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A New Homotopy Perturbation Method for Solving Two-Dimensional Reaction–Diffusion Brusselator System

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Abstract

In this article, a New Homotopy perturbation method (NHPM) is presented to obtain an approximate solution of a class of two-dimensional initial value problems. In this method, the first appropriate approximate solution has been used to reach the exact solution of the equation. Some examples are presented to validate the ability of the proposed method.

Keywords: New Homotopy perturbation method; reaction-diffusion Brusselato; system of partial differential equations.

1. Introduction

Reaction-diffusion systems (RDs) arise frequently in the study of chemical and biological phenomena and naturally they are modeled by partial differential equations. Finding accurate and efficient methods for solving nonlinear system of PDEs have been an active research undertaking for a long time [1-4]. Some authors investigated the numerical solution of Brusselator system [5–8]. Adomian used a decomposition method for the numerical solution of the reaction–diffusion Brusselator system [6]. Twizell et al. developed a second order (in space and time) finite difference method for diffusion free Brusselator system [7]. Wazwaz used modified Adomian decomposition method [5]. Whye-Teong applied the dual-reciprocity boundary element method for numerical solution of this system [8].

Reaction-diffusion Brusselator system with initial conditions has the following general form

$$\begin{cases} \dfrac{\partial u}{\partial t} = B + u^2 v - (A+1)u + \alpha\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right), \\ \dfrac{\partial v}{\partial t} = Au - u^2 v + \alpha\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right). \end{cases} \tag{1}$$

$$\begin{aligned} u(x,y,0) &= f(x,y), \\ v(x,y,0) &= g(x,y). \end{aligned} \tag{2}$$

Where α , A and B are constants.

There are some methods to obtain approximate solutions of these kind of equations. One of them is Homotopy perturbation method. The method introduced by He in 1998[9]. In this method the solution is considered as the summation of an infinite series. This method continuously deforms the difficult problems under study into multiple simple problems which are easy to solve. Homotopy perturbation method can be considered as a universal capable to solve various kinds of non-linear functional equations [10-12].

The article is organized as follows. In Section 2, the new modification of HPM, called NHPM, for solving partial differential equation is presented. The NHPM applied to reaction–diffusion Brusselator system are illustrated in Section 3. Numerical results are presented in section 4.The conclusion of the article appears in Section 5.

2. The Basic Idea of NHPM

Homotopy perturbation method has been well addressed in [13-16], so we introduce the New Homotopy perturbation method in this section.

The general form of a system of PDEs is considered as the following:

$$\frac{\partial u_j}{\partial t} + N_j(u_1, u_2, \ldots, u_n) = g_j(x_1, \ldots, x_{n-1}, t), \quad j = 1, 2, \ldots, n. \tag{3}$$

With the following initial conditions:

$$u_j(x_1, \ldots, x_{n-1}, t_0) = f_j(x_1, \ldots, x_{n-1}), \quad j = 1, 2, \ldots, n.$$

Where $N_1, \ldots, N_n$ are non-linear operators, which usually depend on the functions u_i, $i = 1, 2, \ldots, \text{n}$, and their derivatives with respect to x_i 's, $i = 1, 2, \ldots, n-1, t$ and $g_1, g_2, \ldots, g_n$ are inhomogeneous terms.

For solving system (1), by using NHPM, we construct the following Homotopy:

$$(1-P)\left(\frac{\partial U_j}{\partial t} - u_{j,0}\right) + p\left(\frac{\partial U_j}{\partial t} + N_j(U_1, \ldots, U_n) - g_j\right) = 0, \quad j = 1, 2, \ldots, n. \tag{4}$$

Or

$$\frac{\partial U_j}{\partial t} = u_{j,0} - p\left(u_{j,0} + N_j(U_1, \ldots, U_n) - g_j\right), \quad j = 1, 2, \ldots, n. \tag{5}$$

Applying the inverse operator, $L^{-1} = \int_{t_0}^{t} (.) dt$ to both sides of Eq. (3), we obtain

$$\begin{aligned} U_j(x_1, x_2, \ldots, x_{n-1}, t) &= U_j(x_1, x_2, \ldots, x_{n-1}, t_0) + \int_{t_0}^{t} u_{j0}\, dt \\ &\quad - p\int_{t_0}^{t} \left(u_{j,0} + N_j(U_1, \ldots, U_n) - g_j\right) dt, \quad j = 1, \ldots, n, \end{aligned} \tag{6}$$

where

$$U_j\left(x_1,x_2,\ldots,x_{n-1},t_0\right)=u_j\left(x_1,x_2,\ldots,x_{n-1},t_0\right),\quad j=1,2,\ldots,n\,.$$

Let's present the solution of system (4) in the following form:

$$U_j=U_{j0}+pU_{j1}+p^2U_{j2}+\cdots,\quad j=1,2,\ldots,n, \tag{7}$$

where $U_{i,j},\ \ i=1,\ldots,n,\ \ j=0,1,2,\ldots,$ are functions which should be determined. Suppose that the initial approximations of the solutions of Eqs. (1) are in the following form:

$$u_{i,0}\left(x_1,x_2,\ldots,x_{n-1},t\right)=\sum_{j=0}^{\infty}a_{i,j}\left(x_1,x_2,\ldots,x_{n-1}\right)p_j\left(t\right),\quad ,i=1,2,\ldots,n\,, \tag{8}$$

where $a_{i,j}\left(x_1,x_2,\ldots,x_{n-1}\right),\ \ ,i=1,\ldots,n,\,j=0,1,2,\ldots,$ are unknown coefficients, and $p_0\left(t\right),p_1\left(t\right),p_2\left(t\right),\ldots$ are specific functions.

Substituting (5) and (6) into (4) and equating the coefficients of the terms with identical powers of p, leads to

$$\begin{aligned}
&p^0:U_{i,0}\left(x_1,x_2,\ldots,x_{n-1},t\right)=f_i\left(x_1,x_2,\ldots,x_{n-1}\right)+\sum_{j=0}^{\infty}a_{i,j}\int_{t_0}^{t}p_0\left(t\right)dt,\\
&p^1:U_{i,1}\left(x_1,x_2,\ldots,x_{n-1},t\right)=-\sum_{j=0}^{\infty}a_{i,j}\int_{t_0}^{t}p_0\left(t\right)dt-\int_{t_0}^{t}\left(N_i\left(U_{1,0},\ldots,U_{n,o}\right)-g_i\right)dt,\\
&p^2:U_{i,2}\left(x_1,x_2,\ldots,x_{n-1},t\right)=-\int_{t_0}^{t}\left(N_i\left(U_{1,0},\ldots,U_{n,o},U_{1,1},\ldots,U_{n,1}\right)\right)dt,\\
&\vdots\\
&p^j:U_{i,j}\left(x_1,x_2,\ldots,x_{n-1},t\right)=-\int_{t_0}^{t}\left(N_i\left(U_{1,0},\ldots,U_{n,o},\ldots,U_{1,j-1},\ldots,U_{n,j-1}\right)\right)dt,\\
&\vdots
\end{aligned} \tag{9}$$

By solving these equations in such a way that $U_{i,1}\left(x_1,x_2,\ldots,x_{n-1},t\right)=0,$ then Eqs. (7) yield

$$U_{i,j}\left(x_1,x_2,\ldots,x_{n-1},t\right)=0,\qquad j=2,3,\ldots,n.$$

Therefore, the exact solution may be obtained as follows:

$$\begin{aligned}
u_i\left(x_1,x_2,\ldots,x_{n-1},t\right)&=U_{i,0}\left(x_1,x_2,\ldots,x_{n-1},t\right)\\
&=f_i\left(x_1,x_2,\ldots,x_{n-1}\right)+\sum_{j=0}^{\infty}a_{i,j}\int_{t_0}^{t}p_0\left(t\right)dt,\quad i=1,2,\ldots,n.
\end{aligned} \tag{10}$$

It is worth mentioning that if $g_i\left(x_1,\ldots,x_{n-1},t\right)$, and $u_{i,0}\left(x_1,x_2,\ldots,x_{n-1},t\right)$ are analytic around $t=t_0,$ then their Taylor series are written as

$$u_{i,0}\left(x_1,x_2,\ldots,x_{n-1},t\right)=\sum_{j=0}^{\infty}a_{i,j}\left(x_1,x_2,\ldots,x_{n-1}\right)\left(t-t_0\right)^j ,$$
$$g_i\left(x_1,\ldots,x_{n-1},t\right)=\sum_{j=0}^{\infty}a^*_{i,j}\left(x_1,x_2,\ldots,x_{n-1}\right)\left(t-t_0\right)^j , \tag{11}$$

which can be used in Eqs. (7), where $a_{i,j}\left(x_1,x_2,\ldots,x_{n-1}\right)$, $i=1,2,\ldots,n, j=0,1,2,\ldots,$ are unknown coefficients, that must be computed, and $a^*_{i,j}\left(x_1,x_2,\ldots,x_{n-1}\right)$, $i=1,2,\ldots,n, j=0,1,2\ldots,$ are known ones.

3. New Homotopy Perturbation Method Applied to Reaction-Diffusion Brusselator System

To solve Eq. (1) with initial condition (2), according to the NHPM, the following Homotopies is constructed

$$\begin{cases}(1-p)\left(\dfrac{\partial U}{\partial t}-u_0\right)+p\left(\dfrac{\partial U}{\partial t}-B-U^2V+(A+1)U-\alpha\left(\dfrac{\partial^2 U}{\partial x^2}+\dfrac{\partial^2 U}{\partial y^2}\right)\right)=0,\\ (1-p)\left(\dfrac{\partial V}{\partial t}-v_0\right)+p\left(\dfrac{\partial V}{\partial t}-AU+U^2V-\alpha\left(\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial y^2}\right)\right)=0.\end{cases} \tag{12}$$

Where $p\in[0,1]$ is an embedding parameter, u_0 and v_0 are an initial approximation of the solution of the system.

Applying the inverse operator, $L^{-1}=\int_{t_0}^{t}(.)dt$ to both sides of the equations (12), result in

$$\begin{cases}U(x,y,t)=U(x,y,0)+\int_0^t u_0(x,y,t)dt-p\int_0^t\left(u_0(x,y,t)-B-U^2V+(A+1)U-\alpha\left(\dfrac{\partial^2 U}{\partial x^2}+\dfrac{\partial^2 U}{\partial y^2}\right)\right),\\ V(x,y,t)=V(x,y,0)+\int_0^t v_0(x,y,t)dt-p\int_0^t\left(v_0(x,y,t)-AU+U^2V-\alpha\left(\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial y^2}\right)\right).\end{cases} \tag{13}$$

Suppose that the solutions of system (13) are as assumed in (7); substituting Eqs. (7) Into Eqs. (13), collecting terms with the same powers of match, and equating each coefficient of p match to zero turn to:

$$p^0:\ \begin{cases} U_0(x,y,t)=f(x,y)+\int_0^t u_0(x,y,t)dt, \\ V_0(x,y,t)=g(x,y)+\int_0^t v_0(x,y,t)dt, \end{cases}$$

$$p^1:\ \begin{cases} U_1(x,y,t)=\int_0^t\left(-u_0(x,y,t)+B+U_0^2V_0-(A+1)U_0+\alpha\left(\frac{\partial^2 U_0}{\partial x^2}+\frac{\partial^2 U_0}{\partial y^2}\right)\right), \\ V_1(x,y,t)=\int_0^t\left(-v_0(x,y,t)+AU_0-U_0^2V_0+\alpha\left(\frac{\partial^2 V_0}{\partial x^2}+\frac{\partial^2 V_0}{\partial y^2}\right)\right), \end{cases}$$

$$p^2:\ \begin{cases} U_2(x,y,t)=\int_0^t\left(\left(U_0^2V_1+2U_0U_1V_0\right)-(A+1)U_1+\alpha\left(\frac{\partial^2 U_1}{\partial x^2}+\frac{\partial^2 U_1}{\partial y^2}\right)\right), \\ V_2(x,y,t)=\int_0^t\left(AU_1-\left(U_0^2V_1+2U_0U_1V_0\right)+\alpha\left(\frac{\partial^2 V_1}{\partial x^2}+\frac{\partial^2 V_1}{\partial y^2}\right)\right), \end{cases}$$

$$p^3:\ \begin{cases} U_3(x,y,t)=\int_0^t\left(\left(U_0^2V_2+2U_0U_1V_1+2U_0U_2V_0+U_1^2V_0\right)-(A+1)U_2+\alpha\left(\frac{\partial^2 U_2}{\partial x^2}+\frac{\partial^2 U_2}{\partial y^2}\right)\right), \\ V_3(x,y,t)=\int_0^t\left(AU_2-\left(U_0^2V_2+2U_0U_1V_1+2U_0U_2V_0+U_1^2V_0\right)+\alpha\left(\frac{\partial^2 V_2}{\partial x^2}+\frac{\partial^2 V_2}{\partial y^2}\right)\right), \end{cases} \tag{14}$$

$\vdots$

Assume

$$\begin{cases} u_0(x,y,t)=\sum_{n=0}^{\infty}a_n(x,y)t^n, \quad U(x,y,0)=u(x,y,0), \\ v_0(x,y,t)=\sum_{n=0}^{\infty}b_n(x,y)t^n, \quad V(x,y,0)=v(x,y,0). \end{cases}$$

Now if we solve these equations in such a way that $U_1(x,y,t)=V_1(x,y,t)=0,$ then Eqs. (13) yield to

$$U_j(x,y,t)=V_j(x,y,t)=0, \quad \mathrm{j}=2,3,\ldots.$$

Therefore, the exact solution may be obtained as following form:

$$\begin{cases} u(x,y,t)=U_0(x,y,t)=f(x,y)+\sum_{n=0}^{\infty}a_n\int_0^t t^n dt, \\ v(x,y,t)=V_0(x,y,t)=g(x,y)+\sum_{n=0}^{\infty}b_n\int_0^t t^n dt, \end{cases}$$

where $a_j(x), b_j(x), \quad j=0,1,2,\ldots,$ are unknown coefficients that must be computed.

4. Numerical Results

In this parts two examples are presented to illustrate the validity of method.

Example 1: Consider the nonlinear system with the following initial condition:

$$\begin{cases} \dfrac{\partial u}{\partial t} = u^2 v - 2u + \dfrac{1}{4}(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}), \\ \dfrac{\partial v}{\partial t} = u - u^2 v + \dfrac{1}{4}(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}), \end{cases} \tag{15}$$

$u(x,y,0) = e^{-x-y}, \quad v(x,y,0) = e^{x+y}.$

In this example, we have $A = 1,\ B = 0,\ \alpha = \dfrac{1}{4}.$

Using the model discussed as (14) and solving these equations for $U_1(x,y,t), V_1(x,y,t)$ lead to the following results

$$\begin{aligned} U_1(x,y,t) = & \left(-a_0(x,y) - \frac{1}{2}e^{-x-y}\right)t \\ & + \left(-a_1(x,y) + e^{-2x-2y}b_0(x,y) + \frac{1}{4}\left(a_{0xx}(x,y) + a_{0yy}(x,y)\right)\right)\frac{t^2}{2} \\ & + \Bigg(-a_2(x,y) - 2a_1(x,y) + \frac{1}{2}b_1(x,y)e^{-2x-2y} + \left(a_0(x,y)\right)^2 e^{x+y} \\ & \quad + 2a_0(x,y)b_0(x,y)e^{-x-y} + a_1(x,y) + \frac{1}{4}\left(\frac{1}{2}a_{1xx}(x,y) + \frac{1}{2}a_{1yy}(x,y)\right)\Bigg)\frac{t^3}{3} + \cdots, \\ V_1(x,y,t) = & \left(-b_0(x,y) + \frac{1}{2}e^{x+y}\right)t \\ & + \left(-b_1(x,y) - a_0(x,y) - e^{-2x-2y}b_0(x,y) + \frac{1}{4}\left(b_{0xx}(x,y) + b_{0yy}(x,y)\right)\right)\frac{t^2}{2} \\ & + \Bigg(-b_2(x,y) + \frac{1}{2}a_1(x,y) - \frac{1}{2}b_1(x,y)e^{-2x-2y} - \left(a_0(x,y)\right)^2 e^{x+y} \\ & \quad - 2a_0(x,y)b_0(x,y)e^{-x-y} - a_1(x,y) + \frac{1}{4}\left(\frac{1}{2}b_{1xx}(x,y) + \frac{1}{2}b_{1yy}(x,y)\right)\Bigg)\frac{t^3}{3} + \cdots. \end{aligned} \tag{16}$$

By vanishing of $U_1(x,y,t), V_1(x,y,t)$, the coefficients $a_n(x,y), b_n(x,y), n = 1,2,3,\ldots,$ are determined as follows:

$$\begin{cases} a_0(x,y) = -\dfrac{1}{2}e^{-x-y},\ a_1(x,y) = \dfrac{1}{4}e^{-x-y},\ a_2(x,y) = -\dfrac{1}{16}e^{-x-y},\ldots. \\ b_0(x,y) = \dfrac{1}{2}e^{x+y},\ b_1(x,y) = \dfrac{1}{4}e^{x+y},\ b_2(x,y) = \dfrac{1}{16}e^{x+y},\ldots. \end{cases}$$

Therefore, the exact solution of Eq. (15) are obtained as follows:

$$\begin{cases} u(x,y,t)=U_0(x,y,t)=e^{-x-y}+a_0(x,y)t+\frac{1}{2}a_1(x,y)t^2+\frac{1}{3}a_2(x,y)t^3+\cdots \\ \qquad =\left(1-\frac{1}{2}t+\frac{1}{8}t^2-\frac{1}{48}t^3+\dots\right)e^{-x-y}=e^{-x-y+t/2}, \\ v(x,y,t)=V_0(x,y,t)=e^{x+y}+b_0(x,y)t+\frac{1}{2}b_1(x,y)t^2+\frac{1}{3}b_2(x,y)t^3+\cdots \\ \qquad =\left(1+\frac{1}{2}t+\frac{1}{8}t^2+\frac{1}{48}t^3+\dots\right)e^{x+y}=e^{x+y+t/2}. \end{cases}$$

Example 2: Consider the following system with initial values

$$\begin{cases} \frac{\partial u}{\partial t}=1+u^2v-\frac{3}{2}u+\frac{1}{500}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \\ \frac{\partial v}{\partial t}=\frac{1}{2}u-u^2v+\frac{1}{500}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \end{cases} \tag{17}$$

$$u(x,y,0)=x^2,\quad v(x,y,0)=y^2.$$

According to Eq. (14) for $A=\frac{1}{2}, B=1, \alpha=\frac{1}{500}$, we derive

$$\begin{aligned} U_1(x,y,t)=&\left(-a_0(x,y)+x^4y^2-\frac{3}{2}x^2+\frac{251}{250}\right)t \\ &+\left(-a_1(x,y)+b_0(x,y)x^4+2a_0(x,y)x^2y^2-\frac{3}{2}a_0(x,y)+\frac{1}{500}\left(a_{0xx}(x,y)+a_{0yy}(x,y)\right)\right)\frac{t^2}{2} \\ &+\left(-a_2(x,y)-\frac{3}{4}a_1(x,y)+\frac{1}{2}b_1(x,y)x^4+\left(a_0(x,y)\right)^2y^2+2a_0(x,y)b_0(x,y)x^2\right. \\ &\left.+a_1(x,y)x^2y^2+\frac{1}{500}\left(\frac{1}{2}a_{1xx}(x,y)+\frac{1}{2}a_{1yy}(x,y)\right)\right)\frac{t^3}{3}+\cdots, \end{aligned}$$

$$\begin{aligned} V_1(x,y,t)=&\left(-b_0(x,y)-x^4y^2+\frac{1}{2}x^2+\frac{1}{250}\right)t \\ &+\left(-b_1(x,y)+\frac{1}{2}a_0(x,y)-b_0(x,y)x^4-2a_0(x,y)x^2y^2+\frac{1}{500}\left(b_{0xx}(x,y)+b_{0yy}(x,y)\right)\right)\frac{t^2}{2} \\ &+\left(-b_2(x,y)+\frac{1}{4}a_1(x,y)-\frac{1}{2}b_1(x,y)x^4-\left(a_0(x,y)\right)^2y^2-2a_0(x,y)b_0(x,y)x^2\right. \\ &\left.-a_1(x,y)x^2y^2+\frac{1}{500}\left(\frac{1}{2}b_{1xx}(x,y)+\frac{1}{2}b_{1yy}(x,y)\right)\right)\frac{t^3}{3}+\cdots. \end{aligned}$$

So

$$a_0(x,y)=x^4y^2-\frac{3}{2}x^2+\frac{251}{250},$$
$$a_1(x,y)=-\frac{378}{250}+\frac{2}{250}x^4+\frac{1}{2}x^6+\frac{254}{125}x^2y^2+\frac{9}{4}x^2-x^8y^2-\frac{9}{2}x^4y^2+2x^6y^4,$$
$$a_2(x,y)=\frac{9}{8}+\frac{253}{250}y^2+\frac{3}{250}x^2+\frac{5}{4}x^4-3x^6y^2-6x^2y^2+4x^4y^4,$$
$$\vdots,$$
$$b_0(x,y)=-x^4y^2+\frac{1}{2}x^2+\frac{1}{250},$$
$$b_1(x,y)=-\frac{3}{4}x^2-\frac{2}{250}x^4-\frac{1}{2}x^6-\frac{254}{125}x^2y^2-2x^6y^4+\frac{7}{2}x^4y^2+x^8y^2+\frac{126}{250},$$
$$b_2(x,y)=-\frac{253}{250}y^2-\frac{5}{4}x^4-\frac{3}{250}x^2+5x^2y^2+3x^6y^2-4x^4y^4-\frac{9}{8},$$
$$\vdots$$

This implies that

$$\begin{aligned}u(x,y,t)=U_0(x,y,t)=\mathrm{x}^2&+\left(x^4y^2-\frac{3}{2}x^2+\frac{251}{250}\right)t\\&+\left(-\frac{378}{250}+\frac{2}{250}x^4+\frac{1}{2}x^6+\frac{254}{125}x^2y^2+\frac{9}{4}x^2-x^8y^2-\frac{9}{2}x^4y^2+2x^6y^4\right)\frac{t^2}{2}\\&+\left(\frac{9}{8}+\frac{253}{250}y^2+\frac{3}{250}x^2+\frac{5}{4}x^4-3x^6y^2-6x^2y^2+4x^4y^4\right)\frac{t^3}{3}+\cdots,\end{aligned}$$
$$\begin{aligned}v(x,y,t)=V_0(x,y,t)=y^2&+\left(-x^4y^2+\frac{1}{2}x^2+\frac{1}{250}\right)t\\&+\left(-\frac{3}{4}x^2-\frac{2}{250}x^4-\frac{1}{2}x^6-\frac{254}{125}x^2y^2-2x^6y^4+\frac{7}{2}x^4y^2+x^8y^2+\frac{126}{250}\right)\frac{t^2}{2}\\&+\left(-\frac{253}{250}y^2-\frac{5}{4}x^4-\frac{3}{250}x^2+5x^2y^2+3x^6y^2-4x^4y^4-\frac{9}{8}\right)\frac{t^3}{3}+\cdots.\end{aligned}$$

And this is the limit of infinity of many terms, yields to the exact solution of (17).

5. Conclusions

The main goal of this work has been to derive an approximation for the solutions of reaction-diffusion Brusselator system by applying New Homotopy perturbation method. The number of calculations in the NHPM can be reduced by selecting an appropriate initial approximation in comparison to HPM. The results show that new method is a powerful straightforward method. The computation associated with the example in this article were performed using maple 15.

References

[1] R. Lefever, G. Nicolis, "*Chemical instabilities and sustained oscillations*", J. Theor. Biol. 30 (1971) 267.

[2] G. Nicolis, I. Prigogine, "*Self-Organization in Non-Equilibrium Systems*", Wiley/ Interscience, New York, 1977.

[3] I. Prigogine, R. Lefever, "*Symmetries breaking instabilities in dissipative systems II*", J. Phys. Chem. 48 (1968) 1695–1700.

[4] J. Tyson, “*Some further studies of nonlinear oscillations in chemical systems*”, J. Chem. Phys. 58 (1973) 3919.

[5] AM. Wazwaz, “*The decomposition method applied to systems of partial differential equations and to the reaction–diffusion Brusselator model*”, Appl. Math. Compute. 110 (2000) 251–264.

[6] G. Adomian, “*The diffusion Brusselator equation”*, *Compute*. Math. Appl. 29 (5) (1995) 1-3.

[7] E.H. Twizell, A.B. Gumel, Q. Cao, “*A second-order scheme for the “Brusselator” reaction–diffusion system*”, J. Math. Chem. 26 (1999) 297–316.

[8] Ang Whye-Teong, “*The two-dimensional reaction–diffusion Brusselator system: a dual-reciprocity boundary element solution*”, Eng. Anal. Bound. Elem, 27 (2003) 897–903.

[9] J.H. He, “*A coupling method of a homotopy technique and a perturbation technique for non-linear problems*”, International .Journal of Non-linear Mecha.nics, 35 (2000)37-43.

[10] Momani S., Odibat Z., “*Homotopy Perturbation Method for Nonlinear Partial Differential Equations of Fractional Order*”. Phys. Lett. A, (2007) 365, 345-350.

[11] Abid KAMRAN, Umer HAYAT, Ahmet YILDIRIM, Syed Tauseef MOHYUD-DIN, “*A Reliable Algorithm for Fractional Schrödinger Equations*”, Walailak Journal of Science and Technology, (2013) 10 (4).

[12] Odibat Z., Momani S., “*Modified Homotopy Perturbation Method: Application to Quadratic Riccati Differential Equation of Fractional Order*”. Chaos Solitons Fractals 36, (2008) 167-174.

[13] J.H. He, “*Homotopy perturbation technique*”, Computer Methods in Applied Mechanics and Engineering 178 (1999) 257–262.

[14] Mohamed I. A. Othman, A. M. S. Mahdy, R. M. Farouk, “*Numerical Solution of 12th Order Boundary Value Problems by Using Homotopy Perturbation Method*”, The journal of Mathematics and computer science 1(2010), 14-27.

[15] M. Matinfar, M. Saeidy, “*Application of Homotopy perturbation method for fuzzy integral equations*”, the journal of Mathematics and computer science 1 (2010) 375-385.

[16] M. Rabbani, “*New Homotopy Perturbation Method to Solve Non-Linear Problems*”, the journal of Mathematics and computer science 7 (2013) 272-275.