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Optimal Fuzzy Synchronization of Generalized Lorenz Chaotic Systems

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Abstract

In this article two identical generalized Lorenz systems have been synchronized by a fuzzy controller based on mamdani approach and stability of the proposed scheme has been established by the Lyapunov stability theorem. Controller parameters have been optimized by the genetic algorithm. Effectiveness of proposed method has been demonstrated through computer simulation.

Keywords: Fuzzy controller, Synchronization, Generalized Lorenz system, Genetic Algorithm.

1. Introduction

Synchronization of chaotic systems has received a significant attention, since Pecora and Carroll presented the chaos synchronization method for synchronizing two identical chaotic systems with different initial values [1]. Chaos synchronization can be applied widely in the fields of physics and

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engineering systems, such as power converters, chemical reactions, biological systems, information processing, and especially in secure communication [2–6]. So far, different techniques and methods have been proposed to achieve chaos synchronization such as impulsive control [7,8], adaptive control [9,10], sliding mode control [11–14], fuzzy control [15], optimal control [16], backstepping control [18,19], and so on. The Lorenz system is one of the paradigms of chaos, because it exhibits a wide variety of nonlinear dynamics phenomena such as bifurcations and chaos. Recently Lu et al. [20] have proposed the generalized chaotic system which includes the Lorenz and the Chen systems. The solution bounds of generalized Lorenz chaotic system (GLCS) are investigated based on the time-domain approach [21]. In that paper, the synchronization of two identical GLCSs with unknown parameters has been considered.

A fuzzy control system is a control system based on fuzzy logic [22]. Fuzzy logic is widely used in machine control and recently for chaos control and synchronization.

The genetic algorithm (GA) is a search heuristic that mimics the process of natural evolution. This algorithm is usually used to generate solutions to optimization and search problem. To optimize the controller parameters, different optimization techniques including genetic algorithm can be used.

In this work two identical GLCSs have been synchronized by a fuzzy controller whose parameters have been optimized via the GA.

2. Problem Statement

Consider two identical generalized Lorenz chaotic systems as given below:

Master system

$$\begin{cases} \dot{x}_1 = \left(10 + \frac{25}{29}k\right)(x_2 - x_1) \\ \dot{x}_2 = \left(28 - \frac{35}{29}k\right)x_1 + (k-1)x_2 - x_1x_3 \\ \dot{x}_3 = \left(-\frac{8}{3} - \frac{k}{87}\right)x_3 + x_1x_2 \end{cases} \tag{1}$$

This nonlinear system shows chaotic behavior when $k \in [0,1]$. The following figures (Fig1, Fig2 and Fig3) show chaotic attractor for (k=0.5).

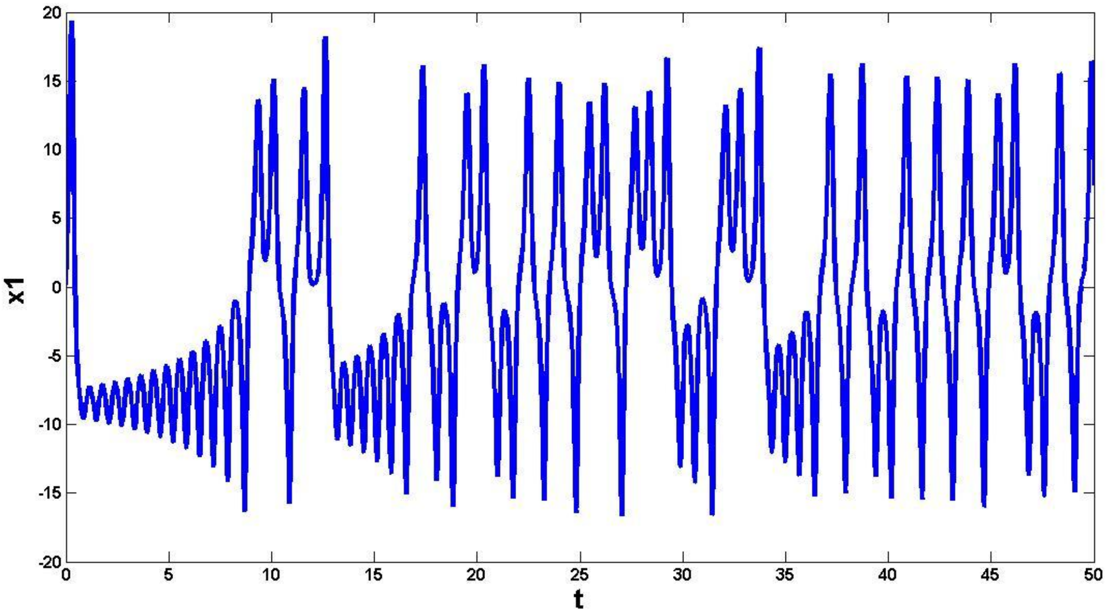

Fig.1. Chaotic behavior (x_1 vs. t)

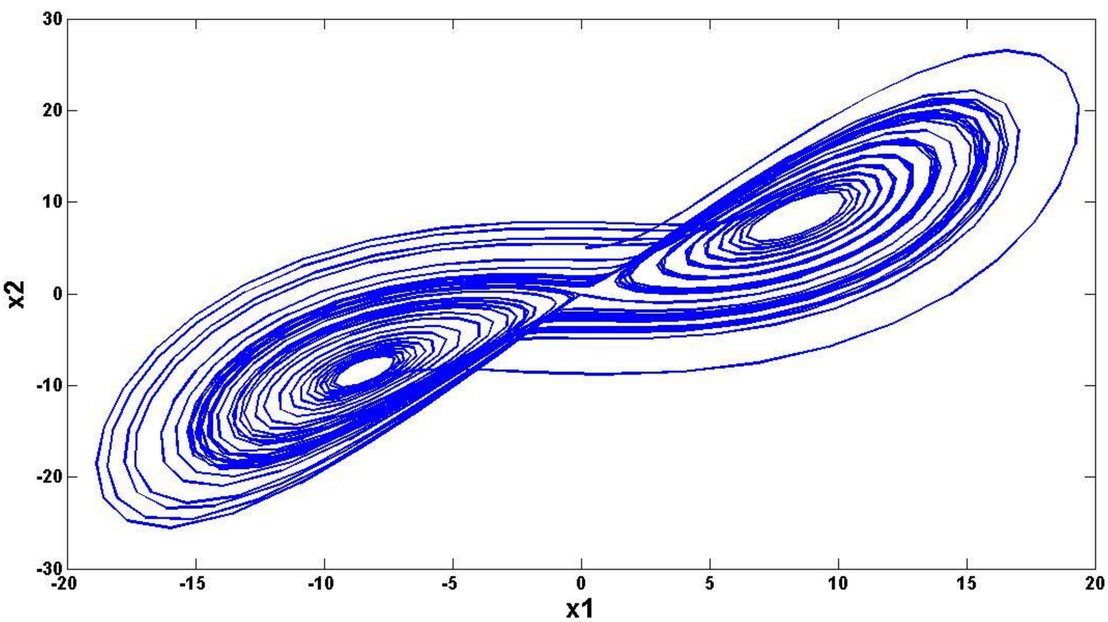

Fig.2. Chaotic attractor (x_1 vs. x_2)

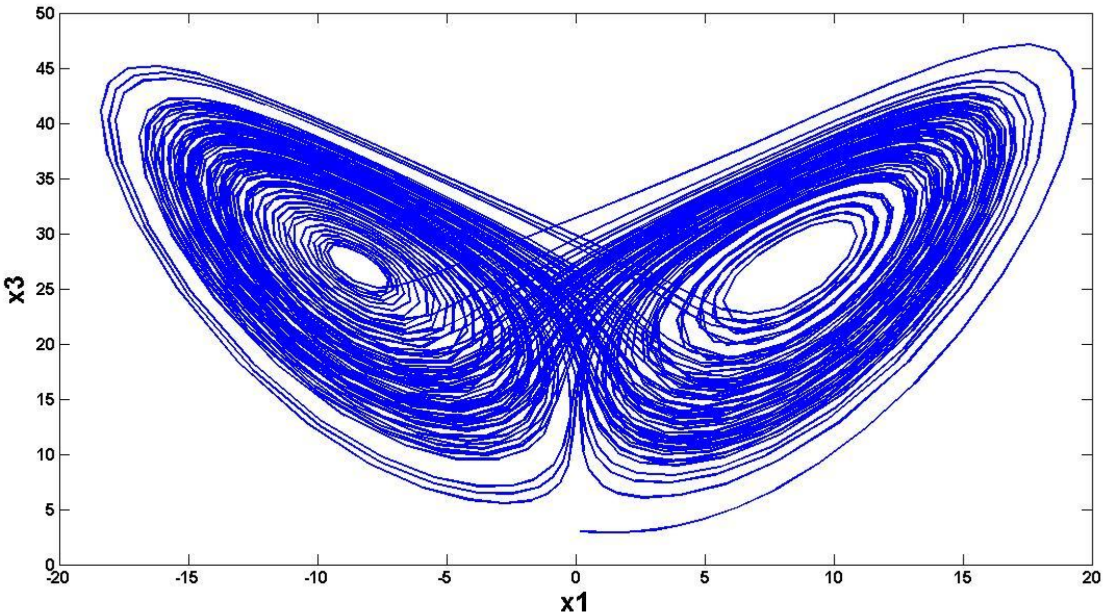

Fig.3. Chaotic attractor (x_1 vs. x_3)

Slave system

$$\begin{cases} \dot{y}_1 = \left(10 + \frac{25}{29}k\right)(y_2 - y_1) \\ \dot{y}_2 = \left(28 - \frac{35}{29}k\right)y_1 + (k-1)y_2 - y_1 y_3 + u \\ \dot{y}_3 = \left(-\frac{8}{3} - \frac{k}{87}\right)y_3 + y_1 y_2 \end{cases} \tag{2}$$

Where u is the manipulated variable.

The objective is synchronizing the above chaotic systems. The synchronization errors have been defined as:

$$\begin{cases} e_1 = x_1 - y_1 \\ e_2 = x_2 - y_2 \\ e_3 = x_3 - y_3 \end{cases} \tag{3}$$

The synchronization errors dynamics are given below:

$$\begin{cases} \dot{e}_1 = \left(10 + \frac{25}{29}k\right)(e_2 - e_1) \\ \dot{e}_2 = \left(28 - \frac{35}{29}k\right)e_1 + (k-1)e_2 + x_1x_3 - y_1y_3 + u \\ \dot{e}_3 = \left(-\frac{8}{3} - \frac{k}{87}\right)e_3 + e_1x_2 + e_2y_1 \end{cases} \quad (4)$$

u is chosen as below:

$$u_l = x_1x_3 - y_1y_3 + u = u - u_{eq} \Rightarrow u = u_l + u_{eq} \;,\; u_{eq} = y_1y_3 - x_1x_3 \quad (5)$$

If the first two equations of (4) were asymptotically stable or $(e_1, e_2 \longrightarrow 0)$ via the input variable (u_l) then the last equation (4) reduces to:

$$\dot{e}_3 = \left(-\frac{8}{3} - \frac{k}{87}\right)e_3 \quad (6)$$

e_3 *will be* a stable internal error dynamics and $(e_3 \longrightarrow 0)$. So (4) reduced to

$$\begin{cases} \dot{e}_1 = \left(10 + \frac{25}{29}k\right)(e_2 - e_1) \\ \dot{e}_2 = \left(28 - \frac{35}{29}k\right)e_1 + (k-1)e_2 + u_l \end{cases} \quad (7)$$

The above equation can be transformed to the following form:

$$\begin{cases} \dot{\bar{e}}_1 = \bar{e}_2 \\ \dot{\bar{e}}_2 = \left(10 + \frac{25}{29}k\right)\left(27 - \frac{6}{29}k\right)\bar{e}_1 + \left(-11 + \frac{4}{29}k\right)\bar{e}_2 + u_l = a\bar{e}_1 + b\bar{e}_2 + u_l \end{cases} \quad (8)$$

By the following state transformation:

$$\begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{10+\frac{25}{29}k} & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \Rightarrow \begin{cases} \bar{e}_1 = \frac{1}{10+\frac{25}{29}k} e_1 \\ \bar{e}_2 = -e_1 + e_2 \end{cases} \quad (9)$$

3. Controller Design

The described systems (1) , (2) have been synchronized via the input variable (u_l) that has been calculated by a fuzzy controller.

The fuzzy controller structure has been determined as follows:

$$u_l = FLC(\bar{e}_1, \bar{e}_2)$$

$$u_l = \frac{\sum_{i=1,\mu_i\neq 0}^{n} \mu_i \cdot u_{li}}{\sum_{i=1,\mu_i\neq 0}^{n} \mu_i} \qquad (10)$$

$$\mu_i = \min\left(\mu_{X_1}(\bar{e}_1), \mu_{X_2}(\bar{e}_2)\right)$$

The fuzzy membership functions are shown in Fig.2.

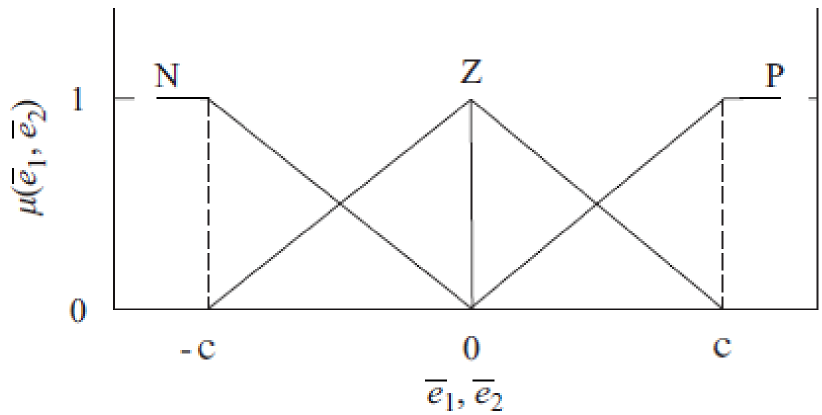

Fig.4 Membership functions

A typical fuzzy rule is as follows:

$$Rule\ i: if\ (\bar{e}_1\ is\ X_1)\ \&\ (\bar{e}_2\ is\ X_2)\ then\ u_{li} = f_i(\bar{e}_1, \bar{e}_2) \qquad (11)$$

Table 1. Fuzzy rule table

Rule	Antecedent $\bar{e}_1$	Antecedent $\bar{e}_2$	Consequent u_{Li}
1	P	P	u_{L1}
2	P	Z	u_{L2}
3	P	N	u_{L3}
4	Z	P	u_{L4}
5	Z	Z	u_{L5}
6	Z	N	u_{L6}
7	N	P	u_{L7}
8	N	Z	u_{L8}
9	N	N	u_{L9}

The rule table consequences of fuzzy module have been obtained based on a Lyapunov function. For controller design the following Lyapunov function has been used:

$$V = \frac{(\bar{e}_1^2+\bar{e}_2^2)}{2} \;\Longrightarrow\; \dot{V} = \bar{e}_1\dot{\bar{e}}_1 + \bar{e}_2\dot{\bar{e}}_2 \tag{12}$$

The control action has been selected such that the time derivative of the Lyapunov function becomes negative.

The consequents in Table.1 have been determined as follows:

Case1. $\bar{e}_2 > 0 : \; \bar{e}_2 \in P$

$\bar{e}_2(\bar{e}_1 + \dot{\bar{e}}_2) < 0 \;\Longrightarrow\; \dot{\bar{e}}_2 < -\bar{e}_1 \;\Rightarrow\; \dot{\bar{e}}_2 = a\bar{e}_1 + b\bar{e}_2 + u_l < -\bar{e}_1 \;\Rightarrow\; u_l < -(1+a)\bar{e}_1 - b\bar{e}_2$

$u_l = -(1+a)\bar{e}_1 - b\bar{e}_2 - \alpha_i \,, \;\; \alpha_i > 0$

$$\boxed{u_{li} = -(1+a)\bar{e}_1 - b\bar{e}_2 - \alpha_i \,, \quad i = 1,4,7} \tag{13}$$

Case2. $\bar{e}_2 < 0 : \; \bar{e}_2 \in N$

$\bar{e}_2(\bar{e}_1 + \dot{\bar{e}}_2) < 0 \;\Longrightarrow\; \dot{\bar{e}}_2 > -\bar{e}_1 \;\Rightarrow\; \dot{\bar{e}}_2 = a\bar{e}_1 + b\bar{e}_2 + u_l > -\bar{e}_1 \;\Rightarrow\; u_l > -(1+a)\bar{e}_1 - b\bar{e}_2$

$u_l = -(1+a)\bar{e}_1 - b\bar{e}_2 + \alpha_i \,, \;\; \alpha_i > 0$

$$\boxed{u_{li} = -(1+a)\bar{e}_1 - b\bar{e}_2 + \alpha_i \,, \quad i = 3,6,9} \tag{14}$$

Case3. $\bar{e}_2 \in Z \,, \bar{e}_1 \in P$

$\bar{e}_2(\bar{e}_1 + \dot{\bar{e}}_2) < 0 \;\Longrightarrow\; \bar{e}_1 + \dot{\bar{e}}_2 = -sgn(e_2) \Longrightarrow\; \dot{\bar{e}}_2 = a\bar{e}_1 + b\bar{e}_2 + u_l < -sgn(e_2)$

$\Longrightarrow\; u_l < -a\bar{e}_1 - b\bar{e}_2 - sgn(e_2) \;\Longrightarrow\; u_l = -a\bar{e}_1 - b\bar{e}_2 - sgn(e_2) - \alpha_i \,, \;\; \alpha_i > 0$

$$\boxed{u_{l2} = -a\bar{e}_1 - b\bar{e}_2 - sgn(e_2) - \alpha_2} \tag{15}$$

Case4. $\bar{e}_2 \in Z \,, \bar{e}_1 \in N$

$\bar{e}_2(\bar{e}_1 + \dot{\bar{e}}_2) < 0 \;\Longrightarrow\; \bar{e}_1 + \dot{\bar{e}}_2 = -sgn(e_2) \Longrightarrow\; \dot{\bar{e}}_2 = a\bar{e}_1 + b\bar{e}_2 + u_l > -sgn(e_2)$

$\Longrightarrow\; u_l > -a\bar{e}_1 - b\bar{e}_2 - sgn(e_2) \;\Longrightarrow\; u_l = -a\bar{e}_1 - b\bar{e}_2 - sgn(e_2) + \alpha_i \,, \;\; \alpha_i > 0$

$$\boxed{u_{l8} = -a\bar{e}_1 - b\bar{e}_2 - sgn(e_2) + \alpha_8} \tag{16}$$

Case5. $\bar{e}_2 \in Z \,, \bar{e}_1 \in Z \;\;\Longrightarrow\;\; u_{l5} = 0$

$$\boxed{u_{l5} = 0} \tag{17}$$

4. Results

Controller parameters have been optimized based on the following objective function which includes synchronization errors and a penalty term on the input:

$$f = \int_0^t \left(\bar{e}_1^2 + \bar{e}_2^2 + u_i^2\right) dt \quad (18)$$

The optimized controller parameters have been calculated via the GA as follows:

$b = 0.18859$
$\alpha_1 = 17.15259$
$\alpha_2 = 18.01591$
$\alpha_3 = 15.68801$
$\alpha_4 = 28.42200$
$\alpha_6 = 14.71820$
$\alpha_7 = 3.07242$
$\alpha_8 = 24.65027$
$\alpha_9 = 29.92193$

For k=0.5, and the initial states of the master system (0.1, 1.5, -0.6) and initial states of the slave system (-0.3, -1, 4), the synchronization errors have been shown in Fig.5. As can be seen, all errors have converged to zero and the two chaotic systems have been synchronized.

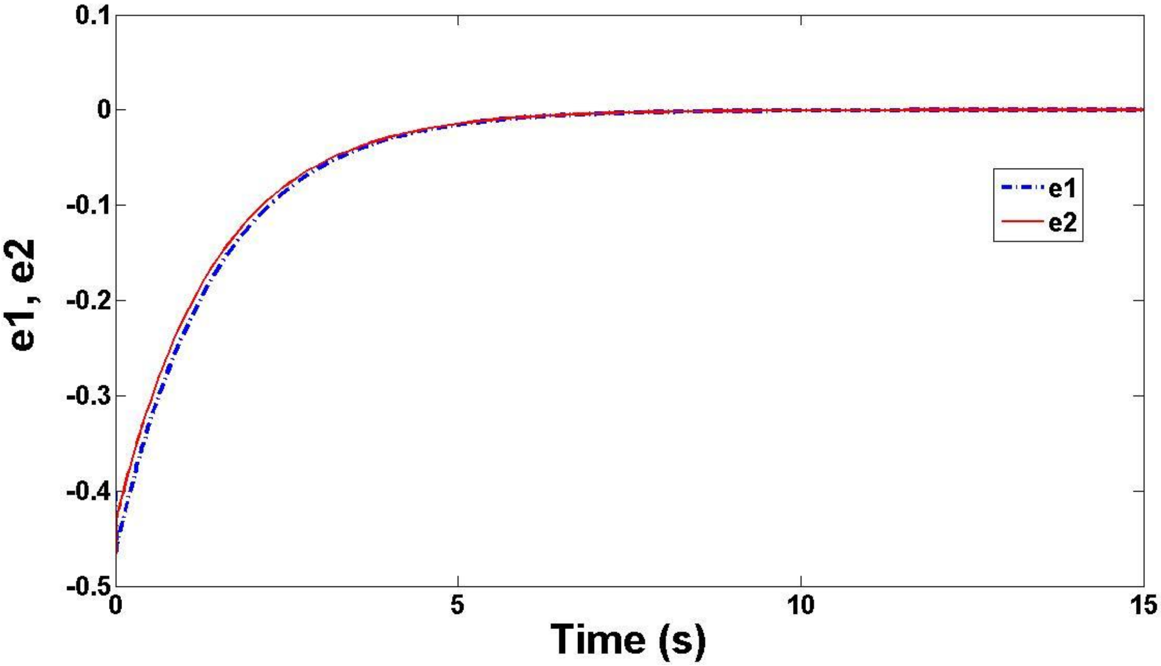

Fig.5a. First and second state synchronization errors

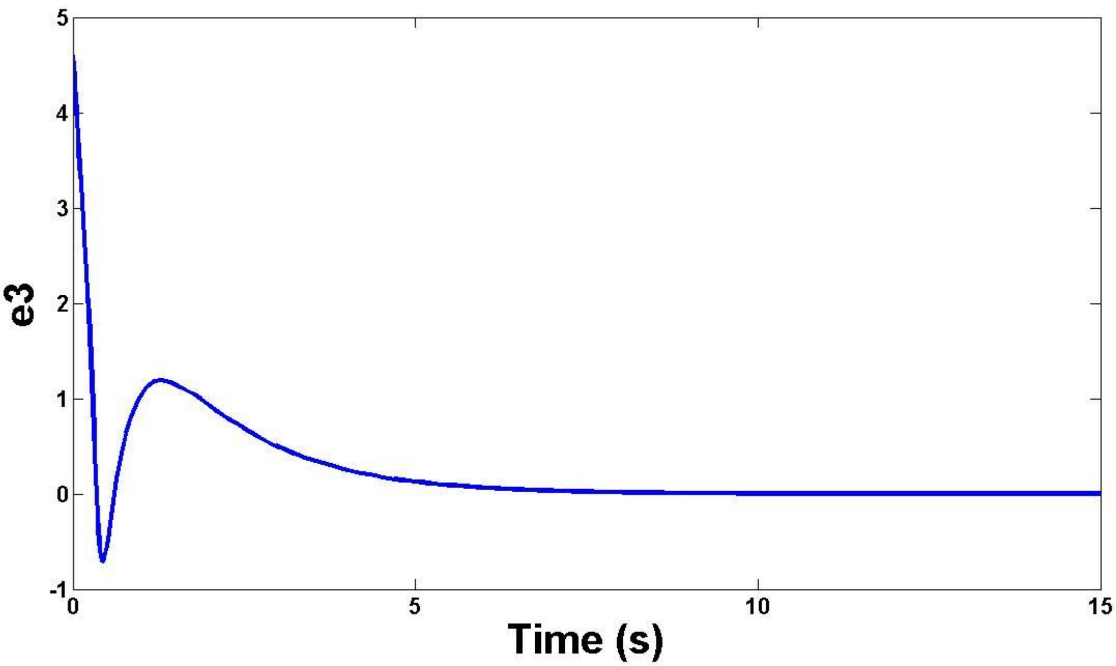

Fig.5b. Third state synchronization error

5. Conclusion

In this paper two identical generalized Lorenz systems have been synchronized by a fuzzy controller based on mamdani approach. Stability of the proposed scheme has been established by the Lyapunov stability theorem. Then controller parameters have been optimized via the genetic algorithm. Simulation results show the effectiveness of proposed method.

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