Journal of mathematics and computer Science 11 (2014) 309 - 318

Contents list available at JMCS
Journal of Mathematics and Computer Science
Journal Homepage: www.tjmcs.com

Solution of Impulsive Differential Equations with Boundary Conditions in Terms of Integral Equations

Mohsen Rabbani[1]
Department of Mathematics, Sari Branch, Islamic Azad University, Sari, Iran
mrabbani@iust.ac.ir

Article history:
Received October 2013
Accepted February 2014
Available online July 2014

Abstract

In this article, we indroduce solution of impulsive differential equations with boundary conditions by using vareational interation method (VIM) in terms of integral equations. For finding the above solution, at first we obtian a solve for differential equations with boundary conditions.

Keywords: Impulsive, Differential, Equation, Integral.

1 Introduction

Impulsive problems are discuased in [4,8]. In this paper we use variational iteration method to obtian solution of impulsive problem. In order to, it is need for a reviewing on VIM.

In [7] Inokuti proposed a general Lagrange multiplier method for solving nonlinear differential equations the following form:

$$Lu + NU = g(x), \tag{1}$$

where L is a linear operator, N is a nonlinear operator, $g(x)$ is a known analytic function and u is an unknown function that to be determined. Also on the supposition that u_0 is the solution of $LU = 0$, in some of the special point such as $x = 1$, in other words,

$$u_{cor}(1) = u_0(1) + \int_0^1 \lambda(Lu_0 + Nu_0 - g)dx, \tag{2}$$

where λ is a general lagrange multiplier and optimum value. The Inokuti method is modified by He as follows,

[1] corresponding author

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\left(Lu_n + N\tilde{u}_n - g\right)ds, \tag{3}$$

where u_0 is an initial approximation and $\tilde{u}_n$ is a restricted variation, such that $\delta\,\tilde{u}_n = 0$ in (see $[5,6]$), the above integral in Eq.(3) is called a correction function and index of n denotes the n th approximation. Also Eq.(3) is called variational iteration method (VIM). In $[5,6]$ this method is used for solving nonlinear problems. The variational iteration method is effective and easy for linear problem because exact solution can be given by only one iteration. In the above process Eq.(3) is written in following form after the λ is obtained.

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\left(Lu_n(s) + Nu_n(s) - g(s)\right)ds. \tag{4}$$

In Eq.(4) by using $u_0(x)$ as an initial approximation, we obtain a sequence of approximations of exact solution of Eq.(1). For illustrating of effectively, easily and accurately a large class of nonlinear problems with approximations which converge quickly. we give number of application of variational iteration method. This method is used to solve Burger and coupled Burger equations in $[1]$, (VIM)is used for solving Fokker-Planck equations in $[3]$. Also in $[2]$, (VIM) is applied for solving nonlinear system of ordinary differential equations. Thus, we can say variational iteration method is a well known method to solve nonlinear equations.

3. Solution of differential equations with boundary conditions

We will solve the following problems;

$$\begin{cases} -x'' = f(t,x), & t \in J = [0,1] \\ x(0) = x(1), \; x'(0) = x'(1), \end{cases} \tag{5}$$

where $f \in C[J \times E, E]$.

$$\begin{cases} -x'' = g(t,x,x'), & t \in J = [0,1], \\ x(0) = x(1), \; x'(0) = x'(1). \end{cases} \tag{6}$$

where $g \in C[J \times E \times E, E]$.

$$\begin{cases} -x'' + Mx = y, & t \in J = [0,1]; \; M > 0, y \in C[J,E] \\ x(0) = x(1), \; x'(0) = x'(1). \end{cases} \tag{7}$$

To solve Eqs.(5-6), at first we solve Eq.(7), where $x(t)$ is unknown function, M is constant and $y(t)$ is known. we proof solution of Eq.(7) is given the following form,

$$x(t)=\int_0^1 G^*(t,s)y(s)ds, \tag{8}$$

Such that $x(t)\in C^2[J,E]$, also

$$G^*(t,s)=\begin{cases}\overset{-1}{\beta}\left[e^{\sqrt{M}(t-s)}+e^{\sqrt{M}(1-s+t)}\right], & t\le s\\ \overset{-1}{\beta}\left[e^{\sqrt{M}(t-s)}+e^{\sqrt{M}(1-t+s)}\right], & t>s\end{cases}, \ \beta=2\sqrt{M}\left(e^{\sqrt{M}}-1\right). \tag{9}$$

Proof: According to (VIM) in Eqs.(3-4), for finding optimal value of λ, we construct corrction functional by effect δ on the both sides of (4), so we have

$$\begin{aligned}\delta x_{n+1}(t)&=\delta x_n(t)+\delta\int_0^t \lambda(t,s)\left[-x_n^{''}(s)+Mx_n(s)-y(s)\right]ds\\ &=\delta x_n(t)-\delta\int_0^t \lambda(t,s)\,x_n^{''}(s)ds+M\int_0^t \lambda(t,s)\,\delta x_n(s)\,ds-\int_0^t \lambda(t,s)\delta y(s)ds,\end{aligned} \tag{10}$$

By considering $\delta y(s)=0$ and integrating by parts stationary conditions, we have

$$=\delta x_n(t)-\delta\left(\lambda(t,s)x_n^{'}(s)\Big|_{S=0}^{S=t}-\int_0^t \lambda_S(t,s)x_n^{'}(s)ds\right)+M\int_0^t \lambda(t,s)\delta x_n(s)ds$$

$$=\delta x_n(t)-\delta\left(\lambda(t,t)x_n^{'}(t)-\lambda(t,0)x_n^{'}(0)\right)+\lambda\int_0^t \lambda_S(t,s)x_n^{'}(s)ds+\int_0^t M\,\lambda(t,s)\delta x_n(s)ds$$

$$=\delta x_n(t)-\lambda(t,t)\delta x_n^{'}(t)+\lambda(t,0)\delta x_n^{'}(0)+\delta\int_0^t \lambda_S(t,s)x_n^{'}(s)ds+\int_0^t M\,\lambda(t,s)\delta x_n(s)ds,$$

because $\delta x_n^{'}(0)=0$, we can write,

$$=\delta x_n(t)-\lambda(t,t)\,\delta x_n^{'}(t)+\delta\left(\lambda_S(t,s)x_n(s)\Big|_{s=0}^{s=t}-\int_0^t \lambda_{ss}(t,s)\,\delta x_n(s)ds\right)+\int_0^t M\lambda(t,s)\,\delta x_n(s)ds$$

$$=\delta x_n(t)-\lambda(t,t)\,\delta x_n^{'}(t)+\lambda_s(t,t)\,\delta x_n(t)-\lambda_s(t,0)\,\delta x_n(0)-\int_0^t \lambda_{ss}(t,s)\,\delta x_n(s)ds+\int_0^t M\lambda(t,s)\,\delta x_n(s)ds$$

$$=\left(1+\lambda_s(t,t)\right)\delta x_n(t)-\lambda(t,t)\,\delta x_n^{'}(t)-\int_0^t \left(\lambda_{ss}(t,s)-M\lambda(t,s)\right)\delta x_n(s)ds,$$

in (10), since $\delta x_{n+1}(t)=0$, so according to right side of (10) we have a differential equations as follows;

$$\begin{cases}\lambda_{ss}(t,s)-M\,\lambda(t,s)=0,\\ \lambda_s(t,t)=-1,\\ \lambda(t,t)=0.\end{cases} \tag{11}$$

Easley we can find the solution of Eq.(11), as;

$$\lambda(t,s)=\frac{1}{2\sqrt{M}}\left[-e^{\sqrt{M}(s-t)}+e^{\sqrt{M}(t-s)}\right], \tag{12}$$

By substituting $\lambda(t,s)$ in (4) we will get a sequence of approximate of exact solution,

$$\begin{aligned}
x_{n+1}(t) &= x_n(t)+\frac{1}{2\sqrt{M}}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]\left[-x_n''(s)+Mx_n(s)-y(s)\right]ds \\
&= x_n(t)-\frac{1}{2\sqrt{M}}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]x_n''(s)ds \\
&\quad +\frac{\sqrt{M}}{2}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]x_n(s)ds-\frac{1}{2\sqrt{M}}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]y(s)ds, \\
&= x_n(t)-\frac{1}{2\sqrt{M}}\left[\left(e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right)x_n'(s)\right]_{s=0}^{s=t}-\frac{\sqrt{M}}{2\sqrt{M}}\int_0^t \left[e^{\sqrt{M}(t-s)}+e^{\sqrt{M}(s-t)}\right]x_n'(s)ds \\
&\quad +\frac{\sqrt{M}}{2}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]x_n(s)ds-\frac{1}{2\sqrt{M}}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]y(s)ds,
\end{aligned}$$

Also, we have

$$\begin{aligned}
&= x_n(t)-\frac{1}{2\sqrt{M}}\left[\left(e^{\sqrt{M}(0)}-e^{\sqrt{M}(0)}\right)x_n'(t)-\left(e^{\sqrt{M}t}-e^{-\sqrt{M}t}\right)x_n'(0)\right] \\
&\quad -\frac{1}{2}\left[\left(e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right)x_n(s)\right]_{s=0}^{s=t}+\frac{\sqrt{M}}{2}\int_0^t \left[-e^{\sqrt{M}(t-s)}+e^{\sqrt{M}(s-t)}\right]x_n(s)ds \\
&\quad +\frac{\sqrt{M}}{2}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]x_n(s)ds-\frac{1}{2\sqrt{M}}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]y(s)ds, \\
&= x_n(t)+\frac{1}{2\sqrt{M}}\left(e^{\sqrt{M}t}-e^{-\sqrt{M}t}\right)x_n'(0)-x_n(t)+\frac{1}{2}\left(e^{\sqrt{M}t}+e^{-\sqrt{M}t}\right)x_n(0) \\
&\quad -\frac{1}{2\sqrt{M}}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]y(s)ds,
\end{aligned}$$

At last, we obtain

$$\begin{aligned}
x_{n+1}(t) &= \frac{1}{2\sqrt{M}}\left(e^{\sqrt{M}t}-e^{-\sqrt{M}t}\right)x_n'(0)+\frac{1}{2}\left(e^{\sqrt{M}t}+e^{-\sqrt{M}t}\right)x_n(0) \\
&\quad -\frac{1}{2\sqrt{M}}\int_0^t \left[e^{\sqrt{M}(t-s)}-e^{\sqrt{M}(s-t)}\right]y(s)ds.
\end{aligned} \tag{13}$$

In Eq. (13), to find $x_n(0)$ and $x_n'(0)$, we use boundary condition of Eq.(7),

$$\begin{cases} x(0) = x(1) \Rightarrow x_{n+1}(0) = x_{n+1}(1), \\ x'(0) = x'(1) \Rightarrow x'_{n+1}(0) = x'_{n+1}(1), \end{cases} \tag{14}$$

By differentiating of Eq. (13) and using Eq. (14), we have a system the following form,

$$\begin{cases} \left(e^{\sqrt{M}} + e^{-\sqrt{M}} - 2\right) x_n(0) + \dfrac{1}{\sqrt{M}}\left(e^{\sqrt{M}} - e^{-\sqrt{M}}\right) x'_n(0) = \dfrac{1}{\sqrt{M}} \int_0^1 \left[\left(e^{\sqrt{M}(1-s)} - e^{\sqrt{M}(s-1)}\right)\right] y(s)ds \\ \left(e^{\sqrt{M}} - e^{-\sqrt{M}}\right) x_n(0) + \dfrac{1}{\sqrt{M}}\left(e^{\sqrt{M}} + e^{-\sqrt{M}} - 2\right) x'_n(0) = \dfrac{1}{\sqrt{M}} \int_0^1 \left[\left(e^{\sqrt{M}(1-s)} + e^{\sqrt{M}(s-1)}\right)\right] y(s)ds. \end{cases} \tag{15}$$

By assume $\beta = 2\sqrt{M}\left(e^{\sqrt{M}} - 1\right)$, the (15) system have a solution as follows;

$$\begin{aligned} x'_n(0) &= \overset{-1}{\beta}\sqrt{M} \int_0^1 \left[\left(e^{\sqrt{M}(1-s)} - e^{\sqrt{M}s}\right)\right] y(s)ds, \\ x_n(0) &= \overset{-1}{\beta} \int_0^1 \left[\left(e^{\sqrt{M}(1-s)} + e^{\sqrt{M}s}\right)\right] y(s)ds. \end{aligned} \tag{16}$$

By substituting Eq.(16) in Eq.(13), we obtain

$$\begin{aligned} x_{n+1}(t) = {} & \frac{1}{2\sqrt{M}}\left(e^{\sqrt{M}t} - e^{-\sqrt{M}t}\right)\overset{-1}{\beta}\sqrt{M} \int_0^1 \left[\left(e^{\sqrt{M}(1-s)} - e^{\sqrt{M}s}\right)\right] y(s)ds \\ & + \frac{1}{2}\left(e^{\sqrt{M}t} + e^{-\sqrt{M}t}\right)\overset{-1}{\beta} \int_0^1 \left[\left(e^{\sqrt{M}(1-s)} + e^{\sqrt{M}s}\right)\right] y(s)ds \\ & - \frac{1}{2\sqrt{M}} \int_0^t \left[\left(e^{\sqrt{M}(t-s)} - e^{\sqrt{M}(s-t)}\right)\right] y(s)ds, \end{aligned}$$

By considering to $\dfrac{1}{2\sqrt{M}} = \left(e^{\sqrt{M}} - 1\right)\overset{-1}{\beta}$, then $\overset{-1}{\beta} = \dfrac{1}{2\sqrt{M}\left(e^{\sqrt{M}} - 1\right)}$, in this way we have,

$$x_{n+1}(t) = \overset{-1}{\beta} \int_0^t \left[e^{\sqrt{M}(t-s)} + e^{\sqrt{M}(1-t+s)}\right] y(s)ds + \overset{-1}{\beta} \int_t^1 \left[e^{\sqrt{M}(s-t)} + e^{\sqrt{M}(1-s+t)}\right] y(s)ds. \tag{17}$$

So,

$$x_{n+1}(t) = \int_0^1 G^*(t,s) y(s)ds,$$

Where

$$G^*(t,s)=\begin{cases}\overset{-1}{\beta}\left[e^{\sqrt{M}\,(s-t)}+e^{\sqrt{M}\,(1-s+t)}\right], & S\ge t\\ \overset{-1}{\beta}\left[e^{\sqrt{M}\,(t-s)}+e^{\sqrt{M}\,(1-t+s)}\right], & S<t\end{cases}$$

By considering of Eq. (7), we can write $x''(s)=y(s)-M\,x(s),$ and for obtaining solution of Eqs. (5-6), we use $M\,x(s)$ the following form;

$$x(t)=\int_0^1 G^*(t,s)\left[f\left(s,x(s)\right)+Mx(s)\right]ds, \tag{18}$$

and

$$x(t)=\int_0^1 G^*(t,s)\left[g\left(s,x(s),x'(s)\right)+Mx(s)\right]ds. \tag{19}$$

4. Solution of impulsive differential equations with boundary conditions

We consider to an impulsive (D.E) as follows;

$$\begin{aligned}&-x''=f(t,x),\ \forall\ 0\le t\le 1, t\ne t_k\ (k=1,2,\dots,m)\\ &\Delta x\Big|_{t=t_k}=I_k\left(x(t_k)\right),\\ &\Delta x'\Big|_{t=t_k}=0,\\ &x(0)=x'(1)=\theta,\end{aligned} \tag{20}$$

Where, $0<t_1<t_2<\dots<t_m<1, f\in C[JXP,P],\ J=[0,1], f(t,\theta)=\theta,\ I_k\in C[P,P],$ $I_k(\theta)=\theta(k=1,2,\dots,m),$ P is a cone in E and $J'=J\setminus\{t_1,t_2,\dots,t_m)\}$, such that $t_i, i=1,\dots,m$ are if $x\in PC[J,P]\cap C^2[J',E]$ be a solution of (20), then it is the following form;

$$\begin{cases}x(t)=\int_0^1 C(t,s)f\left(s,x(s)\right)ds+\sum\limits_{0<t_k<t} I_k\left(x(t_k)\right)\\ where\\ G(t,s)=\begin{cases}t, & t\le s\\ s, & t>s\end{cases}=\min\{t,s\}.\end{cases} \tag{21}$$

Proof: For showing $x(t)$ to (21) Eq form we use variational iteration method so,

$$x_{n+1}(t)=x_n(t)+\delta\int_0^t \lambda(t,s)\left[-x_n''(s)-f\left(s,x(s)\right)\right]ds, \tag{22}$$

by effect δ on the both sides of (21) Eq, we have,

$$\delta x_{n+1}(t)=\delta x_n(t)+\delta\int_0^t \lambda(t,s)\left[-x_n''(s)-f\ (s,x(s))\right]ds$$
$$=\delta x_n(t)-\delta\int_0^t \lambda(t,s)x_n''(s)ds-\int_0^t \lambda(t,s)\underbrace{\delta f\ (s,x(s))ds}_{0},$$
$$=\delta x_n(t)-\delta\int_0^t \lambda(t,s)x_n''(s)ds,$$

by assumption $x_n''(s)$ and $x_n'(s)$ are continuous in $t\neq t_k$, so, we can write,

$$=\delta x_n(t)-\delta(\int_0^{t_1} \lambda(t,s)x_n''(s)-f\ (s,x(s))ds+\int_{t_1}^{t_2} \lambda(t,s)x_n''(s)ds+$$
$$\int_{t_m}^{t} \lambda(t,s)x_n''(s)ds)$$
$$=\delta x_n(t)\sum_{i=0(t_0=0,t_{m+1}=t)}^{m}\delta\int_{t_i}^{t_{i+1}} \lambda(t,s)x_n''(s)ds$$
$$=\delta x_n(t)-\sum_{i=0}^{m}\delta(\lambda(t,s)x_n'(s)\Big|_{s=t_i}^{s=t_{i+1}}-\int_{t_i}^{t_{i+1}} \lambda(t,s)x_n'(s)ds$$
$$=\delta x_n(t)-\sum_{i=0}^{m}\Big[\Big(\lambda(t,t_{i+1})\delta x_n'(t^-,t_{i+1})-\lambda(t,t_i)\delta x_n'(t_i^+)\Big)-$$
$$\lambda s(t,s)\delta x_n(s)\int_{t_i}^{t_{i+1}}+\int_{t_i}^{t_{i+1}} \lambda ss(t,s)\delta x_n(s)ds\Big]$$
$$=\delta x_n(t)-\sum_{i=0}^{m}\lambda(t,t_{i+1})\delta x_n'(t^-,t_{i+1})+\sum_{i=0}^{m}\lambda(t,t_i)\delta x_n'(t^+,t_i)+\sum_{i=0}^{m}\lambda s(t,t_i)\delta x_n(t^+,t_i)$$
$$-\sum_{i=0}^{m}\lambda s(t,t_i)\delta x_n(t_i^+)-\sum_{i=0}^{m}\int_{t_i}^{t_{i+1}} \lambda ss(t,s)\delta x_n(s)ds$$

mean of $x_n^1(s)\Big|_{t_i}^{t_{i+1}}$ is $\lim\limits_{s\to t_{i+1}^-} x_n^1(s)-\lim\limits_{s\to t_{i+1}^+} x_n^1(s)=x_n^1(t_{i+1}^-)-x_n^1(t_{i+1}^+)$

$$=\delta x_n(t)-\Big(\lambda(t,t_1)\delta x_n'(t_1^-)+\lambda(t,t_2)\delta x_n'(t_2^-)+\ldots+\lambda(t,t)\delta x_n'(t^-)\Big)$$
$$+\Big(\lambda(t,0)\delta x_n'(0)+\lambda(t,t_1)\delta x_n'(t_1^+)+\ldots+\lambda(t,t_m)\delta x_n'(t_m^t)\Big)$$
$$+\Big(\lambda s(t,t_1)\delta x_n(t_1^-)+\lambda s(t,t_2)\delta x_n(t_2^-)+\ldots+\lambda s(t,t)\delta x_n(t^-)\Big)$$
$$-\Big(\lambda s(t,0)\delta x_n(0^+)+\lambda s(t,t_1)\delta x_n(t_1^+)+\ldots+\lambda s(t,t_m)\delta x_n(t^+)\Big)$$
$$-\Big(\int_0^{t_1} \lambda ss(t,s)\delta x_n(s)ds+\ldots+\int_{t_m}^{t} \lambda ss(t,s)\delta x_n(s)ds\Big)$$

in order to we have,

$$= \delta x_n(t) - \Big(\lambda(t,t_1)\delta\big(x_n'(t_1^+) - x_n'(t_1^-)\big) + \ldots + \lambda(t,t_m)\delta\big(x_n(t_m^+) - x_n(t_m^-)\big)$$

$$\lambda s(t,t_1)\big(sx_n(t_1^+) - \delta x_n(t_1^-)\big) + \ldots + \lambda s(t,t_m)\big(\delta x_n(t_m^+) - \delta x_n(t_m^-)\big)$$

$$+ \lambda s(t,t)\delta x_n(t) - \lambda(t,t)\delta x_n'(t) - \int_0^t \lambda ss(t,s)\delta x_n(s)ds$$

$$= \delta x_n(t) - \Big(\lambda(t,t_1)\delta\Delta x_n'\big|_{t=t_1} + \ldots + \lambda(t,t_m)\delta\Delta x_n'\big|_{t=t_m} \Big) - \lambda(t,t)\delta x_n'(t)$$

$$\lambda s(t,t_1)\big(\delta sx_n(t_1^+) - \delta x_n(t_1)\big) + \ldots + \lambda s(t,t_m)\big(\delta x_n(t_m^+) - \delta x_n(t_m)\big)$$

$$+ \lambda s(t,t)\delta x_n(t) - \int_0^t \lambda ss(t,s)\delta x_n(s)ds$$

So, we have,

$$\delta x_{n+1}(t) = \big(1 + \lambda s(t,t)\big)\delta x_n(t) - \int_0^t \lambda ss(t,s)\delta x_n(s)ds - \lambda s(t,t)\delta x_n'(t)$$

Similarly (11) Eq we can write,

$$\begin{cases} \lambda ss(t,s) = 0, \\ \lambda s(t,t) = 1, \\ \lambda s(t,t) = 0. \end{cases}$$

so

$$\lambda(t,s) = (t-s). \qquad (23)$$

by substituting (23) Eq in (22) Eq, we obtain,

$$x_{n+1}(t) = x_n(t) + \int_0^1 (t-s)\big[-x_n''(s) - f\,\big(s, x(s)\big)\big]ds$$

$$= x_n(t) - \Big(\int_0^{t_1} (t-s)x_n''(s)ds + \ldots + \int_{t_{m-1}}^{t_m} (t-s)x_n''(s)ds + \int_{t_m}^{t} (t-s)x_n''(s)ds \Big)$$

$$- \int_0^1 (t-s)f\,\big(s, x(s)\big)ds \qquad f \in C\,[JxP, P]$$

$$= x_n(t) - \sum_{i=0}^{m+1} \int_{t_i}^{t_{i+1}} (t-s)x_n''(s)ds - \int_0^1 (t-s)f\,\big(s, x(s)\big)ds$$

$$= x_n(t) - \sum_{i=0}^{m} \Big((t-s)x_n'(s)\int_{s=t_i}^{s=t_{i+1}} + \int_{t_i}^{t_{i+1}} x_n'(s) \Big) - \int_0^1 (t-s)f\,\big(s, x(s)\big)ds$$

where $\int_{t_i}^{t_{i+1}} x_n'(s)ds \qquad x_n(t_{i+1}^-) - x_n(t_i^+)$ so,

$$= x_n(t) - \sum_{i=0}^{m}\Big((t - t_{i+1})x'_n(t_{i+1}^-) - (t - t_i)x'_n(t_i^+) + x_n(t_{i+1}) - x_n(t_i^+)\Big)$$
$$- \int_0^1 (t-s) f\,\big(s, x(s)\big)ds$$
$$= x_n(t) - \big[(t - t_1)x'_n(t_1^-) - (t-0)x'_n(0)$$
$$(t - t_2)x'_n(t_2^-) - (t - t_1)x'_n(t_1^+)$$
$$\vdots \qquad\qquad \vdots$$
$$(t - t)x'_n(t^-) - (t - t_m)x'_n(t_m^+)$$
$$-\Big[\big(x_n(t_1^-) - x_n(0^+)\big) + \big(x_n(t_2^-) - x_n(t_1^+)\big) + \ldots + \big(x_n(t^-) - x_n(t_m^+)\big)$$
$$- \int_0^1 (t-s) f\,\big(s, x(s)\big)ds$$
$$= x_n \cancel{(t)} - \sum_{i=1}^{m}(t - t_i)\Delta x'_n\Big|_{t=t_i} + t x'_n(0) + \sum_{k=1}^{m}\Delta x\Big|_{t=t_k} - \cancel{x_n}(t) - \int_0^1 (t-s) f\,\big(s, x(s)\big)ds$$
$$= x_n(t) - x_n(t) + \theta + \theta + \sum_{0<t_k<t} I_k\,\big(x(t_k)\big) - \int_0^1 (t-s) f\,\big(s, x(s)\big)ds$$

at last, we obtian

$$x_{n+1}(t) = \int_0^1 (s-t) f\,\big(s, x(s)\big)ds + \sum_{0<t_k<t} I_k\,\big(x(t_k)\big) \qquad (24)$$

(24) Eq is equal to (21).

4 Conclusion

In this article we intraduce simple and directe method for obtianing solution of impulsive differantial equations in terms of integral equations such that in some refferences such as [8] the above result is find by the complex and dificult method.

References

[1] M.A, Abdou, A.A. Soliman, *variational iteration method for solving Burger's and coupled Burger's equations*, J.comput.Appl.Math, 181 (2005) 245-251.

[2] J. Biazar, H. Ghazvini, *he's variational iteration method for solving linear and nonlinear systems of ordinary differential equations*, Applied mathematics and computation, 191 (2007) 287-297.

[3] M. Dehghan, M. Tatari, *the use of He's variational iteration method for solving the Fokker-Planck equation*,Phys.scripta,74 (2006) 310-316.

[4] D.Guo and X. Liu, Extremal solutions of nonlinear impulsive integro-differential equations in Banach Spaces, J. Math. Appl. 177 (1993), 538-552.

[5] J.H. He, *variational iteration method for nonlinear and it's applications*, Mechanics and practice, 20, (1) (1998) 30-32(in chinese)

[6] J.H. He, *variational iteration method - a kind of nonlinear analytical technique:Some examples*, Int.Journal of Nonlinear Mechanics, 34 (1999) 699-708.
[7] M. Inokuti, *general use of the Lagrange multiplier in in nonlinear mathematical physics*,in: S.Nemat-nasser(Ed.), Variational Method in Mechanics of solids, Progamon press, oxford,(1978) 156-162.
[8] X. Liu, Monotone iterative technique for impulsive differential equations in a Banach space. J. Math. Phy. Sci. 24(1990), 183-191.