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Solving equal-width wave-Burgers equation by (G'/G)-expansion method

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Abstract

In this paper, we apply the (G'/G)-expansion method to give traveling wave solutions of the third order equal-width wave-Burgers (EW-Burgers) equation. This method is direct, concise and effective and its applications are promising, and it appears to be easier and faster by a symbolic computation system like Maple or Matlab. This work highlights the power of the (G'/G)-expansion method in providing generalized solitary wave solutions of different physical structures.

Keywords: The (G'/G)-expansion method; Nonlinear evolution equations; EW-Burgers equation.

1. Introduction

Nonlinear partial differential equations have an important role in the study of nonlinear physical phenomena in science and technology. This paper investigates the exact solutions of The third order equal-width wave-Burgers (EW-Burgers) equation using the (G'/G)-expansion method. The EW-Burgers equation is special case of the generalized regularized long-wave (GRLW) equation which can be written as following

$$u_t + \alpha\, u_x + \gamma\, (u^p)_x - \delta\, u_{xx} - \beta\, u_{xxt} = 0, \qquad -\infty < x < +\infty, \qquad t > 0. \qquad (1)$$

Where α, γ, δ and β are given non-negative real constants and $p \geq 2$ is a constant. For $\beta = \delta = \gamma = 0$ and $\alpha \neq 0$, Eq. (1) reduces to the first-order linear wave equation. For $\alpha = \beta =$

$\delta = 0$ and $\gamma \neq 0$, Eq. (1) reduces to the first-order nonlinear wave equation. For $\alpha = \beta = \gamma = 0$and $\delta \neq 0$, Eq. (1) reduces to the one-dimensional heat transfer equation. For $\beta = \gamma = 0$, $\alpha \neq 0$ and $\delta \neq 0$, Eq. (1) reduces to the one-dimensional linear advection–diffusion equation. For $\alpha = \beta = 0$, $\gamma \neq 0$ and $\delta \neq 0$, Eq. (1) reduces to the one-dimensional nonlinear advection–diffusion equation. For $\alpha = \beta = 0$, $\gamma \neq 0$, $\delta \neq 0$ and $p = 2$ Eq. (1) reduces to the one-dimensional nonlinear Burgers' equation. For $\alpha = \delta = 0$, $\gamma = 1$, $\beta \neq 0$ and $p = 2$ Eq. (1) reduces to the EW equation. For $\alpha = 1, \delta = 0$, $\gamma \neq 0, \beta \neq 0$ and $p = 2$ Eq. (1) reduces to the RLW equation. For $\alpha = 0$, $\gamma = 1, \delta \neq 0, \beta \neq 0$ and $p = 2$ Eq. (1) reduces to the EW-Burgers equation. The EW-Burger equation is an important mathematical model arising in many different physical contexts to describe many phenomena which are simultaneously involved in nonlinearity, dissipation, dispersion, and instability, especially at the description of turbulence processes [1]. Recently, several direct methods such as Exp-function method [2,3], sine-cosine method [4,5], tanh-coth method [6], the homogeneous balance method [7], varitional iteration method and Adomian decomposition method [8], wavelet operational method [9], F-expansion method [10,11] and others have been proposed to obtain exact solutions of nonlinear partial differential equations. Using these methods many exact solutions, including the solitary wave solutions, shock wave solutions and periodic wave solutions are obtained for some kinds of nonlinear evolution equations. The application of (G'/G)-expansion method to obtain more explicit traveling wave solutions to many nonlinear differential equations has been developed by many researchers [12,13,14,15]. The (G'/G)-expansion method is based on the assumption that the travelling wave solutions can be expressed by a polynomial in (G'/G). It has been shown that this method is straightforward, concise, basic and effective.

The structure of this paper is organized as follows. In the following section, the basic (G'/G)-expansion method isintroduced. Application of (G'/G)-expansion method to EW-Burgers equation is presented in Section 3. Section 4 ends this work with a brief conclusion.

2. The basic (G'/G)-expansion method

We suppose that the given nonlinearpartial differential $u(x, t)$ to be in the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \dots) = 0, \tag{2}$$

whereP is a multivariate polynomial in its arguments. In the following, it is explained the essential steps for implementing (G'/G)-expansion method.

Step1. Takingthe change of variable $\xi = x - wt$, gives$u(x, t) = U(\xi)$, where w is a constant parameter to be determined later.Substituting $\xi = x - wt$into the Eq. (2) yields an ODE for $U(\xi)$ of the form

$$Q(U, U', -wU', U'', -wU'', w^2U'', \dots) = 0. \tag{3}$$

So, if possible, integrate Eq. (3), term by term one or more times. This introduces one or more constants ofintegration.

Step2. Introduce the approach

$$U(\xi) = \sum_{j=0}^{N} b_j \left(\frac{G'}{G}\right)^j, \tag{4}$$

where $G = G(\xi)$ satisfies the differential equation

$$G'' + \lambda G' + \mu G = 0, \tag{5}$$

hereN is a positive integer (to be determined). λ, μandb_j, $j = 0, \dots, N$, are real constants with $b_N \neq 0$. The $\left(\frac{G'}{G}\right)$ satisfies in the following differential equations

$$\frac{d\left(\frac{G'}{G}\right)}{d\xi} = -\left[\mu + \lambda\left(\frac{G'}{G}\right) + \left(\frac{G'}{G}\right)^2\right], \tag{6}$$

and so,

$$\frac{d}{d\xi} = -\left[\mu + \lambda\left(\frac{G'}{G}\right) + \left(\frac{G'}{G}\right)^2\right]\left(\frac{d}{d\left(\frac{G'}{G}\right)}\right), \tag{7}$$

and

$$\frac{d^2}{d\xi^2} = \left(\lambda + 2\left(\frac{G'}{G}\right)\right)\left[\mu + \lambda\left(\frac{G'}{G}\right) + \left(\frac{G'}{G}\right)^2\right]\left(\frac{d}{d\left(\frac{G'}{G}\right)}\right) + \left[\mu + \lambda\left(\frac{G'}{G}\right) + \left(\frac{G'}{G}\right)^2\right]^2\left(\frac{d^2}{d\left(\frac{G'}{G}\right)^2}\right). \tag{8}$$

Substituting Eq. (4), (7) and (8) into the ODE from step 1, yields an algebraic equation in powers ofthe$\left(\frac{G'}{G}\right)$. Then, the positive integer N is determined by the balance of linear and nonlinear terms of the highest order in the resulting algebraic equation.

Step3. With N being determined, the coefficients of each power of $\left(\frac{G'}{G}\right)$ in the algebraic equation from Step 2 put equal to zero.This yields a system of algebraic equations involvingb_j,$j = 0, \dots, N$,$w$and the integration constants. Finally, the general solution of Eq. (5) is to be substitutedinto Eq.(4).

3. Application of the (G'/G)-expansion method to EW-Burgers equation

To look for travelling wave solutions of Eq. (1), we use the wave transformation $\xi = x - wt$ and change Eq. (1) into the form of an ODE

$$-wU' + \alpha UU' - \delta U'' + w\beta U''' = 0. \tag{9}$$

Integrating it with respect to ξ and setting the constant of integration to zero, we obtain

$$-wU + \frac{1}{2}\alpha U^2 - \delta U' + w\beta U'' = 0. \tag{10}$$

Now, we make an approach(5) for the solution of Eq. (9). Balancing the terms U^2 and U'' in Eq. (10), then we get $2N = N + 2$ which yields the leading term order $N = 2$. Therefore, we can write the solution of Eq. (10) in the form

$$U(\xi) = b_0 + b_1\left(\frac{G'}{G}\right) + b_2\left(\frac{G'}{G}\right)^2. \tag{11}$$

Substituting Eq. (11) into Eq. (10), collecting the coefficients of $\left(\frac{G'}{G}\right)^j$, $j = 0, \dots, 4$, and set it to zero we obtain the systemof algebraic equations forb_0, b_1, b_2 and w. Then, solving the system by Maple, we obtain the following answers:

$$\left\{w = -\frac{\delta(\lambda^2 - 4\mu)}{\lambda(-1 + \beta(\lambda^2 - 4\mu))}, \quad b_0 = \frac{12\delta\mu}{\alpha\lambda(-1 + \beta(\lambda^2 - 4\mu))}, \quad b_1 = \frac{12\delta}{\alpha(-1 + \beta(\lambda^2 - 4\mu))}, \quad b_2 = \frac{12\delta}{\alpha\lambda(-1 + \beta(\lambda^2 - 4\mu))}\right\},$$

$$\left\{w = -\frac{\delta(\lambda^2 - 4\mu)}{\lambda(1 + \beta(\lambda^2 - 4\mu))}, b_0 = -\frac{2\delta(2\mu + \lambda^2)}{\alpha\lambda(1 + \beta(\lambda^2 - 4\mu))}, \quad b_1 = -\frac{12\delta}{\alpha(1 + \beta(\lambda^2 - 4\mu))}, \quad b_2 = -\frac{12\delta}{\alpha\lambda(1 + \beta(\lambda^2 - 4\mu))}\right\}, \tag{12}$$

whereλ and μ are arbitrary constants.Substituting Eqs. (12) intoEq. (11) yields:

$$U(\xi) = \begin{cases} \dfrac{12\delta}{\alpha\lambda(-1 + \beta(\lambda^2 - 4\mu))}\left[\mu + \lambda\left(\dfrac{G'}{G}\right) + \left(\dfrac{G'}{G}\right)^2\right], \\ -\dfrac{12\delta}{\alpha\lambda(1 + \beta(\lambda^2 - 4\mu))}\left[2\mu + \lambda^2 + \lambda\left(\dfrac{G'}{G}\right) + \left(\dfrac{G'}{G}\right)^2\right], \end{cases} \tag{13}$$

where $\xi = x - \left(-\frac{\delta(\lambda^2-4\mu)}{\lambda(\mp 1+\beta(\lambda^2-4\mu))}\right)t.$

Substituting the general solutions of Eq. (5) into Eq. (13) we have three types of travelling wave solutions of the EW-Burger equation as follows:

1) When $\lambda^2 - 4\mu > 0$,

$$U_1^{\pm} = \begin{cases} \dfrac{3\delta(\lambda^2-4\mu)}{\alpha\lambda(-1+\beta(\lambda^2-4\mu))}\left[\left(\dfrac{c_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi)+c_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi)}{c_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\right)+c_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\right)}\right)^2-1\right], \\ -\dfrac{60\delta\mu}{\alpha\lambda(1+\beta(\lambda^2-4\mu))}-\dfrac{3\delta(\lambda^2-4\mu)}{\alpha\lambda(1+\beta(\lambda^2-4\mu))}\left[\left(\dfrac{c_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\xi)+c_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi)}{c_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\right)+c_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\right)}\right)^2+3\right], \end{cases} \tag{14}$$

2) When $\lambda^2 - 4\mu < 0$,

$$U_2^{\pm} = \begin{cases} \dfrac{3\delta(4\mu-\lambda^2)}{\alpha\lambda(1+\beta(4\mu-\lambda^2))}\left[\left(\dfrac{-d_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi)+d_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi)}{d_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi\right)+d_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi\right)}\right)^2-1\right], \\ -\dfrac{60\delta\mu}{\alpha\lambda(1-\beta(4\mu-\lambda^2))}-\dfrac{3\delta(4\mu-\lambda^2)}{\alpha\lambda(-1+\beta(4\mu-\lambda^2))}\left[\left(\dfrac{-d_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\xi)+d_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi)}{d_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi\right)+d_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi\right)}\right)^2+3\right], \end{cases} \tag{15}$$

3) When $\lambda^2 - 4\mu = 0$,

$$U_3^{\pm} = \begin{cases} -\dfrac{12\delta}{\alpha\lambda}\left(\dfrac{1}{(\xi-x_0)^2}\right), \\ -\dfrac{12\delta}{\alpha\lambda}\left(\mu+\lambda^2+\dfrac{1}{(\xi-x_0)^2}\right), \end{cases} \tag{16}$$

where c_1, c_2 in Eq. (14), d_1, d_2 in Eq. (15) and x_0 in Eq. (16) are arbitrary real constants.

4. Conclusions

In this paper, we apply the (G'/G)-expansion method to third order EW-Burgers equation and obtain traveling wave solutions. Therefore, the (G'/G)-expansion method is entirely efficient and well suited for finding exact solutions of the EW-Burgers equation. The advantage of this method over other methods is that we can obtain the exact solution by using a simple computer program. The computations associated in this work were performed by using Maple 17.

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