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A new method for solving fuzzy MCDM problems

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Abstract

Solving multi-criteria fuzzy decision making problems is one of the most important objects that scolars deal with. In situations that the information about criteria weights for alternatives is completely unknown, choosing the best alternative is more difficult. In this paper, by using of intuitionistic fuzzy sets(IFSs), we combine the concepts of entropy, correlation coefficient of two IFSs and ideal solution to determine the criteria weights and then evaluate the weighted correlation coefficient between an alternative and the ideal solution. According to this value, the alternatives can be ranked. Practicality and effectiveness of this technique, in comparison with other similar methods, persuade the decision makers to use of it.

Keywords: fuzzy decision making, correlation coefficient, intuitionistic fuzzy.

1. Introduction

In this section of this paper, we introduce the concept of an intuitionistic fuzzy set (IFS) and some of preliminaries.

The concept of intuitionistic fuzzy set, as a generalization of the concept of fuzzy set, have received more and more attention since its appearance (1986). By use of this concept we propose a solving

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method for fuzzy multi-criteria decision making (FMCDM) problems. At first we briefly introduce some basic concepts about intuitionistic fuzzy sets.

Definition 1. An IFS A in a finite nonempty set X, called universe of discourse, is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x) | x \in X\rangle\} \tag{1}$$

where $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ and $0 \le \mu_A(x) + \nu_A(x) \le 1$. The values $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and nonmembership degree of the element x to the set A[1].

Definition 2. For an IFS A in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, we define

$$\bar{\mu}_A = \frac{1}{n}\sum_{i=1}^{n} \mu_A(x_i)\, , \bar{\nu}_A = \frac{1}{n}\sum_{i=1}^{n} \nu_A(x_i), \tag{2}$$

where $\bar{\mu}_A$, $\bar{\nu}_A$ denote the average membership and nonmembership grades of A, respectively.

Definition 3. Let A and B be two arbitrary IFSs in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$. The correlation coefficient of A and B is given by [2]:

$$\rho_{(A,B)} = \frac{1}{2}(\rho_1 + \rho_2) \tag{3}$$

where

$$\rho_1(A,B) = \frac{\sum_{i=1}^{n}(\mu_A(x_i)-\bar{\mu}_A)(\mu_B(x_i)-\bar{\mu}_B)}{\sqrt{\sum_{i=1}^{n}(\mu_A(x_i)-\bar{\mu}_A)^2 \sum_{i=1}^{n}(\mu_B(x_i)-\bar{\mu}_B)^2}}, \tag{4}$$

$$\rho_2(A,B) = \frac{\sum_{i=1}^{n}(\nu_A(x_i)-\bar{\nu}_A)(\nu_B(x_i)-\bar{\nu}_B)}{\sqrt{\sum_{i=1}^{n}(\nu_A(x_i)-\bar{\nu}_A)^2 \sum_{i=1}^{n}(\nu_B(x_i)-\bar{\nu}_B)^2}}. \tag{5}$$

Definition 4. The intuitionistic entropy of an IFS A in universe of discourse X is defined as follows [3]:

$$E(A) = \sum_{i=1}^{n}(1 - \mu_A(x_i) - \nu_A(x_i)). \tag{6}$$

2. The process of algorithm

In this section, we present a hybrid methodology using the concepts of correlation coefficient and ideal solution for solving intuitionistic FMCDM problems which the information about criteria weights for alternatives is completely unknown.

Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of alternatives and let $C = \{C_1, C_2, \dots, C_n\}$ be a set of criteria. The evaluation of the alternative A_i is represented by the following IFS:

$$A_i = \{\langle C_j, \mu_{A_i}(C_j), \nu_{A_i}(C_j) | C_j \in C\rangle\} \tag{7}$$

where $0 \le \mu_{A_i}(C_j) + \nu_{A_i}(C_j) \le 1$, (j=1,2,…,n), (i=1,2,…,m).

If the information about weight w_j of the criterion C_j (j=1, 2, …, n) is completely unknown, we can use the following formula of entropy weights for determining the criteria weights:

$$w_j = \frac{1-H_j}{n-\sum_{j=1}^{n} H_j} \tag{8}$$

where $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$,

$$H_j = \frac{1}{m}\, E(C_j) = \frac{1}{m}\sum_{i=1}^{m}(1 - \mu_{A_i}(C_j) - \nu_{A_i}(C_j)) \tag{9}$$

and $0 \le H_j \le 1$ (j=1,2,…,n).

Based on the entropy theory, if the entropy value for each criterion is smaller across alternatives, it should persuade decision makers that such a criterion should be evaluated as a bigger weight; otherwise, such a criterion will be judged unimportant by most decision makers.

Similar to TOPSIS method, we give the ideal solution for the ranking order of the alternatives:

$$A^{\star} = \{\langle C_j, 1, 0\rangle | C_j \in C\}. \tag{10}$$

where j=1,2,…,n.

By use of the equation (3), the correlation coefficient between an alternative A_i and the ideal alternative $A^\star$
with entropy weights for criteria can be measured. A new formula is defined as follows to calculate the weighted correlation coefficient W_i (i=1, 2, …,m) between an alternative A_i and the ideal alternative $A^\star$:

$$W_i(A_i, A^\star) = \frac{1}{2}(\rho_1(A_i, A^\star) + \rho_2(A_i, A^\star)) \quad (11)$$

where

$$\rho_1(A_i, A^\star) = \frac{\sum_{j=1}^{n} w_j (\mu_{A_i}(C_i) - \bar{\mu}_{A_i})}{\sqrt{\sum_{j=1}^{n} w_j (\mu_{A_i}(C_i) - \bar{\mu}_{A_i})^2}}, \quad (12)$$

$$\rho_2(A_i, A^\star) = \frac{\sum_{j=1}^{n} w_j (\nu_{A_i}(C_i) - \bar{\nu}_{A_i})}{\sqrt{\sum_{j=1}^{n} w_j (\nu_{A_i}(C_i) - \bar{\nu}_{A_i})^2}}. \quad (13)$$

The larger the value of weighted correlation coefficient W_i, the better the alternative A_i, because the alternative A_i is closer to ideal alternative $A^\star$. It is obvious that $-1 \le W_i(A_i, A^\star) \le 1$ and if $A_i = A^\star$ then $W_i(A_i, A^\star) = 1$ $(i = 1, 2, \dots, m)$ and $W_i(A_i, A^\star) = W_i(A^\star, A_i)$. According to the new represented formula, all the alternatives can be ranked so that the best alternative can be selected. To illustrate the application of this method we represent a practical example and then compare the result with similar papers that use of this example.

3. Example

Suppose we have a FMCDM problem with four alternatives and three criterions. We want to select the best option. This example can be extended to any FMCDM problem with other information about alternatives that criteria weights are completely unknown. We compared the solving method of this example with several other similar methods in different references. One can observe that the convenience in calculations can prove the effectiveness of this solving method in comparison with other similar solving methods ([4], [5]).

In order to obtaining the degrees to which alternative A_i satisfies and does not satisfy criterion C_j (j=1, 2, 3; i =1, 2, 3, 4), we can use the statistical method. Suppose n experts are expected to answer ''yes'' or ''no'' or ''I don't know'' to the question whether alternative A_i satisfies criterion C_j.

The information obtained from answers of experts, are shown in table 1.

Table 1. Information obtained from the answers of experts

	C_1	C_2	C_3
A_1	⟨45, 35⟩	⟨50,30⟩	⟨20,55⟩
A_2	⟨65,25⟩	⟨65,25⟩	⟨55,15⟩
A_3	⟨45,35⟩	⟨55,35⟩	⟨55,20⟩
A_4	⟨75,15⟩	⟨65,20⟩	⟨35,15⟩

By using equation (8) entropy weights for determining each criterion weight w_j (j=1, 2, …, n) are :
$w_1 = 0.356$, $w_2 = 0.3613$ and $w_3 = 0.2827$, respectively.

By applying equations (2), (12), (13), (11), respectively, we can compute $\bar{\mu}_{A_i}$, $\bar{\nu}_{A_i}$, $\rho_1(A_i, A^\star)$, $\rho_2(A_i, A^\star)$ to obtain $W_i(A_i, A^\star)$ (i=1,2,3,4) as follows:

$W_1(A_1, A^\star) = 0.00185$, $W_2(A_2, A^\star)$= 0.2801, $W_3(A_3, A^\star)$= 0.0395, $W_4(A_4, A^\star)$= 0.11355.

Therefore the ranking order of all of the alternatives is $A_2 \succ A_4 \succ A_3 \succ A_1$. Thus the alternative A_2 is the best choice.

The new formula that we use in this article, is more correct than one defined in [6]. By comparing this solving method with several other similar methods ([4], [5]), one can observe the simplicity, effectiveness and more precise results of it.

4. Conclusion

In this paper we have proposed the method for solving FMCDM problems with unknown information on criteria weights in which criteria values are given by intuitionistic fuzzy numbers. We have combined the concepts of entropy, correlation coefficient of two IFSs and the ideal alternative to obtain a new formula for rank the alternatives and select the best one. This method, in comparison with other solving methods of FMCDM problems that deal with unknown information about criteria weights, is more practical and effective and can efficiently help the decision maker(s).

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