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Fuzzy Adaptive PSO Approach for Portfolio Optimization Problem

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Abstract

The mean-variance model of Markowitz is the most common and popular approach in the investment selection; besides, the mathematical planning model proposed by Markowitz is the most effective method of the optimal portfolio selection. However, if there are a lot of investing assets and a lot of market's restrictions, the common optimizing methods are not useful. Moreover, the portfolio optimization problem cannot be solved easily by applying the mathematical methods. In the present study, the heuristic Fuzzy Adaptive Particle Swarm Optimization (PSO) method is proposed to solve three highly applied models of the portfolio problem. Therefore, to fulfil this task the efficient frontier of the investment is drawn by applying the price information of the 50 shares accepted in Tehran stock market from October of 2009 to October of 2013. Results of this study manifest the efficiency of the used method in relation to other heuristic methods.

Keywords: portfolio optimization, fuzzy adaptive particle swarm optimization, mean-variance model, the efficient frontier

1. Introduction

The principal problem in portfolio optimization is the optimal and correct selection of assets which includes both risky and non-risky assets that can be prepared with the distinct amount of investment. In modern theory, Markowitz [1] has defined the portfolio as the mathematical formula. In Markowitz mean-variance model, the mean represents the expected efficiency and the variance shows the risk of the portfolio. After introducing Markowitz model, some researchers such as H. Konno, H. Yamazaki [2] have modified and improved this model and proposed mean-absolute deviation model or MAD. The only difference of the new method is that, the absolute deviation represents the risk.

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Moreover, R. Mansini and M.G Speranza [3] have assumed the investment division infinitely in their model whereas in the real world, shares are transacted distinctively. Therefore, the descriptive Markowitz model has transformed into a complex real number programming model with the related restrictions of the least transactions. In addition, some other researchers focus on shares multi-period optimization. In this case, it is supposed that investors invest continuously rather than investing in a single period. For instance U. Celikyurt and N. Meade [4] consider many different factors such as economic, social and political ones in the portfolio optimization and they model the random under studied market by considering the mentioned factors and by applying the Markov chain approach.

Applying the heuristic algorithms in solving the optimization problem has been used recently. Yin Peng-Yeng, Jing-Yu Wang [5] have used the PSO method in the non-linear problem of allocating resources; besides, have compared its efficiency with the genetic algorithm. Consequently, the results have shown that PSO technique is more applicable than genetic algorithm. Furthermore, in [6], by combining PSO and GA techniques Yan-Wei et al have selected a multi-period portfolio by applying the risk variance factor. They go on to claim that the combination of PSO and GA is much more effective than applying each of these techniques. In [7], Cura Tunchan has used PSO in the constrained portfolio optimization problem; He has chosen the weekly prices of the number of shares in different world's markets within 5 years from 1992 to 1997 and has drawn the effective boundary of investment by applying this technique. Additionally, in [8], has been used GA algorithm and applied PSO algorithm on its attained answers. Then, they claim that this algorithm is the more effective algorithm in relation to the previous algorithms. Also, in [9], Fernandez and Gomez have used neurotic networks in portfolio selection. In the present paper, the modified model of Markowitz known as cordiality constrained Mean-variance model or CCMV has used.

2. The Portfolio Optimization Problem

The amount of the risk and the efficiency of the asset properties are two principal characteristics in making any decision on investment. Generally, investors seek minimizing the risk of investment in the distinct level of the efficiency or maximizing the efficiency at the distinct presupposition of the risk. In 1952, Harry, M. Markowitz proposed his model based on the mathematical planning for portfolio selection which is based on following presuppositions [1]:

1. Investors avoid any risk and expect increasing in their investments and their wealth ultimate expecting curve is minimizing.

2. Investors have selected their portfolio based on the expected mean and variance of the efficiency; therefore, their indifference curve is a function of the expected efficiency rate and the expected variance.

3. Any option of investment is able to be divided infinitely.

4. Investors have the horizon of one time or one period that is the same for all investors.

5. At the distinct level of the risk, investors prefer the higher efficiency and reversely at the distinct level of the efficiency, they demand the lowest risk.

Indeed, the Markowitz investment problem indicates that any typical investor considers the high expected efficiency and the uncertainty of the efficiency as two important factors in his or her investment. Here, the investor has N valuable document .Therefore, the efficiency of each document is considered as a random variable with the mean μ and the variance σ^2. Moreover, if the covariance between the efficiency of the share and the share is determined and if the investor has some money to invest between stocks, an important question is posed. This question asks the how-ness of allocating the investment to documents that makes the attained expected portfolio highly accepted.

1. The determination of the effective portfolio set, the effective portfolio is a portfolio with the lowest efficiency variance among all portfolios which have the same expected efficiency or it is a portfolio with the highest expected efficiency among all portfolios which have the same variance.

2. The selection from the effective set indicates selecting a portfolio which can prepare the most appropriate combination of the risk and efficiency for investors.

The present presupposition of this paper is that assets are not correlated to each other completely; therefore, the aim is to select the best combination of the financial properties to make the investment portfolio efficiency maximized and to have the portfolio risk minimized.

Markowitz problem is like a quadratic programming with the variance minimizing object function of the properties' set. In this problem, the efficiency is constant. Besides, this problem has an effective method based on which the sum of all properties weights equals to one. In the presupposition of the problem, the weight of each property is a real non negative number; therefore, the Markowitz mean-variance model is calculated as (1):

$$\begin{aligned} & Min \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} \\ & S.to: \sum_{i=1}^{N} x_i \mu_i = R^*, \\ & \qquad \sum_{i=1}^{N} x_i = 1, \\ & \qquad x_i \geq 0 \quad (i=1,\dots,N) \end{aligned} \tag{1}$$

Considering the fifth assumption of the Markowitz model, one can be renamed as the cordiality constrained mean-variance or (CCMV) by correcting the function of problem (1) and by the help of the first constrained of the first model of Fernandez and Gomez [9]. The first model is as follow:

$$\begin{aligned} & Minimize\ \lambda[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}] - (1-\lambda)\sum_{i=1}^{N} x_i \mu_i \\ & S.to: \quad \sum_{i=1}^{N} x_i = 1, \qquad\qquad \text{model1} \\ & \qquad x_i \geq 0. \ (i=1,\dots,N) \end{aligned}$$

By adding the high and low boundaries to stock i ratio in the portfolio, the number (1) model of the new problem is introduced as the number (2) model:

$$
\begin{aligned}
&Minimize\ \lambda[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j\,\sigma_{ij}] - (1-\lambda)\sum_{i=1}^{N} x_i \mu_i \\
&S.to: \quad \sum_{i=1}^{N} x_i = 1, \qquad\qquad \text{model 2} \\
&\qquad \varepsilon_i \le x_i \le \delta_i,\ (i = 1,\dots,N) \\
&\qquad x_i \ge 0. \qquad (i = 1,\dots,N)
\end{aligned}
$$

Consequently, if the new constraint which is related to the number of chosen properties is added to this problem, the number (3) model of the problem is introduced:

$$
\begin{aligned}
&Minimize\ \lambda[\sum_{i=1}^{N}\sum_{j=1}^{N} z_i x_i z_j x_j\,\sigma_{ij}] - (1-\lambda)\sum_{i=1}^{N} z_i x_i \mu_i \\
&S.to: \quad \sum_{i=1}^{N} x_i = 1, \\
&\qquad \sum_{i=1}^{N} z_i = K, \qquad\qquad \text{model 3} \\
&\qquad \varepsilon_i z_i \le x_i \le \delta_i\, z_i,\ (i = 1,\dots,N) \\
&\qquad x_i \ge 0, \qquad (i = 1,\dots,N) \\
&\qquad z_i \in \{0,1\}.
\end{aligned}
$$

In model (3), z_i is the variable of decision which must be made on investing in each stock. If this variable equals to 1, it indicates that each stock is placed in the portfolio. The aim of solving these models is to draw the Efficient Frontier of the investment. It is clear that the introduced models are the combinations of the planning problem and the quadratic programming. In order to solve these problems more precisely, there are not effective algorithms in mathematical programming. Therefore, in the present study, Fuzzy Adaptive PSO algorithm is used to determine the optimized portfolio and to draw the efficient frontier of the investment.

3. Fuzzy Adaptive PSO

In the common PSO algorithm, each particle finds for an optimal solution to the objective function in the search space. Each particle updates its position based on its previous position and new information regarding velocity. Its best location found in the search space so far is called p_{best} , and the best location found for all the particles in the population is called g_{best}. The approach of the PSO is shown as follows [10]:

Step1. Initialize position and velocity vectors.

Step2. Evaluate fitness function value.

Step3. Update p_{best} and g_{best}.

Step4. Modify search points.

Update the velocity vector for particle i using the following equation:

$$V_i^{k+1} = wV_i^k + c_1 \times random_1 \times \left(p_{best_i} - X_i^k\right) + c_2 \times random_2 \times \left(g_{best_i} - X_i^k\right)$$

Where V_i^k is the current velocity of particle i at iteration k, w is the inertia weight, c_1 and c_2 are the acceleration coefficients, $random_1$ and $random_2$ are random numbers between 0 and 1, X_i^k is the current position of particle i at iteration k, p_{best_i} is the best value of particle i and g_{best} is the best evaluation value among the p_{best}. The inertia weight w is given by

$$w = w_{max} - \frac{w_{max} - w_{min}}{itera_{max}} \times k$$

Where w_{max} is the initial weight, w_{min} is the final weight, $itera_{max}$ is the maximum iteration number, k is the current iteration number. Update the position of each particle using the equation, $X_i^{k+1} = X_i^k + V_i^{k+1}$, And check $X_{min} \le X_i^{k+1} \le X_{max}$ where X_{min} and X_{max} represent lower and upper bounds, respectively.

Step5. Termination.

If the maximum number of iterations or the goal is reached, then the procedure is terminated and the latest g_{best} is the optimal solution. Otherwise, proceed to step2. Now, let df denote the difference between the $f(g_{best})$'s for two consecutive iterations, i.e.at iteration k

$$df^k = f\left(g_{best}{}^{k-1}\right) - f\left(g_{best}{}^{k}\right) \ge 0, \qquad (2)$$

When the difference is small, particularly in the beginning, the coefficient c_1 should be enlarged to extend the search region to obtain a global result and the coefficient c_2 should be decreased to a smaller value to prevent a premature termination of searching. An increase in df implies that the particles are converging to an optimal point. Therefore, in order to speed up the convergence, the coefficient c_1 is set to a smaller value and the coefficient c_2 is increased to a larger value. In between these two extreme cases, i.e. df is medium, then the coefficients c_1 and c_2 are kept at medium values. Based on the above inference, the following fuzzy rules are suggested:

1) If df is small, then c_1 is big and c_2 is small.
2) If df is medium, then c_1 is medium and c_2 is medium.
3) If df is big, then c_1 is small and c_2 is big.

Let $itera_{max}$ denote the maximum number of iterations, which is given and divided into $\tilde{p}$ partitions, depicted by p_j , j=0, 1,…, $\tilde{p}$ -1. Suppose that partition p_j consist of n_j iterations, then: $itera_{max} = \sum_{j=0}^{\tilde{p}-1} n_j$.

After n_j iterations of modifying the position of the particles in partition p_j . Note that there are n_j new df 's being generated by (2) after n_j iterations. Accordingly, the values of d_1 , d_2 and $\tilde{d}$ are updated by

$$\tilde{d} = max\left\{\text{the previous } \tilde{d}\left(ignored \text{ if } j=0\right) \text{and the } n_j \text{ new } df's\right\},\ d_1 = \alpha\bar{d},\ d_2 = \beta\bar{d}$$

Where α and β with $0 < \alpha \le \beta < 1$ are given a priori. Note that the membership function for the medium df becomes triangular if $\alpha = \beta$. As for the first n_0 interactions, because d_1 , d_2

and $\tilde{d}$ are not yet available, we will adopt the consequence part of fuzzy Rule 1 by choosing a larger constant for c_1 and a smaller constant for c_2 [10].

The main difference is that the proposed Fuzzy Adaptive PSO utilizes fuzzy inferences to adaptively adjust acceleration coefficients instead of using fixed constants as in the standard PSO and is presented as follows:

Step1. Provide a maximum iteration number $itera_{max}$ and divide it into $\tilde{p}$ partitions $p_j, j = 0,1,..., \tilde{p}-1$, assuming that partition p_j consists of n_j iterations.

Step2. For the first n_0 iterations in partition p_0 :

a) Initialize each particle in the population by randomly selecting its location and velocity vectors within the search range and select a larger constant for c_1 and a smaller constant for c_2 . Evaluate the fitness function to gain the initial p_{best} of each particle and the initial g_{best} in the population.

b) Modify the search points by (1)-(3).

c) Each particle is evaluated using the objective function of the target problem to determine its best value so far (p_{best}) and the globally best value so far (g_{best}). Then, substitute the previous and the current f (g_{best})'s into (4) to obtain a new df and store it.

d) If then n_0 iterations in partition p_0 are completed, determine the values of d_1, d_2 and $\tilde{d}$ by utilizing $n_0$dfs in (6) and proceed to step3. Otherwise, go to step 2-b.

Step3. For each of the n_j iterations in partition $p_j, j = 1,..., \tilde{p}-1$:

a) Use the latest df in the fuzzy inference rules 1-3 and defuzzify the outputs by the centroid method to determine a new set of acceleration coefficients c_1 and c_2 .

b) Update the search points by using the new c_1 and c_2 in (1)-(3).

c) Evaluate the objective function of each particle to determine its best value so far (p_{best}) and the globally best value so far (g_{best}). Then, calculate a new df by (4) and store the new df .

d) If the n_j iterations in partition p_j are completed, update the new values of d_1 , d_2 and $\tilde{d}$ by substituting the corresponding $n_j dfs$ and the latest $\tilde{d}$ into (6) and proceed to step4. Otherwise, go to step 3-a.

Step4. If the maximum iteration number $itera_{max}$ or the goal is reached, then the procedure is terminated and the latest g_{best} is the optimal solution. Otherwise, step3 is continued.

4. Case Study

In the present study, the portfolio optimization problem of stocks is answered in three different models by applying the Fuzzy Adaptive PSO technique. The statistical society includes all active companies in Tehran stock market from the October of 2009 to the October of 2013. To fulfil this aim the price information of 50 accepted companies' stocks has been used. This number is selected based on the regular and comprehensive information of the chosen companies. The information related to the mean and the standard deviation of different stocks is mentioned in table1. Considering the mentioned information, the cost efficiency mean of all chosen stocks is negative in the under studied efficiency.

Table1. Price Information

Number of days traded	Return standard deviation	Return average	Price average	Stock number	Number of days traded	Return standard deviation	Return average	Price average	Stock number
1015	0.0253320	0.10083	2131.21	26	1026	0.0213432	0.07465	1300.67	1
1072	0.0204661	0.05885	4088.67	27	950	0.0311813	–0.00591	1349.52	2
981	0.0228405	0.06018	5497.98	28	1123	0.0139251	0.04135	1002.69	3
1066	0.0262698	–0.04539	2827.83	29	779	0.0240394	0.00413	847.0565	4
992	0.0202733	0.02062	1159.18	30	846	0.0201253	0.08112	2026.02	5
1051	0.0312888	–0.00021	2833.30	31	1095	0.024784	0.02511	2207.64	6
603	0.0437742	0.23595	1829.778	32	1022	0.0425357	0.05776	7567.32	7
1105	0.0419705	–0.04738	6421.28	33	943	0.0337611	0.11867	16532.42	8
992	0.0456699	–0.06723	3635.56	34	1126	0.0267414	0.05715	2134.15	9
934	0.01 97288	0.16996	4217.45	35	1073	0.0260185	0.00654	1076.13	10
1073	0.0254758	0.09681	1268.04	36	992	0.0220033	0.11752	2192.93	11
1052	0.0381612	0.01173	2867.81	37	1020	0.0273769	0.08292	1621.899	12
715	0.0240062	–0.09725	964	38	1087	0.0280214	–0.03429	3440	13
988	0.0312131	0.16478	1315.56	39	989	0.0257256	0.04786	3923.68	14
1039	0.0175326	0.09948	3130.83	40	953	0.0290539	0.04814	695.2067	15
863	0.0427488	0.01772	5092.79	41	1162	0.0319661	–0.01964	4309.32	16
969	0.0278913	0.00583	943.7337	42	1082	0.0231125	–0.05703	1134.43	17
1140	0.0230989	0.01640	1393.69	43	883	0.0245320	0.17607	13014.47	18
765	0.0338280	–0.16443	2187.39	44	1014	0.0337892	–0.03513	4497.01	19
815	0.0218729	0.05112	13069.59	45	1017	0.0272021	0.12422	902.7414	20
908	0.0517853	–0.11364	10839.92	46	1076	0.0231704	–0.04322	2369.82	21
1044	0.0202835	0.07982	1669.13	47	1116	0.0190829	0.02806	2448.677	22
1137	0.0253991	–0.02965	2837.38	48	1105	0.0170739	0.09564	1223.429	23
949	0.0279197	0.09718	6032.81	49	972	0.0290891	0.01888	10199.64	24
1009	0.0215035	0.05851	5470.77	50	1180	0.0280090	0.06501	2001.85	25

In the proposed algorithm, $K = 10$, $\varepsilon_i = 10$ and $\delta_i = 1$ in which $i = 1, 2..., N$ have been used. Considering $\Delta\lambda = 0.02$, this algorithm must be repeated 51 times for each model; therefore, the exact 51 points of the efficient frontier can be attained. Results obtained from applying this algorithm along with the efficient frontier of the mean-variance model of the standard Markowitz are drawn in three different figures for each model, respectively.

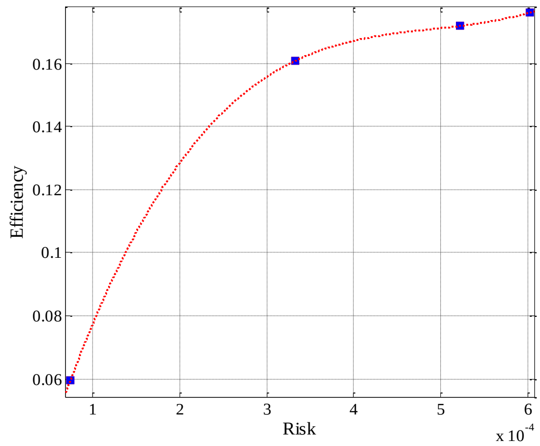

Fig1. the effective border calculated from applying model 1

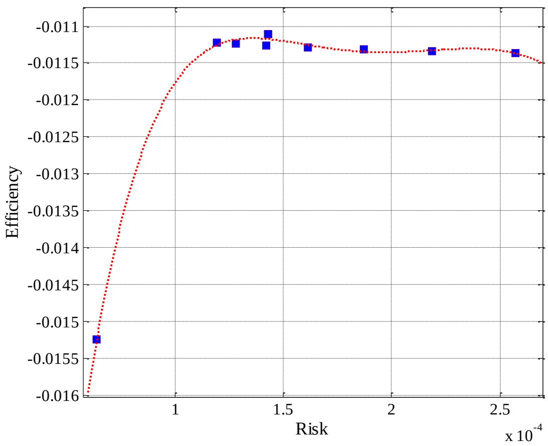

Fig 2. the effective border calculated from applying model 2

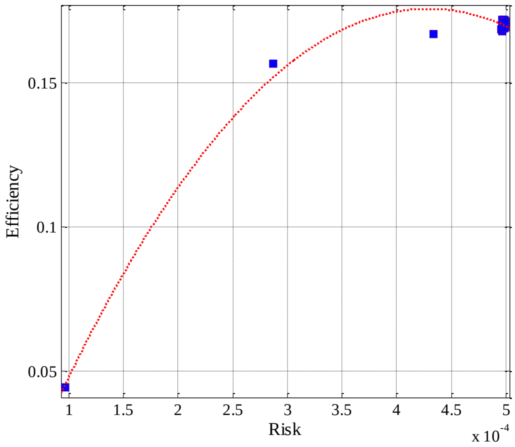

Fig3. the effective border calculated from applying model 3

5. Conclusion

The aim of this study is selecting the portfolio and determining the investment efficient frontier**.** Here, the Markowitz mean –variance model has been answered by applying Fuzzy Adaptive PSO method in three different levels under some imposed restraints. Since model 3 which is a combined problem of both the real programming and the quadratic programming is difficult to solve, the present heuristic algorithm has been chosen. Results of the study have proved the efficiency of the proposed method.

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