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Comparison of Fuzzy Numbers with Ranking Fuzzy and Real Number

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Abstract

Ranking fuzzy numbers play as a key tool in many applied models in the world and in particular decision-making procedures. We are going to present a new method based on the ranking the fuzzy number and real number. The problem of ranking the fuzzy number and real number is proposed with ranking function and then this approach to extend the ranking of two fuzzy numbers with ranking function. The proposed method is illustrated by some numerical examples and in particular the results of ranking by the proposed method and some common and existing methods for ranking fuzzy sets is compared to verify the advantage of the new approach. We will see that against of most existing ranking approaches where for two fuzzy sets are the exact ranking, the above men sioned method can give a ranking fuzzy numbers with acceptance rate smaller as fuzzy.

Keywords: Fuzzy numbers, Ranking.

1 -Introduction

Fuzzy ranking is a topic which has been studied by many researchers. In [6], Wang and Kree introduced reasonable properties for the ordering of fuzzy quantities .In [18], Abbasbandy and Asady defined a sign distance of fuzzy numbers and proposed a ranking method with a fuzzy origin for fuzzy numbers and distance of fuzzy numbers with respect to it' s origin. In [23], Yao and Wu defined a sign distance of fuzzy numbers and proposed a ranking method. In [20], Allah- viranloo and Adabitabar Firozja introduced a new metric distance on fuzzy numbers which was used for ranking fuzzy numbers by comparing with two crisp numbers max and min. Tran and Duckste in [15], compared the fuzzy numbers using a fuzzy distance measure. Some researchers for ranking fuzzy numbers introduced a defuzzification methods in [16, 13]. Fortemps and Roubens in [1], proposed a ranking method based on area compensation. Some of the other researchers have proposed functions for ranking. In Modarres and in [2], Sadi-Nezhad method a fuzzy number is evaluated by a function called preference function and ranked by preference ratio. Wang et al. In [22],

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defined the maximal and minimal reference sets of fuzzy numbers to measure LR deviation degree and then the transfer coecient was defined to measure the relative variation of LR deviation degree of fuzzy number and then proposed the ranking method based on the LR deviation degree and relative variation of fuzzy numbers. In [20], Asady proposed a revised method of ranking LR fuzzy number based on deviation degree with Wang's method. In [21], Wang and Luo presented a ranking approach for fuzzy numbers called area ranking based on positive and negative ideal points, which defined two indices for the purpose of ranking. In [19], Abbasbandy and Hajjari introduced a ranking of trapezoidal fuzzy numbers based on the left and the right spreads at some α-levels of trapezoidal fuzzy numbers. Each method has a shortcoming. Adabitabar firozja et al. In [11], proposed a ranking function for ranking real numbers and fuzzy number with acceptance rate and then to extended for ranking two fuzzy numbers. In this paper, we proposed a new ranking function for ranking real numbers and fuzzy number with acceptance rate and then to extended for ranking two fuzzy numbers. The paper is organized as follows: The background on fuzzy concepts is presented in section 2. A comparison between one real number and a fuzzy number with its properties is introduced in Section 3. Subsequently, in Section 4 ranking of two fuzzy numbers and its properties is considered, Numerical Examples in section 5, finally, conclusion are drawn in Section 6.

2 - Background

A fuzzy set $\tilde{A}$ is a generalized left right fuzzy numbers (GLRFN) defined by Dubois and Prade[8], and denoted as $\tilde{A}=(a_1, a_2, a_3, a_4)_{LR}$, if it 's membership function satisfies the following:

$$\mu_{\tilde{A}}(x)=\begin{cases} L(\frac{a_2-x}{a_2-a_1}), & a_1\le x\le a_2 \\ 1 & a_2\le x\le a_3 \\ R(\frac{x-a_2}{a_4-a_3}), & a_3\le x\le a_4 \end{cases} \tag{1}$$

Where L and R are strictly decreasing functions defined on [0,1] and satisfying the conditions:

$$\begin{array}{ll} L(t)=R(t)=1 & if \quad t\le 0 \\ L(t)=R(t)=0 & if \quad t>0 \end{array} \tag{2}$$

Trapeziodal fuzzy numbers (TrFN) are special cases of GLRFN with $L(t)=R(t)=1-t.$
A $\alpha-$level interval of fuzzy number $\tilde{A}$ is denoted as:

$$[\tilde{A}]_\alpha=[A_L(\alpha),A_u(\alpha)]=\left[a_2-(a_2-a_1)L^{-1}(\alpha),a_3+(a_4-a_3)R^{-1}(\alpha)\right] \tag{3}$$

And if $\lambda\in R$ then

$$\tilde{A}+\lambda=(a_1+\lambda,a_2+\lambda,a_3+\lambda,a_4+\lambda)_{LR}$$
$$\lambda\tilde{A}=\begin{cases}(\lambda a_1,\lambda a_2,\lambda a_3,\lambda a_4)_{LR}, & \lambda\ge 0\\ (\lambda a_4,\lambda a_3,\lambda a_2,\lambda a_1)_{LR}, & \lambda<0\end{cases} \tag{4}$$

We denote by F_{LR} the set of generalized left right fuzzy numbers.

3 - Comparison between crisp and fuzzy number

Let $\tilde{A} \in F_{LR}$ and $x \in R$ then we consider the following problem:

What do we say about whether x is greater to $\tilde{A}$?

To solve this problem, Adabitabar frozja et al. In [3] proposed a ranking function for ranking of fuzzy numbers and real numbers but, for two real numbers x, y where:

$$x, y \in [a_2 - (a_2 - a_1)L^{-1}(\tfrac{1}{2}), a_3 - (a_4 - a_3)R^{-1}(\tfrac{1}{2})]. \qquad (5)$$

If $x < y$ then $\tilde{A} \approx x$ and $\tilde{A} \approx y$, where this is not a good result.

In this paper, we proposed new ranking functions for comparison between crisp and fuzzy number. We extend the natural ordering relation ≤ on real numbers by increasing real function $L(\tilde{A},.): R \to [0,1]$, and $L(\tilde{B},.): R \to [0,1]$, decreasing real function to introduce the ordering relation crisp and fuzzy number with the characteristic functions $L(\tilde{A},x)$, (acceptance rate larger x of $\tilde{A}$) and $L(\tilde{B},x)$, (acceptance rate larger x of $\tilde{B}$) as follow:

$$L(A,x) = \frac{\int_{-\infty}^{x} \mu_A(t)dt}{\int_{-\infty}^{+\infty} \mu_A(t)dt}, \qquad L(B,x) = \frac{\int_{-\infty}^{x} \mu_B(t)dt}{\int_{-\infty}^{+\infty} \mu_B(t)dt} \qquad (6)$$

Where with $\tilde{A} = (0,1,2,3)_{LR}$, Figure 1 shows the diagram of $L(\tilde{A},x)$, $L(x,\tilde{A})$.

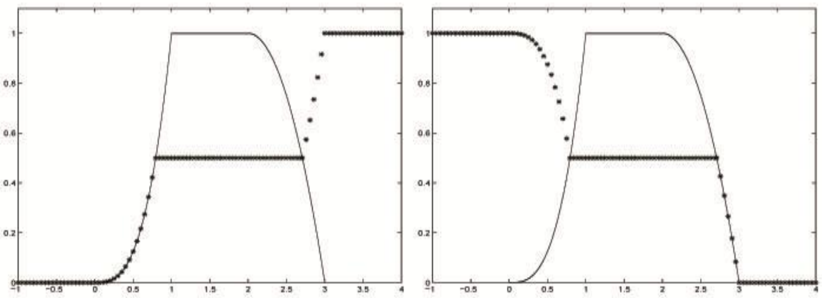

Figure 1: $\tilde{A} = (0,1,2,3)_{LR}$ − and $L(\tilde{A},x)$ ∗∗∗ and $L(x,\tilde{A})$ ∗∗∗.

3.1- Some properties

Let $\tilde{A} \in F_{LR}$ and $K \in R$, then

Proposition 1.

$$L(k\tilde{A},kx) = \begin{cases} L(\tilde{A},x), & 0 \le k \\ L(x,\tilde{A}), & k > 0 \end{cases}, \quad L(k\tilde{B},kx) = \begin{cases} L(\tilde{B},x) & 0 \le k \\ L(x,\tilde{B}) & k > 0 \end{cases} \qquad (7)$$

Proposition 2. $L(\tilde{A}+x,x+k)=L(\tilde{A},x), \qquad L(\tilde{B}+x,x+k)=L(\tilde{B},x) \qquad k\in R$ (8)

Proposition 3.If $x<y$ then

$$L(\tilde{A},x)<L(\tilde{A},y), \qquad L(x,\tilde{A})>L(y,\tilde{A}) \tag{9}$$

Regarding to equations (5) proofs is evident.

4- Ranking of two fuzzy numbers

Let $\tilde{A}\in F_{LR}$ and $\tilde{B}\in F_{LR}$ we consider the following problem:

What do we say about whether $\tilde{A}$ is greater (smaller) than or equal to $\tilde{B}$?

In order to solve this problem, Adabitabar frozja et al. [3] proposed a ranking function for ranking of two fuzzy numbers but in this paper we proposed the new ranking function for ranking of two fuzzy numbers. We have used ordering relations $L(\tilde{A},x)$ and $L(\tilde{B},x)$ where we defined in (6) and denoted ordering relation $\tilde{A}$ and $\tilde{B}$ by $R(\tilde{A},\tilde{B})$ and defined such as follow:

$$R(\tilde{A},\tilde{B})=\int_{supp(A)\cup supp(B)} L(\tilde{A},x)-L(\tilde{B},x)dx \tag{10}$$

Where $L(\tilde{A},x)$ defined fin equations (6) and $L(\tilde{A},\tilde{B})$ is acceptance rate smaller $\tilde{A}$ of x. In this paper, to compare of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ all values in Supp (A,B) with the membership degree compare with $\tilde{A}$ and vice versa.

4.1- Some properties

For $\tilde{A}$ and $\tilde{B}\in F_{LR}$ and $k\in R$

Proposition 4. $R(A,A)=0$

Proof: Recording to equation (10).

$$R(\tilde{A},\tilde{A})=\int_{supp(A)\cup supp(A)} (L(\tilde{A},x)-L(\tilde{A},x))dx=0 \tag{11}$$

Proposition 5. $R(A,B)=-R(B,A)$

Proof: Recording to equation (10)

$$(\tilde{A},\tilde{B}) \ = \int_{supp(A)\cup supp(B)} L(A,x)-L(B,x)dx$$

$$= \int_{supp(A)\cup supp(B)} L(B,x)-L(A,x)dx = -R(\tilde{A},\tilde{B}) \tag{12}$$

Proposition 6: $R(\tilde{A}+k,\tilde{B}+k)=R(\tilde{A},\tilde{B}),\quad k\in R$

Proof: By considering to equations (8) and (10).

$$R(\tilde{A}+k,\tilde{B}+k)=\int_{supp(\tilde{A}+k)\cup supp(\tilde{B}+k)}(\tilde{A}+k,\tilde{B}+k)dx \tag{13}$$

If $x=k+t$ then $dx=dt$, A cording to t= $supp(A)\cup supp(B)$, x= $supp(A+k)\cup supp(B+k)$

we have :

$$=\int_{supp(A+k)\cup supp(B+k)}\left(\frac{\int_{-\infty}^{t+k}\mu_{A+k}(t+k)dt}{\int_{-\infty}^{+\infty}\mu_{A+k}(t+k)dt}-\frac{\int_{-\infty}^{t+k}\mu_{B+k}(t+k)dt}{\int_{-\infty}^{+\infty}\mu_{B+k}(t+k)dt}\right)d(t+k)$$

$$=\int_{supp(A)\cup supp(B)}\left(\frac{\int_{-\infty}^{t}\mu_{A}(t)dt}{\int_{-\infty}^{+\infty}\mu_{A}(t)dt}-\frac{\int_{-\infty}^{t}\mu_{B}(t)dt}{\int_{-\infty}^{+\infty}\mu_{B}(t)dt}\right)d(t)==R(\tilde{A},\tilde{B})$$

Proposition 7: $R(kA,kA)=\begin{cases}R(A,B) & k\geq 0\\ -R(A,B) & k<0\end{cases}$ (14)

Proof: From equations (8), (10) and also $k\geq 0, k\in R$ we can write

$$R(kA,kB)=\int_{supp(kA)\cup supp(kB)}\frac{\int_{-\infty}^{x}\mu_{kA}(x)dx}{\int_{-\infty}^{+\infty}\mu_{kA}(x)dx}-\frac{\int_{-\infty}^{x}\mu_{kB}(x)dx}{\int_{-\infty}^{+\infty}\mu_{kB}(x)dx}\,dx \tag{15}$$

According to $t=supp(A)\cup supp(B),\ x=supp(kA)\cup supp(kB),$ we have:

$$=\int_{supp(kA)\cup supp(kB)}\frac{\int_{-\infty}^{kt}\mu_{kA}(kt)d(kt)}{\int_{-\infty}^{+\infty}\mu_{kA}(kt)d(kt)}-\frac{\int_{-\infty}^{kt}\mu_{kB}(kt)d(kt)}{\int_{-\infty}^{+\infty}\mu_{kB}(kt)d(kt)}dt=R(A,B)$$

Definition 1.We define the ranking of $\tilde{A}\in F_{LR}$, $\tilde{B}\in F_{LR}$ and $x\in R$ by $L(\tilde{A},x),L(\tilde{B},x)$:

1) If $R(\tilde{A},\tilde{B})<0\leftrightarrow \tilde{A}\geq\tilde{B}$

2) If $R(\tilde{A},\tilde{B})=0\leftrightarrow \tilde{A}\approx\tilde{B}$

3) If $R(\tilde{A},\tilde{B})>0\leftrightarrow \tilde{A}\leq\tilde{B}$

5. Numerical Examples

We have considered some examples constructed and discussed in [4,16] for comparing the current method with some other ranking methods, where results of some other ranking rules [4] are shown in Table 1 and results of current method are shown the following form:

Set1: $A_1=(0.4\ ,0.9\ ,1\ ),\ A_2=(0.4,0.5,1\ ),\ A_3=(0.4,0.5,1\ )$ then $R(A_1,A_2)=-0.066$, $R(A_2,A_3)=-0.066$ and $R(A_1,A_3)=-0.133$ therefore, $A_1\geq A_2,\ A_2\geq A_3\ ,\ A_1\geq A_3$.

Set 2: $A_1=(0.2,0.5,07)\ A_1=(0.2,0.5,08)$, $A_2=(0.4,0.5,06)$, then there for, $A_1\approx A_2$.

Set 3: $A_1=(0.5,0.7,09)$, $A_2=(0.3,0.7,09)$, $A_3=(0.3,0.4,07,0.9)$ so, $R(A_1,A_2)=-0.066$, $R(A_2,A_3)=-0.214$ and $R(A_1,A_3)=-0.148$, therefore, $A_1\geq A_2,\ A_2\geq A_3,\ A_1\geq A_3$.

The proposed method has been used by continuous function for ranking and gives the acceptance rate.

6 -Conclusions

In this paper, initially we proposed a ranking approach for ranking one crisp real number and one fuzzy number with membership function then have extended this approach for ranking two fuzzy numbers. For comparing of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ on Supp($\tilde{A}$) to the membership degree, is compared with $\tilde{B}$ also, on Supp($\tilde{B}$) with the membership degree, is compared with $\tilde{A}$. Thus, we obtain some useful properties. Some numerical examples have been presented in order to compare the proposed method of ranking with some of the approaches. This method was used a continuous function for ranking and gives the acceptance rate.

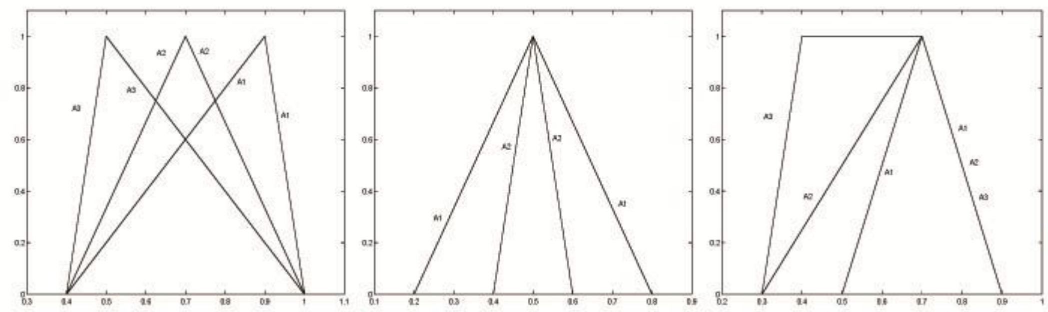

Figure 2: Set 1- $\{\tilde{A}_1,\tilde{A}_2,\tilde{A}_3\}$ Set 2- $\{\tilde{A}_1,\tilde{A}_2,\tilde{A}_3\}$ Set 3- $\{\tilde{A}_1,\tilde{A}_2,\tilde{A}_3\}$

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Table 1: Comparison on fuzzy numbers by some methods.

Methods		Set1			Set2		Set3	
Y ager								
F1	0.760	0.700	0.630	0.500	0.500	0.700	0.630	0.570
F2	0.90	0.76	0.660	0.610	0.540	0.750	0.750	0.750
F3	0.800	0.700	0.600	0.600	0.500	0.700	0.650	0.570
Ba,Kw	1.000	0.740	0.600	1.000	1.000	1.000	1.000	1.000
Bald								
$1:P$	0.420	0.330	0.330	0.270	0.270	0.370	0.270	0.270
G	0.550	0.400	0.340	0.300	0.240	0.420	0.350	0.350
$r:a$	0.280	0.230	0.220	0.200	0.230	0.270	0.190	0.19
Kerre	1.000	0.860	0.760	0.910	0.910	1.000	0.910	0.750
Jain								
$k = 1$	0.900	0.760	0.660	0.730	0.670	0.820	0.820	0.820
$k = 2$	0.840	0.650	0.540	0.600	0.480	0.710	0.710	0.710
$k = 1/2$	0.950	0.860	0.780	0.830	0.800	0.890	0.890	0.890
Dub,Pra								
PD	1.000	0.740	0.600	1.000	1.000	1.000	1.000	1.000
PSD	0.740	0.230	0.160	0.730	0.240	0.500	0.500	0.500
ND	0.630	0.380	0.180	0.270	0.760	0.670	0.350	0.000
NSD	0.260	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Lee, Li									
U.m	0.760	0.700	0.630	0.500	0.500	0.700	0.630	0.570	
U.G	...	...	....	0.120	0.040	...	...	...	
P.m	0.800	0.700	0.600	0.500	0.500	0.700	0.650	0.580	
P.G	...	...	...	0.090	0.030	...	...	...	
For,Rou									
F0	0.800	0.700	0.600	0.500	0.500	0.700	0.650	0.575	
Tran,Duc									
$DM; f:x$	0.187	0.308	0.442	0.505	0.501	0.304	0.342	0.457	
$Dm; f:x$	0.838	0.704	0.573	0.505	0.501	0.702	0.671	0.585	
$DM; f:1$	0.231	0.316	0.416	0.510	0.501	0.307	0.365	0.445	
$Dm; f:1$	0.808	0.707	0.611	0.510	0.501	0.703	0.658	0.590	
$Tof - Moh\ \gamma 2$									
$d(.,M); s=1$	0.341 2	0.500	0.6588	0.500	0.500	0.3789	0.421	0.5455	
$\gamma 2$ $d(.,m); s=1$	0.6588	0.500	0.3412	0.500	0.500	0.6211	0.5784	0.4545	
$\gamma 2$ $d(.,M); s=\alpha$	0.2976	0.500	0.7024	0.500	0.500	0.3378	0.3956	0.5481	
$\gamma 2$ $d(.,m); s=\alpha$	0.7024	0.500	0.2976	0.500	0.500	0.6622	0.6044	0.4519	
MahmodiNejad etal.[13]	0.1019	0.1837	0.2958	0.274	0.190	0.1396	0.2451	0.3636	
Wangetal.[21] *Wang*[20]	0	0.0476	0.1364	0.1429	0.1567 0	0.0246	0.2		
$Risk(\alpha)RIA1$									
	0.208	0.375	0.542	0.375	0.45	0.500	0.385	0.458	0.556
	0.333	0.500	0.667	0.500	0.500	0.500	0.458	0.583	0.667
	0.458	0.625	0.792	0.625	0.542	0.500	0.531	0.708	0.778
$Risk(\alpha)RIA2$									
	0.115	0.375	0.729	0.375	0.458	0.500	0.282	0.533	0.682
	0.167	0.500	0.833	0.500	0.500	0.500	0.333	0.667	0.750
	0.271	0.625	0.885	0.625	0.542	0.500	0.394	0.762	0.808