta)(t) = sup_{t=r-s} min\{\eta(r), \delta(s)\}, \qquad t \in \mathbb{R}$$

$$(\eta \odot \delta)(t) = sup_{t=rs} min\{\eta(r), \delta(s)\}, \qquad t \in \mathbb{R}$$

$$(\eta \oslash \delta)(t) = sup_{t=\frac{r}{s}} min\{\eta(r), \delta(s)\}, \qquad t \in \mathbb{R}$$

Definition 2.1 [6, Definition 2.2] Let X be a linear space over $\mathbb{R}$. Suppose $\|.\|: X \longrightarrow \mathbb{F}^+$ is a mapping satisfying

$$(i) \|x\| = \bar{0} \ if \ and \ only \ if \ x = 0,$$

$$(ii) \|rx\| = |r| \|x\|, x \in X, r \in R,$$

$$(iii) \ for \ all \ a, y \ \in X, \|x + y\| \preccurlyeq \ \|x\| \ \oplus \ \|y\|$$

and

$$(A'): x \neq 0 \implies \|x\|(t) = 0, \forall t \leq 0.$$

$(X, \|.\|)$is called a fuzzy normed linear space and $\|.\|$ is called fuzzy norm on X. In the rest of this paper we use the previous definition of fuzzy norm. We note that

(i)condition(A') in Definition 2.1 is equivalent to the condition

(A''): For all $x(\neq 0) \in X$ and each $\alpha \in (0,1], \|x\|_\alpha^- > 0, where\ [\|x\|]_\alpha = [\|x\|_\alpha^-, \|x\|_\alpha^+]$ and $(ii) \|x\|_\alpha^i, i = 1,2$, are crisp norms on X.

Theorem 2.2 [5, Theorem 2.7] Let $(X, \|.\|)$ be an fuzzy normed linear space, $\alpha \in (0,1], \varepsilon > 0, N(\varepsilon, \alpha) = \{x: \|x\|_\alpha^+ < \varepsilon\}$. Then $(X, \|.\|)$ is a hausdorff topological vector space, whose neighborhood base of origin 0 is $\{N(\varepsilon, \alpha): \varepsilon > 0, \alpha \in (0,1]\}$.

Corollary 2.3 [5, Corollary 2.9] The two families of open $\{N(\varepsilon, \alpha): \varepsilon > 0, \alpha \in (0,1]\}$ and $\{N(\alpha, \alpha): \alpha \in$ *0,1* are equivalent 0-neighborhood bases.

According to J. Xiao and X. Zhu [7], $(X, \|.\|)$ be an fuzzy normed linear space, $A \subseteq X$and $x_0 \in X$. $x_0$is called a point of closure of A if $\{x_0 + N(\alpha, \alpha)\} \cap A \neq \phi$ for every $\alpha \in (0,1]$; $\bar{A}$ denotes the set of all points of closure of A. A is called a fuzzy closed set if $A = \bar{A}$. A is called separable if there is a countable set B such that $A \subseteq \bar{B}$. A is called a fuzzy bounded set if for each $\alpha \in (0,1]$ there exists $M = M(\alpha) > 0$ such that $A \subset N(M, \alpha)$.

A sequence $\{x_n\}$ in X is said converge to $x \in X$ if and only if for each $\alpha \in (0,1], \lim_{n\to\infty} \|x_n - x\|_\alpha^+ = 0$. In this case we write $\lim_{n\to\infty} x_n = x$. Also a sequence $\{x_n\}$ is called a Cauchy sequence if for each $\alpha \in (0,1], \lim_{m,n\to\infty} \|x_n - x_m\|_\alpha^+ = 0$. A fuzzy normed linear space $(X, \|.\|)$is said to be complete if every Cauchy sequence in X converges in X.

Theorem 2.4 [5, Theorem 2.10] Let $(X, \|.\|)$ be an fuzzy normed linear space, $A \subseteq X$. A is a fuzzy bounded subset of X if and only if A is a bounded subset of X.

Theorem 2.5 [7, Theorem 2.10] Let $(X, \|.\|)$ be an fuzzy normed linear space, $A \subseteq X$.Then the following are equivalent:

(1) A is compact (i.e., every fuzzy open cover of A has a finite subcover).
(2) A is sequentially compact (i.e., every sequence of points of A has a subsequence coverged to a point of A).

Let $(X, \|.\|)$ and $(Y, \|.\|)$Be two fuzzy normed linear spaces. A function $T: X \to Y$ is said to be weakly fuzzy continuous at $x_0 \in X$ if for a given $\varepsilon > 0, \exists \delta \in \mathbb{F}^+, \delta \succ \bar{0}$, such that

$$\left\|T_x - T_{x_0}\right\|_\alpha^- < \varepsilon \quad whenever \quad \|x - x_0\|_\alpha^+ < \delta_\alpha^+$$

$$\left\|T_x - T_{x_0}\right\|_\alpha^+ < \varepsilon \quad whenever \quad \|x - x_0\|_\alpha^- < \delta_\alpha^-$$

3 The main results

In this section, we introduce fuzzy compact operators between fuzzy normed spaces and examine some properties of the operators.

Definition 3.1 [5, Definition 3.1] T is said to be a fuzzy compact operator if T is a weakly fuzzy continuous operator and for each fuzzy bounded set $A \subset X, \overline{T(A)}$ is a fuzzy compact set of X.

Example 3.2 Let $(X_1, \|.\|_1)$ and $(X_2, \|.\|_2)$ be two ordinary normed spaces, and $T : X_1 \to X_2$ be a compact operator. Then it is easy to see $T : (X_1, \|.\|) \to (X_2, \|.\|)$ is a fuzzy compact operator, where $(X_1, \|.\|)$ and $(X_2, \|.\|)$ Respectively, i.e.,

$$\|x_i\|(t) = \begin{cases} 1 & t = \|x_i\|_i, \\ 0 & t \neq \|x_i\|_i. \end{cases}$$

$$x_i \in X_i \; for \; i = 1,2.$$

Example 3.3 Let C[0,1] be the set of all real valued continuous functions on [0,1] with the fuzzy

$$\|f\|(t) = \begin{cases} 1 & t = \|f\|_u, \\ 0 & t \neq \|f\|_u. \end{cases}$$

Where

$$\|f\|_u = sup_{x \in [0.1]} |f(x)|$$

if$K(x, y)$ with $x, y \in [0,1]$ is a real valued continuous functions, then the operator

$$T : C[0,1] \to C[0,1]$$

define by

$$(T\varphi)(x) = \int_0^1 K(x,y)\varphi(y)dx$$

Where $\varphi \in C[0,1]$ is a fuzzy compact operator.

Theorem 3.4 Let $(X, \|.\|)$ and $(Y, \|.\|)$ be fuzzy normed spaces and $T: X \to Y$ be a weakly fuzzy continuous operator, then T is fuzzy compact if and if it maps every fuzzy bounded sequence $\{x_n\}$ in X an to a sequence $\{T(x_n)\}$ in Y which has a fuzzy convergent subsequence.

Proof. Suppose that T is fuzzy compact operator, and $\{x_n\}$is a fuzzy bounded sequence in $(X, \|.\|)$. The fuzzy closure of $\{T(x_n): n \in N\}$ is a fuzzy compact set. So $\{T(x_n)\}$ has a fuzzy convergent subsequnce by theorem 2.5. Conversely, let A be a fuzzy bounded subset of $(X, \|.\|)$. We show that the fuzzy closure of T (A) is fuzzy compact. Let $\{x_n\}$ be a sequence in the fuzzy closure of T (A). There exists $\{y_n\}$ in T (A) such that $\lim_{n\to\infty} \|x_n - y_n\| = \bar{0}$. Let $y_n = T(z_n)$, where $z_n \in A$. Since A ia a fuzzy bounded set, so is $\{z_n\}$. On the other hand, $T(z_n)$ has a fuzzy convergent subsequence $\{y_{n_k}\} = \{T(z_{n_k})\}$. Let $y_{n_k} \to y$ for some $y \in Y$. We have

$$\|x_{n_k} - y\| \preccurlyeq \|x_{n_k} - y_{n_k}\| \oplus \|y_{n_k} - y\|$$

$$\lim_{k\to\infty} \|x_{n_k} - y\| \preccurlyeq \lim_{k\to\infty} \|x_{n_k} - y_{n_k}\| \oplus \lim_{k\to\infty} \|y_{n_k} - y\|$$

Hence

$$\lim_{k\to\infty} \|x_{n_k} - y\| = \bar{0}$$

Now $\{x_{n_k}\}$is a fuzzy convergent subsequence of $\{x_n\}$. Thus the fuzzy clouser of T (A) is a fuzzy compact set.

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