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On asymptotically lacunary statistical equivalent functions via ideals

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Abstract

The goal of this paper is to introduce $\mathcal{I}_\theta$-asymptotically statistical equivalent by taking nonnegative two real-valued Lebesgue measurable functions $\gamma(\nu)$ and $\mu(\nu)$ in the interval $(1, \infty)$ instead of sequences and we establish some inclusion relations.

Keywords: Asymptotical equivalent functions, ideal convergence, lacunary sequence, $\mathcal{I}$-statistical convergence.

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1. Introduction and preliminaries

Quite recently, Das et al. [1] studied well known summability methods by using ideal and introduced new notions, namely ideal statistical convergence and ideal lacunary statistical convergence.

In 1993 Marouf [5] introduced definitions of asymptotically equivalent sequences and asymptotic regular matrices. Later on Patterson [6] extended these concepts by presenting an asymptotically statistical equivalent. In [8] Savaş introduced $\mathcal{I}_\theta$-asymptotically statistical equivalent sequences. Also Gümüs and Savaş [14] generalized $\mathcal{I}_\theta$-asymptotically statistical equivalent sequences. In [10], Savaş studied generalized summability methods of functions and also introduced statistically convergent functions via ideals, (see, [11]).

The notion of $\mathcal{I}$-convergence was studied by Kostyrko et al. [4]. Some works on ideals can be found in [3, 9, 12, 13].

The main objective is to present $\mathcal{I}_\theta$-asymptotically statistical equivalent and $\mathcal{I}$-asymptotically statistical equivalent by taking nonnegative two real-valued Lebesgue measurable functions in the interval $(1, \infty)$. Furthermore we prove some interesting theorems.

Definition 1.1 ([5, Marouf])**.** Let $\gamma = (\gamma_i)$ and $\mu = (\mu_i)$ be two nonnegative sequences. If

$$\lim_i \frac{\gamma_i}{\mu_i} = 1,$$

then we say that $\gamma = (\gamma_i)$ and $\mu = (\mu_i)$ are *asymptotically equivalent* and it is denoted by $x \sim y$.

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Definition 1.2 ([2, Fridy])**.** Let $\gamma = (\gamma_i)$ be a sequence, if for every $\varphi > 0$,

$$\lim_n \frac{1}{n}\{\text{the number of } i \leqslant n : |\gamma_i - \beta| \geqslant \varphi\} = 0,$$

then we say that $\gamma = (\gamma_i)$ is statistically convergent to β.

The next definition is natural combination of Definitions 1.1 and 1.2.

Definition 1.3 ([6, Patterson])**.** Let $\gamma = (\gamma_i)$ and $\mu = (\mu_i)$ be two nonnegative sequences. If for every $\varphi > 0$,

$$\lim_n \frac{1}{n}\left\{\text{the number of } i \leqslant n : \left|\frac{\gamma_i}{\mu_i} - \beta\right| \geqslant \varphi\right\} = 0,$$

then we say that $\gamma = (\gamma_i)$ and $\mu = (\mu_i)$ are asymptotically statistical equivalent of multiple β and it is denoted by $\gamma \overset{S_\beta}{\sim} \mu$ and simply asymptotically statistical equivalent if $\beta = 1$.

The following definitions and notions will be needed.

Definition 1.4 ([4])**.** A non-empty family $\mathcal{J} \subset 2^Y$ of subsets of a nonempty set Y is said to be an ideal in Y if the following conditions hold

(i). $R, S \in \mathcal{J}$ implies $R \cup S \in \mathcal{J}$;
(ii). $R \in \mathcal{J}$, $S \subset R$ implies $S \in \mathcal{J}$.

Definition 1.5 ([3])**.** A non-empty family $\mathcal{F} \subset 2^{\mathbb{N}}$ is said to be a filter of $\mathbb{N}$ if the following conditions hold:

(i). $\emptyset \notin \mathcal{F}$;
(ii). $R, S \in \mathcal{F}$ implies $R \cap S \in \mathcal{F}$;
(iii). $R \in \mathcal{F}$, $S \subset R$ imply $S \in \mathcal{F}$.

If $\mathcal{J}$ is proper ideal of $\mathbb{N}$ (i.e., $\mathbb{N} \notin \mathcal{J}$), then the family of sets $\mathcal{F}(\mathcal{J}) = \{K \subset \mathbb{N} : \exists R \in \mathcal{J} : K = \mathbb{N} \backslash R\}$ is a filter of $\mathbb{N}$. It is called the filter associated with the ideal.

Definition 1.6 ([3, 4])**.** A proper ideal $\mathcal{J}$ is said to be admissible if $\{n\} \in \mathcal{J}$ for each $n \in \mathbb{N}$.

Given $\mathcal{J} \subset 2^{\mathbb{N}}$ be a nontrivial ideal in $\mathbb{N}$. The sequence (γ_i) is said to be $\mathcal{J}$-convergent to β, if for each $\varphi > 0$ the set $A(\varphi) = \{n \in \mathbb{N} : |\gamma_i - \beta| \geqslant \varphi\}$ belongs to $\mathcal{J}$ (see, [3, 4]). Following these results we introduce two new notions $\mathcal{J}_\theta$-asymptotically statistical equivalent of multiple β and strong $\mathcal{J}_\theta$-asymptotically equivalent of multiple β.

By a lacunary $\theta = (l_s)$; $s = 0, 1, 2, \ldots$, where $l_0 = 0$, we shall mean an increasing sequence of non-negative integers with $\tau_s = l_s - l_{s-1} \to \infty$ as $s \to \infty$. The intervals determined by θ will be denoted by $J_s = (l_{s-1}, l_s]$ and the ratio $\frac{l_s}{l_{s-1}}$ will be denoted by q_s.

Patterson and Savaş [7] introduced the following definition.

Definition 1.7. Let $\theta = (l_s)$ be a lacunary sequence, two nonnegative sequences $\gamma = (\gamma_i)$ and $\mu = (\mu_i)$ are said to be asymptotically lacunary statistical equivalent of multiple β provided that for every $\varphi > 0$

$$\lim_s \frac{1}{\tau_s}\left|\left\{i \in J_s : \left|\frac{\gamma_i}{\mu_i} - \beta\right| \geqslant \varphi\right\}\right| = 0,$$

where the vertical bars indicate the number elements in the enclose set.

The following definitions are given in [1].

Definition 1.8. A sequence $\gamma = (\gamma_i)$ is said to be $\mathcal{J}$-statistically convergent to β or $S(\mathcal{J})$-convergent to β if, for any $\varphi > 0$ and $\psi > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{n}|\{s \leqslant n : |\gamma_i - \beta| \geqslant \varphi\}| \geqslant \psi\right\} \in \mathcal{J}.$$

In this case, we write $\gamma_i \to \beta(S(\mathcal{J}))$. The class of all $\mathcal{J}$-statistically convergent sequences will be denoted by $S(\mathcal{J})$.

Definition 1.9. Let θ be a lacunary sequence. A sequence $\gamma = (\gamma_i)$ is said to be $\mathcal{J}$-lacunary statistically convergent to β or $S_\theta(\mathcal{J})$-convergent to β if, for any $\varphi > 0$ and $\psi > 0$,

$$\left\{s \in \mathbb{N} : \frac{1}{\tau_s}|\{i \in J_s : |\gamma_i - \beta| \geqslant \varphi\}| \geqslant \psi\right\} \in \mathcal{J}.$$

In this case, we write $\gamma_i \to \beta(S_\theta(\mathcal{J}))$. The class of all $\mathcal{J}$-lacunary statistically convergent sequences will be denoted by $S_\theta(\mathcal{J})$.

Definition 1.10. Let θ be a lacunary sequence. A sequence $\gamma = (\gamma_i)$ is said to be strong $\mathcal{J}$-lacunary convergent to β or $N_\theta(\mathcal{J})$-convergent to β if, for any $\varphi > 0$

$$\left\{s \in N : \frac{1}{\tau_s}\sum_{i \in J_s} |\gamma_i - \beta| \geqslant \varphi\right\} \in \mathcal{J}.$$

In this case, we write $\gamma_i \to \beta(N_\theta(\mathcal{J}))$. The class of all strong $\mathcal{J}$-lacunary statistically convergent sequences will be denoted by $N_\theta(\mathcal{J})$.

We now introduce the following definitions.

Definition 1.11. Let θ be a lacunary sequence and $\gamma(\nu)$ be a nonnegative real-valued Lebesgue measurable function in the interval $(1, \infty)$ if

$$\mathcal{J} - \lim_{s \to \infty} \frac{1}{\tau_s} \int_{\nu \in J_s} |\gamma(\nu) - \beta| \, d\nu = 0.$$

Then we say that the function $\gamma(\nu)$ is $N_\theta(\mathcal{J})$-summable to β. If $\mathcal{J} = \mathfrak{I}_{\text{fin}} = \{L \subseteq \mathbf{N} : L \text{ is a finite subset}\}$, $N_\theta(\mathcal{J})$-summability becomes N_θ-summability, which is defined as

$$\lim_{s \to \infty} \frac{1}{\tau_s} \int_{\nu \in J_s} |\gamma(\nu) - \beta| \, d\nu = 0.$$

Definition 1.12. A nonnegative real-valued Lebesgue measurable function γ (v) is said to be $\mathcal{J}_\theta$-statistically convergent or $S_\theta(\mathcal{J})$ convergent to β, if for every $\varphi > 0$ and $\psi > 0$,

$$\left\{s \in N : \frac{1}{\tau_s}|\{\nu \in J_s \, |\gamma(\nu) - \beta| \geqslant \varphi\}| \geqslant \psi\right\} \in \mathcal{J}.$$

In this case, we write $S_\theta(\mathcal{J}) - \lim \gamma(\nu) = \beta$ or $\gamma(\nu) \to \beta \ (S_\theta(\mathcal{J}))$. If we take $\mathcal{J} = \mathcal{J}_{\text{fin}}$, then $S_\theta(\mathcal{J})$-convergence reduces to lacunary statistical convergence.

2. New definitions

Definition 2.1. Let θ be a lacunary sequence; and $\mathcal{J}$ be an admissible ideal in $\mathbb{N}$ and $\gamma(\nu)$, $\mu(\nu)$ be two nonnegative real-valued Lebesgue measurable functions in the interval $(1, \infty)$. If for every $\varphi > 0$ and $\psi > 0$,

$$\{s \in \mathbb{N} : \frac{1}{\tau_s}|\{\nu \in J_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \varphi\}| \geqslant \psi\} \in \mathcal{J},$$

then we say that the functions $\gamma(\nu)$ and $\mu(\nu)$ are $\mathcal{J}_\theta$-asymptotically equivalent of multiple β (denoted by $\gamma(\nu) \overset{S_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$) and simply $\mathcal{J}$-asymptotically lacunary statistical equivalent if $\beta = 1$. Furthermore, let $S_\theta^\beta(\mathcal{J})$ denote the set of $\gamma(\nu)$ and $\mu(\nu)$ such that $\gamma(\nu) \overset{S_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$.

If we take $\mathcal{J} = \mathcal{J}_{\text{fin}}, \mathcal{J}_\theta$-asymptotically statistical equivalent coincides with lacunary asymptotically statistical equivalent which is given below.

Definition 2.2. Let θ be a lacunary sequence; and $\mathcal{J}$ be an admissible ideal in $\mathbb{N}$ and $\gamma(\nu)$, $\mu(\nu)$ be two nonnegative real-valued functions which are measurable in the interval $(1, \infty)$. If for every $\varphi > 0$

$$\lim_s \frac{1}{\tau_s} \left| \left\{ \nu \in J_s : \left| \frac{\gamma(\nu)}{\mu(\nu)} - \beta \right| \geqslant \varphi \right\} \right| = 0,$$

then we say that the functions $\gamma(\nu)$ and $\mu(\nu)$ are lacunary asymptotically statistical equivalent of multiple β (denoted by $\gamma(\nu) \overset{S_\theta^\beta}{\sim} \mu(\nu)$), and simply asymptotically statistical equivalent if $\beta = 1$.

Definition 2.3. Let θ be a lacunary sequence; and $\mathcal{J}$ is an admissible ideal in $\mathbb{N}$ and $\gamma(\nu)$, $\mu(\nu)$ be two nonnegative real-valued Lebesgue measurable functions in the interval $(1, \infty)$. If

$$\left\{ s \in \mathbb{N} : \frac{1}{\tau_s} \int_{\nu \in J_s} \left| \frac{\gamma(\nu)}{\mu(\nu)} - \beta \right| d\nu \geqslant \varphi \right\} \in \mathcal{J},$$

we say that the functions $\gamma(\nu)$ and $\mu(\nu)$ are strongly $\mathcal{J}_\theta$-asymptotically equivalent of multiple β (denoted by $\gamma(\nu) \overset{N_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$) and strong simply $\mathcal{J}$-asymptotically lacunary equivalent if $\beta = 1$. Let $N_\theta^\beta(\mathcal{J})$ denote the set of $\gamma(\nu)$ and $\mu(\nu)$ such that $\gamma(\nu) \overset{N_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$.

If $\mathcal{J} = \mathcal{J}_{\text{fin}} = \{L \subseteq \mathbf{N} : L \text{ is a finite subset } \}$, strongly $\mathcal{I}_\theta$-asymptotically equivalent becomes strongly lacunary asymptotically equivalent which is defined as

$$\lim_{s \to \infty} \frac{1}{\tau_s} \int_{\nu \in J_s} \left| \frac{\gamma(\nu)}{\mu(\nu)} - \beta \right| d\nu = 0.$$

3. Main result

Theorem 3.1. *Let* $\theta = \{l_s\}$ *be a lacunary sequence, then*

1. *if* $\gamma(\nu) \overset{N_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$, *then* $\gamma(\nu) \overset{S_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$;
2. *if* $\gamma(\nu)$ *and* $\mu(\nu) \in B(X, Y)$ *and* $\gamma(\nu) \overset{S_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$, *then* $\gamma(\nu) \overset{N_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$;
3. $\gamma(\nu) \overset{S_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu) \cap B(X, Y) = \gamma(\nu) \overset{N_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu) \cap B(X, Y)$,

where $B(X, Y)$, *is set of bounded functions.*

Proof.

Part (1): If $\varphi > 0$ and $\overset{N_\theta^L(\mathcal{J})}{\sim} \mu(\nu)$, then

$$\int_{\nu \in J_s} \left| \frac{\gamma(\nu)}{\mu(\nu)} - \beta \right| d\nu \geqslant \int_{\nu \in J_s \& |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| > \varphi} \left| \frac{\gamma(\nu)}{\mu(\nu)} - \beta \right| d\nu \geqslant \varphi \left| \left\{ \nu \in J_s : \left| \frac{\gamma(\nu)}{\mu(\nu)} - \beta \right| \geqslant \varphi \right\} \right|$$

and so

$$\frac{1}{\tau_s} \int_{\nu \in J_s} \left| \frac{\gamma(\nu)}{\mu(\nu)} - \beta \right| d\nu \geqslant \frac{1}{\tau_s} \left| \left\{ \nu \in J_s : \left| \frac{\gamma(\nu)}{\mu(\nu)} - \beta \right| \geqslant \varphi \right\} \right|.$$

Then, for any $\psi > 0$

$$\left\{s \in \mathbb{N} : \frac{1}{\tau_s}\left|\left\{\nu \in J_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \varphi\right\}\right| \geqslant \psi\right\} \subseteq \left\{s \in N : \frac{1}{\tau_s}\int_{\nu \in J_s}\left|\frac{\gamma(\nu)}{\mu(\nu)} - L\right| d\nu \geqslant \varphi.\psi\right\} \in \mathcal{J}.$$

Hence we have $\gamma(\nu) \overset{S_\theta^L(\mathcal{J})}{\sim} \mu(\nu)$.

Part (2): Suppose $\gamma(\nu)$ and $\mu(\nu)$ are in $B(X, Y)$ and $\gamma \overset{S_\theta^\beta(\mathcal{J})}{\sim} \mu$. Then we can assume that

$$\left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \leqslant M \text{ for all } \nu.$$

Given $\varphi > 0$, we have

$$\begin{aligned}\frac{1}{\tau_s}\int_{\nu \in J_s}\left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| d\nu &= \frac{1}{\tau_s}\int_{\nu \in J_s \& \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| > \varphi}\left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| d\nu + \frac{1}{\tau_s}\int_{\nu \in J_s \& \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| < \varphi}\left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| d\nu \\ &\leqslant \frac{M}{\tau_s}\left|\left\{\nu \in J_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \frac{\varphi}{2}\right\}\right| + \frac{\varphi}{2}.\end{aligned}$$

Consequently, we have

$$\left\{s \in \mathbb{N} : \frac{1}{\tau_s}\int_{\nu \in J_s}\left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| d\nu \geqslant \varphi\right\} \subseteq \left\{s \in N : \frac{1}{\tau_s}\left|\left\{\nu \in J_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \frac{\varphi}{2}\right\}\right| \geqslant \frac{\varphi}{2M}\right\} \in \mathcal{J}.$$

Therefore $\gamma(\nu) \overset{N_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu)$.

Part (3): It follows from (1) and (2). □

Theorem 3.2. *Let $\mathcal{J}$ be an ideal and $\theta = \{l_s\}$ be a lacunary sequence with* $\liminf q_s > 1$*, then*

$$\gamma(\nu) \overset{S^\beta(\mathcal{J})}{\sim} \mu(\nu) \textit{ implies } \gamma(\nu) \overset{S_\theta^\beta(\mathcal{J})}{\sim} \mu(\nu).$$

Proof. Suppose first that $\liminf q_s > 1$, then there exists a $\delta > 0$ such that $q_s \geqslant 1 + \delta$ for sufficiently large s, which implies

$$\frac{\tau_s}{l_s} \geqslant \frac{\delta}{1 + \delta}.$$

If $x \overset{S_\theta^\beta(\mathcal{J})}{\sim} y$, then for every $\varphi > 0$ and for sufficiently large s, we have

$$\frac{1}{\tau_s}\left|\left\{\nu \leqslant l_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \varphi\right\}\right| \geqslant \frac{1}{l_s}\left|\left\{\nu \in J_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \varphi\right\}\right| \geqslant \frac{\delta}{1 + \delta}\frac{1}{\tau_s}\left|\left\{\nu \in J_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \epsilon\right\}\right|.$$

Then, for any $\psi > 0$, we get

$$\left\{s \in \mathbb{N} : \frac{1}{\tau_s}\left|\left\{\nu \in J_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \varepsilon\right\}\right| \geqslant \delta\right\} \subseteq \left\{s \in N : \frac{1}{l_s}\left|\left\{l \leqslant l_s : \left|\frac{\gamma(\nu)}{\mu(\nu)} - \beta\right| \geqslant \varphi\right\}\right| \geqslant \frac{\psi\delta}{(1 + \delta)}\right\} \in \mathcal{J}.$$ □

For the next result we suppose that the lacunary sequence θ satisfies the condition that for any set $C \in F(\mathcal{J})$, $\bigcup\{\nu : l_{s-1} < \nu < l_s, s \in C\} \in F(\mathcal{J})$.

Theorem 3.3. *Let $\mathcal{J}$ be an ideal and $\theta = (l_s)$ be a lacunary sequence with* $\sup q_s < \infty$*, then*

$$\gamma(\nu) \overset{S_\theta^L(\mathcal{J})}{\sim} \mu(\nu) \textit{ implies } \gamma(\nu) \overset{S^L(\mathcal{J})}{\sim} \mu(\nu).$$

Proof. If $\limsup_s q_s < \infty$, then without any loss of generality we can assume that there exists a $B \in (0, \infty)$ such that $q_s < B$ for all $s \geqslant 1$. Assume that $\gamma \overset{S_\theta^\beta(\mathcal{I})}{\sim} \mu$ and for $\varphi, \psi, \psi_1 > 0$ write the sets

$$C = \{s \in \mathbb{N} : \frac{1}{\tau_s}|\{\nu \in J_s : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| < \psi\}$$

and

$$T = \{n \in \mathbb{N} : \frac{1}{n}|\{s \leqslant n : |\frac{\gamma_\nu}{\mu_\nu} - \beta| \geqslant \varphi\}| < \psi_1\}.$$

It is clear that $C \in F(\mathcal{I})$, the filter associated with the ideal $\mathcal{I}$. Further consider that

$$A_j = \frac{1}{\tau_j}|\{s \in J_j : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| < \psi$$

for all $j \in C$. Let $n \in \mathbb{N}$ be such that $l_{s-1} < n < l_s$ for some $s \in C$. Now

$$\begin{aligned}
\frac{1}{n}|\{\nu \leqslant n : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| &\leqslant \frac{1}{l_{s-1}}|\{l \leqslant l_s : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| \\
&= \frac{1}{l_{s-1}}|\{\nu \in J_1 : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| + \cdots + \frac{1}{l_{s-1}}|\{\nu \in J_s : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| \\
&= \frac{l_1}{l_{s-1}}\frac{1}{\tau_1}|\{\nu \in J_1 : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| + \frac{l_2 - l_1}{l_{s-1}}\frac{1}{\tau_2}|\{l \in J_2 : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| + \cdots + \\
&\quad + \frac{l_s - l_{s-1}}{l_{s-1}}\frac{1}{\tau_s}|\{\nu \in J_s : |\frac{\gamma(\nu)}{\mu(\nu)} - \beta| \geqslant \varphi\}| \\
&= \frac{l_1}{l_{s-1}}A_1 + \frac{l_2 - l_1}{l_{s-1}}A_2 + \cdots + \frac{l_s - l_{s-1}}{l_{s-1}}A_s \\
&\leqslant \sup_{j \in C} A_j . \frac{l_s}{l_{s-1}} < B\delta.
\end{aligned}$$

Taking $\delta_1 = \frac{\delta}{B}$ and in view of the fact that $\bigcup\{n : l_{s-1} < n < l_s, s \in C\} \subset T$, where $C \in F(\mathcal{I})$, it follows from our assumption on θ that the set T also belongs to $F(\mathcal{I})$. □

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