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On Generalized Ricci-Recurrent LP-Sasakian Manifolds

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Abstract

The object of the present paper is to study a generalized Ricci-recurrent LP-Sasakian manifold. Here we show that the generalized Ricci-recurrent LP-Sasakian manifold admitting cyclic Ricci tensor is an Einstein manifold.
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1. Introduction

In 1950, A.G.Walker [1] introduced the idea of recurrent manifolds. In 2012, De and Mallick [17] defined almost pseudo concircularly symmetric manifolds. In the same year Taleshian and N. Asghari [18] defined Lorentzian α-Sasakian manifolds. On the otherhand , De and Guha [2] introduced generalized recurrent manifold with the non-zero 1-form A and another non-zero associated 1-form B.Such a manifold has been denoted by GK_n.If the associated 1-form B becomes zero,then the manifold GK_n reduces to a recurrent manifold introduced by Ruse [3] which is denoted by K_n.

The idea of Ricci-recurrent manifold was introduced by Patterson [4]. He denoted such a manifold by R_n.Ricci-recurrent manifolds have been studied by many authors [5], [6], [7].

In 1989, K. Matsumoto [8] introduced the notion of LP-Sasakian manifold. Then I. Mihai and R. Rosca [9] introduced the same notion independently and they obtained several results on this manifold. LP-Sasakian manifolds have also been studied by K. Matsumoto and I. Mihai [10] , U.C. De and et.al., [11].

In 1995, De, Guha and Kamilya [12] introduced and studied a type of Riemannian manifold $(M_{n,}g)(n > 2)$ whose Ricci tensor S of type (0,2) satisfies the condition

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + B(X)g(Y,Z), \tag{1}$$

where A and B are two 1-forms, B is non zero, P, Q are two vector fields such that

$$g(X,P) = A(X), \tag{2}$$

$$g(X,Q) = B(X). \tag{3}$$

for every vector field X. Such a manifold was called a generalized Ricci-recurrent manifold and an n dimensional manifold of this kind was denoted by GR_n. If the 1-form B vanishes identically, then the manifold reduces to a Ricci recurrent manifold introduced by Patterson (1952).

In this paper it is proved that in a generalized Ricci-recurrent LP-Sasakian manifold the vector fields P and Q defined by (2) and (3) are in opposite direction. In the last section it is proved that if a generalized Ricci-recurrent LP-Sasakian manifold admits a cyclic Ricci-tensor then the manifold becomes an Einstein manifold.

2. Preliminaries

An n-dimensional differentiable manifold M_n is called an LP-Sasakian manifolds if it admits a (1,1) tensor field φ, a vector field ξ, a 1-form η and a Lorentzian metric g which satisfy

$$(\nabla_X \varphi)(Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{4}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g .

Let S and r denote respectively the Ricci tensor of type (0, 2) and the scalar curvature of $M_{n.}$.It is known that in an LP-Sasakian manifold M_n , the following relations hold

$$\varphi(X) = 0, \quad \eta(\varphi X) = 0 , \tag{5}$$

$$\eta(\xi) = -1 , \tag{6}$$

$$\varphi^2(X) = X + \eta(X)\xi , \tag{7}$$

$$g(X,\xi) = \eta(X), \tag{8}$$

$$\nabla_X \xi = \varphi(X) , \tag{9}$$

$$(\nabla_X \eta)(Y) = g(X,\varphi Y) = g(\varphi X, Y), \tag{10}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{11}$$

$$S(X,\xi) = (n-1)\eta(X). \tag{12}$$

for any vector fields X, Y.

The above results will be used in the next sections.

3. Generalized Ricci-recurrent LP-Sasakian manifolds

In this section we suppose that a generalized Ricci-recurrent manifold is an LP-Sasakian manifold. Then Ricci tensor S of a generalized Ricci-recurrent manifold will satisfy the condition (1).

We have,

$$(\nabla_X S)(Y,Z) = XS(Y,Z) - S(\nabla_X Y,Z) - S(Y,\nabla_X Z)\ , \tag{13}$$

Therefore from (1) and (13), we get

$$A(X)S(Y,Z) + B(X)g(Y,Z) = XS(Y,Z) - S(\nabla_X Y,Z) - S(Y,\nabla_X Z)\ \ , \tag{14}$$

Putting $Z = \xi$ in relation (14) , we get

$$A(X)S(Y,\xi) + B(X)g(Y,\xi) = XS(Y,\xi) - S(\nabla_X Y,\xi) - S(Y,\nabla_X \xi),$$

Using (8), (9) and (12), we get

$$(n-1)A(X)\eta(Y) + B(X)\eta((Y) = (n-1)[(\nabla_X\eta)(Y) + \eta(\nabla_X Y)] - (n-1)\eta(\nabla_X Y) - S(Y,\varphi(X))$$

$$= (n-1)(\nabla_X\eta)(Y) - S(Y,\varphi(X))\ . \tag{15}$$

Now,

$$(\nabla_X g)(Z,\xi) = Xg(Z,\xi) - g(\nabla_X Z,\xi) - g(Z,\nabla_X\xi)$$

using (8) and (9) in the above equation ,we have

$$(\nabla_X g)(Z,\xi) = X\eta(Z) - \eta(\nabla_X Z) - g\big(Z,\varphi(X)\big)\ .$$

Since, $\quad g\big(X,\varphi(Z)\big) + g(\varphi(X),Z) = 0,$

Therefore, from the above equation ,we get

$$(\nabla_X g)(Z,\xi) = (\nabla_X\eta)(Z) + g(\varphi(Z),X)\ . \tag{16}$$

Again, since $\nabla g = 0.$

Therefore, from equation (16) , we have

$$(\nabla_X\eta)(Z) = -g(\varphi(Z),X). \tag{17}$$

Hence from (15) and (17), we get

$$(n-1)A(X)\eta(Y) + B(X)\eta((Y) = -(n-1)g(\varphi(Y),X) - S(Y,\varphi(X)).$$

Putting $Y = \xi$ in the above relation , we get

$$[(n-1)A(X) + B(X)]\eta(\xi) = -(n-1)g(\varphi(\xi),X) - S(\xi,\varphi(X)) \tag{18}$$

Since for every vector field

$$g(\varphi(X),\xi) = \eta(\varphi(X)\ ) = 0$$

In virtue of (5) , (6) and (12), we get from (18)

$$[(n-1)A(X)+B(X)]\eta(\xi)=0,$$

since $\eta(\xi)=-1$.

Therefore, we get

$$(n-1)A(X)+B(X)=0\;, \tag{19}$$

which leads to state the following theorem:

Theorem 3.1. If a generalized Ricci-recurrent manifold is an LP-Sasakian manifold , then the associated vector fields of the 1-form A and B are in opposite direction.

4. Generalized Ricci-recurrent LP-Sasakian manifold admitting cyclic Ricci-tensor

In this section we suppose that a generalized Ricci-recurrent LP-Sasakian manifold admits a cyclic Ricci tensor S, that is

$$(\nabla_X S)(Y,Z)+(\nabla_Y S)(Z,X)+(\nabla_Z S)(X,Y)=0 \tag{20}$$

then by the virtue of (1) it follows from (20) that

$$A(X)S(Y,Z)+B(X)g(Y,Z)+A(Y)S(Z,X)+B(Y)g(Z,X)$$
$$+A(Z)S(X,Y)+B(Z)g(X,Y)=0\;, \tag{21}$$

Putting $Z=\xi$ in (21) and using (8) and (12) , we get

$$(n-1)A(X)\eta(Y)+B(X)\eta(Y)+(n-1)A(Y)\eta(X)$$
$$+B(Y)\eta(X)+A(\xi)S(X,Y)+B(\xi)g(X,Y)=0\;,$$

which implies that

$$[(n-1)A(X)+B(X)]\eta(Y)+[(n-1)A(Y)+B(Y)]\eta(X)$$
$$+A(\xi)S(X,Y)+B(\xi)g(X,Y)=0. \tag{22}$$

By the virtue of (19), we have from (22)

$$A(\xi)S(X,Y)+B(\xi)g(X,Y)=0$$

which gives

$S(X,Y)=\mu g(X,Y)$

where $\mu=-\frac{B(\xi)}{A(\xi)}$

Hence we can state the following theorem:

Theorem 4.1. If a generalized Ricci-recurrent LP-Sasakian manifold admits a cyclic Ricci tensor, then it becomes an Einstein manifold.

Corollary 4.1. For n-dimensional generalized Ricci-recurrent manifold M_n with cyclic Ricci tensor , we have the following results:

1. If M_n is a Lorentzian β-Kenmotsu manifold [13] and [14] , then

$$A(\xi)S(X,Y) = -2n\beta^2 A(\xi)g(X,Y).$$

2. If M_n is a Lorentzian α-Sasakian manifold [15], then

$$A(\xi)S(X,Y) = 2n\alpha^2 A(\xi)g(X,Y).$$

3. If M_n is a $(LCS)_n$- manifold [16], then

$$A(\xi)S(X,Y) = (n-1)(\alpha^2 - \rho)A(\xi)g(X,Y).$$

Therefore, in view of corollary (4.1) we can state the following theorem:

Theorem 4.2: *Let* M_n be a generalized Ricci-recurrent manifold with cyclic Ricci tensor. If M_n is one of Lorentzian β-Kenmotsu manifold, Lorentzian α-Sasakian manifold and $(LCS)_n$- manifold with non-zero A(ξ) *everywhere*, *then* M_n is Einstein manifold.

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