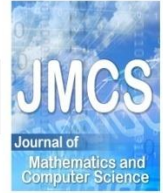


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## Short-Term Electricity Price Forecasting Using Optimal TSK Fuzzy Systems

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### Abstract

Since all financial transactions in restructured power markets are based on electricity prices, it is necessary that the price of electric power be predicted precisely. Some particular features such as: nonlinearity, non-stationary behaviors, as well as volatility of electricity prices make such a prediction a very challenging task. In this paper, a new structure of TSK fuzzy systems is presented that provides high order TSK fuzzy systems from lower orders which have capability of modeling and forecasting chaotic time series. The method used for optimization of fuzzy systems is the Interior point method. Using this method for forecasting electricity price is useful because of its chaotic behavior. The results are compared with RBF neural network and TSK fuzzy system presents better results.

**Keywords:** Forecasting; TSK fuzzy systems; Interior Point Method; RBF neural network.

## 1. Introduction

One of the most essential issues in restructured electric power market to handle all kind of energy transaction (i.e. short-term, midterm and long-term transaction) is price forecasting [1,2,3,4,5]. Various kinds of electricity price prediction methods are being used for different transactions schemes. Day-ahead price forecasting is one of the most important scheme with a crucial importance especially in pool-based power markets [6,7]. During the last two decades, many researchers have focused on the field of price prediction using different approaches that are reviewed in the following: Hsiao suggested an artificial neural network technique to predict long-term electricity price for European Energy Exchange. In his study, autocorrelation function is used for data analysis, while a neural network with {5:7:2} structure is exploited to predict the price in the scheme of one, two, three days and a week as well as three months ahead which performs an automatic regression approach with

respect to prediction error [3]. In [8] a fuzzy-neural network is employed for price forecasting in Spain energy market that results more accurate prediction in comparison with ARMA and Wavelet\_ARIMA. Automatic Dynamic Harmonic Regression is applied to a day-ahead price prediction of Spain energy market in which the reported results show a better performance than ARIMA [7]. In 2005, Wavelet-ARIMA is used in another study for price prediction in Spain energy market. In the first step of WARIMA, historical data of the price pertaining to the previous day is classified into four series. In the second step, an ARMA model for each of the four series is designed to predict 24 values for the day ahead. In the last step, the predicted values by ARIMA are combined using inverse wavelet transform, and final price prediction for the day ahead is obtained [6]. Antonio proposed different techniques such as Transfer Function, and Dynamic Regression for price prediction [7]. ARIMA, Neural Networks, and Wavelet are employed to predict electricity price in PJM market in year 2002 [6]. The results of this study show a better performance of time series techniques in comparison with wavelet and neural networks. The results also show that DR and TF, among the other time series techniques, are more effective than ARIMA models. WL models also behave similar to ARIMA models [1]. Reinaldo & Javier employed GARCH (Generalized Auto Regressive Conditional Heteroskedastic) to predict electricity price in Spain and California markets and showed that GARCH results are more reliable than those of ARIMA. Especially, when electrical load data are included in GARCH model the results may be improved [1]. Neuro-Fuzzy models were employed in a study to predict Locational Marginal Prices. Since LMPs in one region are increased due to power interruption and congestion, linguistic and numerical information are used for prediction and fuzzy logic is therefore employed to transform linguistic data into numerical [5]. A significant feature of hourly electricity price which challenges the prediction process is the high frequency of hourly price signal (see figure 1) [1,9] which demands for the use of TSK fuzzy systems as powerful tools to handle such nonlinear signals. A study was done on the short term forecasting of electricity price using neural network (FPNN) in 2010, the results demonstrated of this method over the probabilistic neural network (PNN) [14].

In this paper various order TSK fuzzy systems have been employed to forecast day ahead electricity price according to PJM market data. Tuning of these TSK fuzzy systems is done by using the Interior Point Method such that the proposed system gives the right output for an specific input. Results show the advantage of TSK fuzzy systems in forecasting electricity price in comparison with RBF neural network. However it is also shown that zero order TSK fuzzy system has the best performance among other TSK systems.

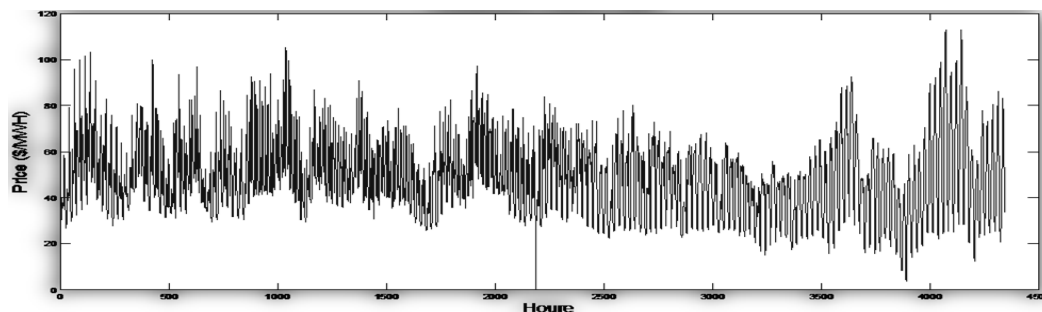


Fig.1: Typical electricity price variations

## 2. Proposed TSK Fuzzy Systems

A typical fuzzy system consists of four fundamental parts [10]: 1- Fuzzifier, 2-Rule Base, 3- Inference Engine, 4- Defuzzifier As shown above, the input (which is the vector  $[x_1 \ x_2 \ : \ : \ x_m]^T$ ) is converted from crisp to fuzzy which is done comparing input value by input membership functions, the result then is employed by inference engine which generates a fuzzy output according to input value and fuzzy rules. finally a real world signal  $y$  is produced by converting fuzzy output back to a crisp value. The important specification of a TSK fuzzy system, is the way it produces output signal. The most popular types of TSK fuzzy systems are: 1- Zero Order (or Constant) and 2- First Order (or Linear) [10]. The  $l$ -th rule of a typical rule base of a Zero order TSK system and a First Order TSK system are as noted below respectively:

$$\text{If } x_1 \text{ is } A_1^l \ \& \ x_2 \text{ is } A_2^l \ \& \ \dots \ \& \ x_m \text{ is } A_m^l \ \text{Then } y = a_0^l$$

$$\text{If } x_1 \text{ is } A_1^l \ \& \ x_2 \text{ is } A_2^l \ \& \ \dots \ \& \ x_m \text{ is } A_m^l \ \text{Then } y = a_0^l + a_1^l x_1 + a_2^l x_2 + \dots + a_m^l x_m$$

Where  $A_i^l$  is a membership function of  $i$ -th input,  $a_0^l$  is the constant value of  $l$ -th rule output and  $a^l$  is the coefficient of  $i$ -th input variable in the linear combination forming output of the  $l$ -th rule  $y^l$ . The total effect of rules is usually calculated by weighted averaging:

$$y = \frac{\sum_{l=1}^r w^l y^l}{\sum_{l=1}^r w^l}$$

Where  $r$  is total number of rules and  $w^l$  is the weight assigned to  $l$ -th rule, which shows how much each rule effects on the output and is calculated according to the dependency of input variables ( $x_1 \dots x_m$ ) to membership functions of  $l$ -th rule ( $A_1^l \dots A_m^l$ )[10]. As it is clear the operation of defuzzifier is almost done by the inference engine and output calculation is much simpler than Mamdani fuzzy system.

## 3. High Order TSK

The idea of higher order TSK fuzzy systems was first introduced by Buckley [11]. Supposing the  $l$ -th rule of a typical TSK fuzzy system from order  $n$  with  $m$  inputs:

$$\text{If } x_1 \text{ is } A_1^l \ \& \ x_2 \text{ is } A_2^l \ \& \ \dots \ \& \ x_m \text{ is } A_m^l \ \text{Then } y = \sum_{\substack{j_1 + \dots + j_m \leq n \\ j_1, j_2, \dots, j_m \geq 0}} (a_{j_1, j_2, \dots, j_m}^l) x_1^{j_1} x_2^{j_2} \dots x_m^{j_m}$$

## 4. Electricity Price Forecasting Using Proposed TSK Fuzzy Systems

In this section TSK fuzzy system, introduced in chapter 3, is used in order to forecast electricity price data in pjm market. in general, there are two methods used in forecasting one-day ahead electricity price.

In this section, the direct method is used to forecast one-day ahead electricity price. That is, using 24 fuzzy models in forecasting 24-hour ahead. The Input data used in training fuzzy system is presented bellow:

$$\begin{cases} X = [P_{h-24}, P_{h-48}, P_{h-72}, P_{h-96}, \\ P_{h-120}, P_{h-144}, P_{h-168}, P_{h-192}, \dots, P_{h-N*24}] \\ y = P_h \end{cases}$$

For example, if  $N = 5$  ,to predict the price of electricity on 24 May 2006, at 1 a.m, Data from 5 days ago at the same time is used. Four days are considered as the input and the day before the forecasting day as the output of the fuzzy system. After the training phase; for predicting the  $P(h+24)$ ,  $P(h)$  is also considered as the input (figure 2). The Membership functions of the input are selected according to figure 3.

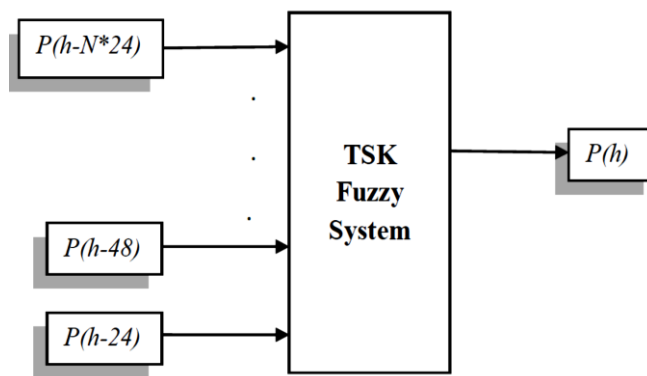


Figure.2: inputs of the fuzzy system

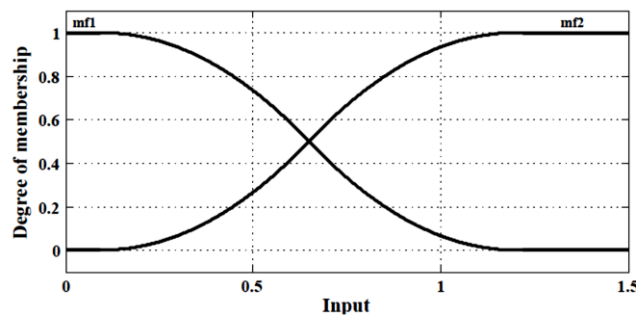


Figure.3: Input Membership function

The interior point method is used to tune the parameters of fuzzy systems [12]. Predicted results for the four weeks of winter, spring, summer and autumn of 2006 are shown in Table 1 to show the Performance of fuzzy system in forecasting, two error criteria of RMSE and MAPE are used.

Table 1. Results for the four weeks of winter, spring, summer and autumn of 2006

Winter Week : March 14, 2006 to March 19, 2006							
Day	N=number of train data	Zero-Order-TSK		First-Order-TSK		Second-Order-TSK	
		RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
Monday	5	3.4694	5.9107	6.6923	13.122	10.471	14.673
Tuesday	5	8.9541	12.616	8.9591	14.896	13.930	17.906
Wednesday	5	4.0447	6.3778	8.9604	12.041	11.963	16.093
Thursday	5	4.3196	7.3617	10.812	18.739	14.621	20.785
Friday	5	6.1919	10.199	7.7608	9.8222	9.7510	12.956
Saturday	5	6.0479	9.5732	9.9936	17.697	12.851	18.541
Sunday	10	8.4940	12.5453	29.460	47.805	24.961	43.760

spring Week : May 14, 2006 to May 19, 2006							
Day	N=number of train data	Zero-Order-TSK		First-Order-TSK		Second-Order-TSK	
		RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
Monday	10	2.0422	5.1259	4.4937	8.5300	39.743	71.990
Tuesday	5	1.7249	4.3821	2.7596	6.3418	11.275	21.696
Wednesday	5	5.3715	7.6428	4.9327	7.5700	5.9805	8.6192
Thursday	5	2.4705	4.6640	1.9365	3.9200	14.312	23.456
Friday	5	2.9729	4.7301	3.8075	6.2792	28.471	51.202
Saturday	10	4.4905	9.0168	7.2440	16.990	16.648	37.676
Sunday	5	4.8952	11.788	3.9345	9.3731	8.557	14.105

Summer Week : August 21, 2006 to August 27, 2006							
Day	N=number of train data	Zero-Order-TSK		First-Order-TSK		Second-Order-TSK	
		RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
Monday	10	6.4310	6.9563	10.984	17.309	14.063	24.479
Tuesday	5	1.8309	2.3835	3.6646	6.7963	11.787	19.899
Wednesday	5	2.5541	4.2171	3.3766	5.1415	9.4467	10.547
Thursday	5	2.6069	4.1190	3.0965	4.6913	10.764	14.370
Friday	5	4.9900	5.7054	4.0628	5.8398	12.417	12.950
Saturday	5	4.5720	8.1743	4.9717	8.4852	8.8767	11.940
Sunday	10	5.6600	10.222	21.204	29.604	22.865	31.860

Fall Week : November 13, 2006 to November 19, 2006							
Day	N=number of train data	Zero-Order-TSK		First-Order-TSK		Second-Order-TSK	
		RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
Monday	10	7.3866	11.203	13.610	20.244	28.340	33.472
Tuesday	5	3.7700	5.3839	7.6137	10.924	30.561	45.018
Wednesday	5	2.2753	3.7518	4.5617	6.4570	23.310	33.264
Thursday	5	2.5290	5.7445	8.0223	12.219	41.810	60.532
Friday	5	4.7729	9.0447	7.3288	13.276	13.850	27.171
Saturday	5	8.2700	16.213	11.979	23.024	14.607	28.490
Sunday	5	5.8154	9.9361	5.8700	10.052	10.663	22.076

According to the results and the Modeling Flowchart, it is concluded that the best model for short term electricity price forecasting is Zero-Order-TSK fuzzy.

RBF neural network [13] is used in forecasting and its comparison with the TSK fuzzy system. While the day before the current day (which is going to be predicted) has the most correlation with it so, eighth days that have more correlation with the day before the current day, are chosen and used as training data. The input vector used for training is defined as follows:

$$\begin{cases} X = [P_{h-24}, P_{h-25}, P_{h-48}, P_{h-49}, P_{h-72}, P_{h-73}, P_{h-96}, P_{h-97}, \\ P_{h-120}, P_{h-121}, P_{h-144}, P_{h-145}, P_{h-168}, P_{h-169}, P_{h-192}, P_{h-193}] \\ y = P_h \end{cases}$$

In this section two days of year 2006 have been selected for instance to be forecasted. Results of day ahead forecast made by fuzzy system and RBF neural network are illustrated on figure 4.

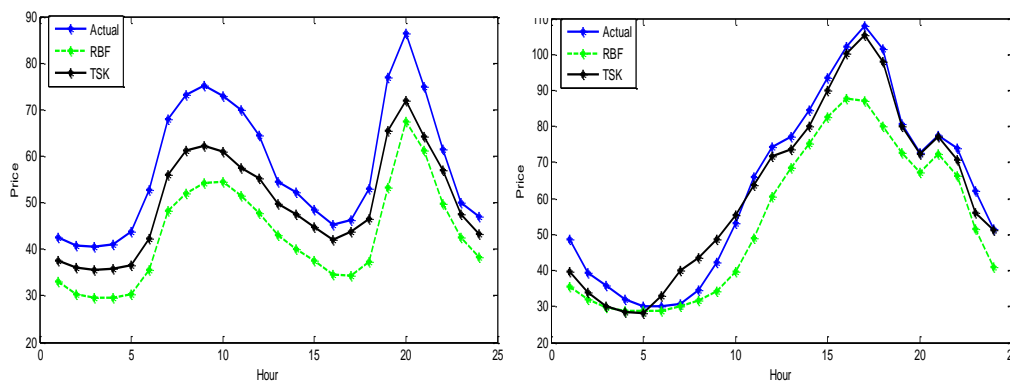


Figure 4. Results of day ahead forecast for 20 march 2006 and 27 may 2006

Now results of forecast for four weeks (selected form winter, spring, summer and fall respectively) made by fuzzy system and RBF neural network, are given in table 2.

Table 2. Results of forecast for four weeks (selected form winter, spring, summer and fall respectively)

Winter Week : March 14, 2006 to March 19, 2006						
Day	TSK			RBF		
	N=number of train data	RMSE	MAPE	N=number of train data	RMSE	MAPE
Monday	10	3.4694	5.9107	17	4.4226	6.4489
Tuesday	5	8.9541	12.616	17	7.2725	10.141
Wednesday	5	4.0447	6.3778	17	6.9525	9.4468
Thursday	10	4.3196	7.3617	17	2.2073	3.2917
Friday	5	6.1919	10.199	17	8.2520	16.260
Saturday	5	6.0479	9.5732	17	7.3135	12.863
Sunday	10	8.4940	12.5453	17	15.105	24.999

spring Week : May 14, 2006 to May 19, 2006						
Day	TSK			RBF		
	N=number of train data	RMSE	MAPE	N=number of train data	RMSE	MAPE
Monday	10	2.0422	5.1259	17	4.5725	9.1932
Tuesday	5	1.7249	4.3821	17	2.3400	5.0720
Wednesday	5	5.3715	7.6428	17	4.4795	7.5081
Thursday	5	2.4705	4.6640	17	3.2597	5.2700
Friday	5	2.9729	4.7301	17	3.8250	6.9822
Saturday	10	4.4905	9.0168	17	7.5669	7.5669
Sunday	5	4.8952	11.788	17	3.9910	11.009

Summer Week : May 21, 2006 to May 27, 2006						
Day	TSK			RBF		
	N=number of train data	RMSE	MAPE	N=number of train data	RMSE	MAPE
Monday	10	6.4310	6.9563	17	8.0660	14.613
Tuesday	5	1.8309	2.3835	17	4.6889	6.8710
Wednesday	5	2.5541	4.2171	17	3.9966	6.7615
Thursday	5	2.6069	4.1190	17	2.2040	3.1946
Friday	5	4.9900	5.7054	17	3.6820	4.4105
Saturday	5	4.5720	8.1743	17	10.769	14.134
Sunday	10	5.6600	10.222	17	6.4744	10.798

Fall Week : November 13, 2006 to November 19, 2006						
Day	TSK			RBF		
	N=number of train data	RMSE	MAPE	N=number of train data	RMSE	MAPE
Monday	10	7.3866	10.203	17	4.2100	10.961
Tuesday	5	3.7700	5.3839	17	6.9772	10.175
Wednesday	5	2.2753	3.7518	17	4.0846	5.5591
Thursday	5	2.5290	5.7445	17	2.9324	5.9996
Friday	5	4.7729	9.0447	17	3.9841	6.3811
Saturday	10	8.2700	16.213	17	5.5070	12.740
Sunday	5	5.8154	9.9361	17	9.8239	20.186

Table 3.

PJM Interconnection: statistical measures for seasons of 2006				
Week	TSK		RBF	
	RMSE	MAPE	RMSE	MAPE
Winter	5.9316	9.5519	7.3607	11.921
Spring	3.4246	6.5906	4.2906	8.1700
Summer	4.0921	5.9682	5.6115	8.6830
Fall	3.7927	8.7539	5.3598	10.2861

## 5. Conclusion

In this paper various order TSK fuzzy systems were employed to forecast day ahead electricity price according to PJM market data. Tuning of these TSK fuzzy systems was done by using the Interior Point Method such that the proposed system gives the right output for an specific input. Results show the advantage of TSK fuzzy systems in forecasting electricity price in comparison with RBF neural network. However it was also seen that zero order TSK fuzzy system had the best performance among other TSK systems.

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