Solution of Newell-Whitehead-Segel equation by variational iteration method with He's polynomials

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Abstract

This article seeks to extend the variational iteration method (VIM) with He's polynomials for the approximate solution of nonlinear Newell-Whitehead-Segel equation (NWSE). Lagrange multiplier in correction functional is determined with the help of variational theory, and then homotopy perturbation method (HPM) is employed to dissolve the nonlinear terms. Thus a successful series is obtained with these iterations which are termed as He's polynomials. Result shows that this method is highly accurate and comes closer very quickly to the exact solution. We formulate three possible cases of NWSE to show the capability and ability of the present method. The valuable outcome discloses that the proposed strategy is very convenient, straightforward and can be utilized to linear and nonlinear problems.

Keywords: NWSE, VIM, Lagrange multiplier, HPM.

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1. Introduction

In natural phenomena, nonlinear partial differential equations perform an meaningful and enhanced role in modeling numerous physical appearances related to solid-state physics, fluid mechanics, plasma physics, population dynamics, chemical kinetics, nonlinear optics, protein chemistry, etc.. These nonlinear models, as well as their analytic solutions, are of enormous interest for appropriate topics. The VIM was come forth by He [9, 19] and favorably applied to autonomous ordinary differential systems [10], hyperbolic differential equations [6], non-homogeneous Cauchy Euler differential equations [26], Lane-Emden problems [7], Klein-Gordon and Sine-Gordon equations [5, 25], and Bratu-type equations [4, 20, 33].

HPM [8, 14, 15] was pioneered, based on the introduction of homotopy in topology, accompanied by the traditional perturbation method for the solution of algebraic equations. This procedure brings

forth a summation of an infinite series with clearly computable expressions, which coincides rapidly to the solution of the problem. HPM obtained the approximate solution not only for small parameters, but also for very large parameters and the initial approximation can be freely selected with possible unknown constants. In the literature, numerous authors have successfully applied HPM for various kind of problems such as inverse problem of diffusion equation [34], linear programming problems [32], blood flows problem [2], nonlinear oscillator problems [3, 12, 35, 38, 39], hybridization of HPM with Laplace transform [23, 27], HPM with expanding parameter [17], HPM with auxiliary parameters [16], and a fractional calculus [18, 22, 36, 37].

Many stripe patterns, e.g., ripples in the sand, stripes of seashells, appear in a variety of spatially extended systems which can be specified by a set of equation called amplitude equations. One of the most meaningful of amplitude equations is the NWSE which describes the presence of the stripe pattern in two dimensional systems.

The NWSE is written as

$$\frac{\partial \vartheta}{\partial t} = \kappa \frac{\partial^2 \vartheta}{\partial x^2} + \alpha \vartheta - \beta \vartheta^{\gamma+1} \tag{1.1}$$

with the initial condition

$$\vartheta(x, 0) = f(x),$$

where α, β are real numbers and κ, γ are positive integers. In Eq. (1.1), the first term on the left hand side, $\frac{\partial \vartheta}{\partial t}$, expresses the variations of $\vartheta(x, t)$ with time at a fixed location, the first term on the right hand side, $\frac{\partial^2 \vartheta}{\partial x^2}$, expresses the variations of $\vartheta(x, t)$ with spatial variable x at a specific time and the remaining terms on the right hand side, $\alpha \vartheta - \beta \vartheta^\gamma$, takes into account the effect of the source term.

Recently, NWSE has been studied by Laplace Adomian decomposition method [31], Adomian decomposition method [24], differential transform method [1], cubic B-spline collection method [40], VIM [30] and HPM [21, 28, 29]. In current paper, we will study the VIM combined with HPM. Further, HPM cab be dissolved into a number of iterations which is computable easily. These iterations are known as so called He's polynomials. The major advantage of this method is its capability of combining the two powerful method to obtain exact solution for nonlinear equations. This paper is organized as follows. In Section 2, we study the basic idea of VIM. In Section 3, we present a brief review of the homotopy perturbation with He's polynomials. In Section 4, we developed the scheme of VIM with He's polynomials. Section 5, presents some numerical test examples on VIM with He's polynomials applied to find the approximate solutions and finally, we summarize our results and draw conclusions in last Section 6.

2. Variational iteration method

To clarify the VIM, we begin by considering a differential equation in the formal form:

$$L\vartheta(x, t) + N\vartheta(x, t) = g(x, t),$$

where L and N are linear and nonlinear operators respectively, and $g(x, t)$ is a known analytical function. The VIM allows us to write a correct functional of the following type:

$$\vartheta_{n+1}(x, t) = \vartheta_n(x, t) + \int_0^t \lambda(s) \left[L\vartheta_n(x, s) + N\tilde{\vartheta}_n(x, s) - g(x, s) \right] ds, \tag{2.1}$$

where λ is a general Lagrange's multiplier, which can be identified optimally via the variational theory and $\tilde{\vartheta}_n$ is a restricted for variation which means $\delta\tilde{\vartheta}_n = 0$, yields the following Lagrange multipliers

$$\lambda = -1, \tag{2.2}$$

Therefore, substituting (2.2) into functional (2.1) we obtain the following iteration formula

$$\vartheta_{n+1}(x,t) = \vartheta_n(x,t) - \int_0^t \left[L\vartheta_n(x,s) + N\tilde{\vartheta}_n(x,s) - g(x,s) \right] ds. \tag{2.3}$$

Thus, Eq. (2.3) is called as a correction functional. The successive approximation ϑ_{n+1}, $n \geqslant 0$ of the solution ϑ will be readily obtained upon using the determined Lagrange multiplier and any selective function ϑ_0. Consequently, the solution is given by

$$\vartheta = \lim_{n \to \infty} \vartheta_n.$$

3. Basic idea of HPM

To illustrate the basic concept of HPM, consider the following non-linear functional equation

$$A(\vartheta) - f(r) = 0, \quad r \in \Omega, \tag{3.1}$$

with boundary conditions

$$B\left(\vartheta, \frac{\partial \vartheta}{\partial n}\right) = 0, \quad r \in \Gamma,$$

where A is a general functional operator, B is a boundary operator, $f(r)$ is a known analytic function, and Γ is the boundary of the domain Ω. The operator A can generally be divided into two operators, L and N, where L is a linear and N being a nonlinear operator. Therefore, Eq. (3.1) can be written as follows

$$L(\vartheta) + N(\vartheta) - f(r) = 0.$$

Using the homotopy technique, we construct a homotopy $v(r,p): \Omega \times [0,1] \to \mathbb{R}$ which satisfies

$$H(v,p) = (1-p)[L(v) - L(\vartheta_0)] + p[L(v) - N(v) - f(r)],$$

or

$$H(v,p) = L(v) - L(\vartheta_0) + pL(\vartheta_0) + p[N(v) - f(r)] = 0, \tag{3.2}$$

where $p \in [0,1]$, is called homotopy parameter, and ϑ_0 is an initial approximation for the solution of Eq. (3.1), which satisfies the boundary conditions. According to HPM, we can use p as a small parameter, and assume that the solution of Eq. (3.2) can be written as a power series in p

$$v = v_0 + pv_1 + p^2 v_2 + \cdots. \tag{3.3}$$

Considering $p = 1$, the approximate solution of Eq. (3.1) will be obtained as follows

$$\vartheta = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots.$$

Substituting (3.3) into (3.2) and equating the terms with identical powers of p, we can obtain a series of equations of the following form:

$$p^0 : v_0 - f(x) = 0, \quad p^1 : v_1 - H(v_0) = 0, \quad p^2 : v_2 - H(v_0, v_1) = 0, \quad p^3 : v_3 - H(v_0, v_1, v_2) = 0, \quad \ldots, \tag{3.4}$$

where $H(v_0, v_1, v_2, \ldots, v_j)$ depends upon $v_0, v_1, v_2, \ldots, v_j$ and are the so-called He's polynomials, which can be calculated by using the formula

$$H(v_0, v_1, v_2, \ldots, v_j) = \frac{1}{j!} \frac{\partial^j}{\partial p^j} N\left(\sum_{i=0}^{j} v_i p^i \right) \Bigg|_{p=0}.$$

It is obvious that the system of nonlinear equations in (3.4) is easy to solve, and the components $v_i, i \geqslant 0$ of the HPM can be completely determined, and the series solutions are thus entirely determined.

4. VIM with He's polynomials

We consider the following equation:

$$L\vartheta(x,t) + N\vartheta(x,t) = g(x,t).$$

The VIM allows us to write a correct functional of the following type:

$$\vartheta_{n+1}(x,t) = \vartheta_n(x,t) + \int_0^t \lambda(s)\big[L\vartheta_n(x,s) + N\tilde{\vartheta}_n(x,s) - g(x,s)\big]\,ds,$$

where λ is a general Lagrange's multiplier. Now, by using the HPM [11, 13], we can construct an equation as follows:

$$\sum_{n=0}^{\infty} p^n \vartheta_n(x,t) = \vartheta_0(x,t) + p\int_0^t \lambda(s)\Big[N\Big(\sum_{n=0}^{\infty} p^n \vartheta_n(x,s)\Big) - g(x,s)\Big]\,ds. \tag{4.1}$$

As it is seen, the procedure is constructed by coupling of VIM and HPM methods. A comparison of like powers of p gives solutions of various orders. By equating the terms of (4.1) with identical powers of p, and taking the limit as p tends to 1, we obtain

$$\vartheta(x,t) = \lim_{p\to 1}\sum_{n=0}^{\infty} p^n \vartheta_n(x,t), = \vartheta_0(x,t) + \vartheta_1(x,t) + \vartheta_2(x,t) + \cdots.$$

5. Numerical applications

5.1. Example 1

In equation (1.1), if $\alpha = 2$; $\beta = 3$, $\kappa = 1$, and $\gamma = 1$, the NWSE is written as:

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial^2 \vartheta}{\partial x^2} + 2\vartheta - 3\vartheta^2 \tag{5.1}$$

with initial condition

$$\vartheta(x,0) = 1. \tag{5.2}$$

The correct functional of Eq. (5.1) is given as

$$\vartheta_{n+1}(x,t) = \vartheta_n(x,t) + \int_0^t \lambda_1(s)\left[\frac{\partial \vartheta_n}{\partial s} - \frac{\partial^2 \tilde{\vartheta}_n}{\partial x^2} - 2\tilde{\vartheta}_n + 3\tilde{\vartheta}_n^2\right]. \tag{5.3}$$

This yields the stationary conditions

$$1 + \lambda_1(s) = 0, \quad \lambda_1'(s = t) = 0.$$

The Lagrange multipliers can be identified as follows:

$$\lambda_1(s) = -1.$$

Substituting these values of the Lagrange multipliers into the functionals (5.3), gives the iteration formulas

$$\vartheta_{n+1}(x,t) = \vartheta_n(x,t) - \int_0^t \left[\frac{\partial \vartheta_n}{\partial s} - \frac{\partial^2 \vartheta_n}{\partial x^2} - 2\vartheta_n + 3\vartheta_n^2\right]ds. \tag{5.4}$$

Applying the HPM on (5.4), we have:

$$\sum_{n=0}^{\infty} p^n \vartheta_n(x, t) = \vartheta_0(x, t) + p \int_0^t \left[\sum_{n=0}^{\infty} p^n \frac{\partial^2 \vartheta_n}{\partial x^2} + 2 \sum_{n=0}^{\infty} p^n \vartheta_n - 3 \sum_{n=0}^{\infty} p^n \vartheta_n^2 \right] ds.$$

We can select $\vartheta_0(x, t) = 1$ by using the given initial value (5.2). Accordingly, we obtain the following successive approximations by comparing the coefficient of like powers of p,

$$p^0 : \vartheta_0(x, t) = 1,$$

$$p^1 : \vartheta_1(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_0}{\partial x^2} + 2\vartheta_0 - 3\vartheta_0^2 \right] ds = -t,$$

$$p^2 : \vartheta_2(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_1}{\partial x^2} + 2\vartheta_1 - 6\vartheta_0\vartheta_1 \right] ds = \frac{4}{2!} t^2,$$

$$p^3 : \vartheta_3(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_2}{\partial x^2} + 2\vartheta_2 - 3\vartheta_1^2 - 6\vartheta_0\vartheta_0 \right] ds = -\frac{22}{3!} t^3,$$

$$p^4 : \vartheta_4(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_3}{\partial x^2} + 2\vartheta_3 - 6\vartheta_1\vartheta_2 - 6\vartheta_0\vartheta_3 \right] ds = \frac{160}{4!} t^4,$$

$$p^5 : \vartheta_5(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_4}{\partial x^2} + 2\vartheta_4 - 3\vartheta_2^2 - 6\vartheta_1\vartheta_3 \right] ds = -\frac{496}{5!} t^5,$$

$$\vdots$$

The series solutions are therefore given by

$$\vartheta(x, t) = \vartheta_0 + \vartheta_1 + \vartheta_2 + \vartheta_3 + \cdots, \qquad \vartheta(x, t) = 1 - t + \frac{4}{2!} t^2 - \frac{22}{3!} t^3 + \frac{160}{4!} t^4 - \frac{496}{5!} t^5 + \cdots,$$

and hence the exact solution becomes

$$\vartheta(x, t) = \frac{-\frac{2}{3} e^{2t}}{\frac{1}{3} - e^{2t}}.$$

5.2. Example 2

In equation (1.1), if $\alpha = \beta = \kappa = \gamma = 1$, the NWSE is written as:

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial^2 \vartheta}{\partial x^2} + \vartheta - \vartheta^2 \tag{5.5}$$

with initial condition

$$\vartheta(x, 0) = \frac{1}{(1 + e^{\frac{x}{\sqrt{6}}})^2}. \tag{5.6}$$

The correct functional of Eq. (5.5) is given as

$$\vartheta_{n+1}(x, t) = \vartheta_n(x, t) + \int_0^t \lambda_1(s) \left[\frac{\partial \vartheta_n}{\partial s} - \frac{\partial^2 \tilde{\vartheta}_n}{\partial x^2} - \tilde{\vartheta}_n + \tilde{\vartheta}_n^2 \right]. \tag{5.7}$$

This yields the stationary conditions

$$1 + \lambda_2(s) = 0, \quad \lambda_2'(s = t) = 0.$$

The Lagrange multipliers can be identified as follows:

$$\lambda_2(s) = -1.$$

Substituting these values of the Lagrange multipliers into the functionals (5.7), gives the iteration formulas

$$\vartheta_{n+1}(x, t) = \vartheta_n(x, t) - \int_0^t \left[\frac{\partial \vartheta_n}{\partial s} - \frac{\partial^2 \vartheta_n}{\partial x^2} - \vartheta_n + \vartheta_n^2 \right] ds. \tag{5.8}$$

Applying the HPM on (5.8), we have:

$$\sum_{n=0}^{\infty} p^n \vartheta_n(x, t) = \vartheta_0(x, t) + p \int_0^t \left[\sum_{n=0}^{\infty} p^n \frac{\partial^2 \vartheta_n}{\partial x^2} + \sum_{n=0}^{\infty} p^n \vartheta_n - \sum_{n=0}^{\infty} p^n \vartheta_n^2 \right] ds.$$

We can select $\vartheta_0(x, t) = 1$ by using the given initial value (5.6). Accordingly, we obtain the following successive approximations by comparing the coefficient of like powers of p,

$$p^0 : \vartheta_0(x, t) = \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^2},$$

$$p^1 : \vartheta_1(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_0}{\partial x^2} + \vartheta_0 - \vartheta_0^2 \right] ds = \frac{5 e^{\frac{x}{\sqrt{6}}}}{3 \left(1 + e^{\frac{x}{\sqrt{6}}}\right)^3} t,$$

$$p^2 : \vartheta_2(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_1}{\partial x^2} + \vartheta_1 - 2\vartheta_0 \vartheta_1 \right] ds = \frac{25 e^{\frac{x}{\sqrt{6}}} \left(-1 + 2e^{\frac{x}{\sqrt{6}}}\right)}{36 \left(1 + e^{\frac{x}{\sqrt{6}}}\right)^4} t^2,$$

$$p^3 : \vartheta_3(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_2}{\partial x^2} + \vartheta_2 - \vartheta_1^2 - 2\vartheta_0 \vartheta_0 \right] ds = \frac{125 e^{\frac{x}{\sqrt{6}}} \left(1 + 4(e^{\frac{x}{\sqrt{6}}})^2 - 7e^{\frac{x}{\sqrt{6}}}\right)}{648 \left(1 + e^{\frac{x}{\sqrt{6}}}\right)^5} t^3,$$

$$\vdots$$

The series solutions are therefore given by

$$\vartheta(x, t) = \vartheta_0 + \vartheta_1 + \vartheta_2 + \vartheta_3 + \cdots,$$

$$\vartheta(x, t) = \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}}}\right)^2} + \frac{5 e^{\frac{x}{\sqrt{6}}}}{3 \left(1 + e^{\frac{x}{\sqrt{6}}}\right)^3} t + \frac{25 e^{\frac{x}{\sqrt{6}}} \left(-1 + 2e^{\frac{x}{\sqrt{6}}}\right)}{36 \left(1 + e^{\frac{x}{\sqrt{6}}}\right)^4} t^2 + \frac{125 e^{\frac{x}{\sqrt{6}}} \left(1 + 4(e^{\frac{x}{\sqrt{6}}})^2 - 7e^{\frac{x}{\sqrt{6}}}\right)}{648 \left(1 + e^{\frac{x}{\sqrt{6}}}\right)^5} t^3 + \cdots$$

and hence the exact solution becomes

$$\vartheta(x, t) = \frac{1}{\left(1 + e^{\frac{x}{\sqrt{6}} - \frac{5}{6}t}\right)^2}.$$

5.3. Example 3

In equation (1.1), if $\alpha = \beta = \kappa = 1$ and $\gamma = 3$, the NWSE is written as:

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial^2 \vartheta}{\partial x^2} + \vartheta - \vartheta^4 \tag{5.9}$$

with initial condition

$$\vartheta(x, 0) = \frac{1}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}}. \tag{5.10}$$

The correct functional for system of Eq. (5.9) is given as

$$\vartheta_{n+1}(x, t) = \vartheta_n(x, t) + \int_0^t \lambda_1(s) \left[\frac{\partial \vartheta_n}{\partial s} - \frac{\partial^2 \tilde{\vartheta}_n}{\partial x^2} - \tilde{\vartheta}_n + \tilde{\vartheta}_n^4 \right]. \tag{5.11}$$

This yields the stationary conditions

$$1 + \lambda_3(s) = 0, \quad \lambda_3'(s = t) = 0.$$

The Lagrange multipliers can be identified as follows:

$$\lambda_3(s) = -1.$$

Substituting these values of the Lagrange multipliers into the functionals (5.11), gives the iteration formulas

$$\vartheta_{n+1}(x, t) = \vartheta_n(x, t) - \int_0^t \left[\frac{\partial \vartheta_n}{\partial s} - \frac{\partial^2 \vartheta_n}{\partial x^2} - \vartheta_n + \vartheta_n^4 \right] ds. \tag{5.12}$$

Applying the HPM on (5.12), we have:

$$\sum_{n=0}^{\infty} p^n \vartheta_n(x, t) = \vartheta_0(x, t) + p \int_0^t \left[\sum_{n=0}^{\infty} p^n \frac{\partial^2 \vartheta_n}{\partial x^2} + \sum_{n=0}^{\infty} p^n \vartheta_n - \sum_{n=0}^{\infty} p^n \vartheta_n^4 \right] ds.$$

We can select $\vartheta_0(x, t) = 1$ by using the given initial value (5.10). Accordingly, we obtain the following successive approximations by comparing the coefficient of like powers of p,

$$p^0 : \vartheta_0(x, t) = \frac{1}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}},$$

$$p^1 : \vartheta_1(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_0}{\partial x^2} + \vartheta_0 - \vartheta_0^4 \right] ds = \frac{7 e^{\frac{3x}{\sqrt{10}}}}{5 \left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}} t,$$

$$p^2 : \vartheta_2(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_1}{\partial x^2} + \vartheta_1 - 4\vartheta_0^3 \vartheta_1 \right] ds = \frac{49 e^{\frac{3x}{\sqrt{10}}} \left(-3 + 2 e^{\frac{3x}{\sqrt{10}}}\right)}{100 \left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{8}{3}}} t^2,$$

$$p^3 : \vartheta_3(x, t) = \int_0^t \left[\frac{\partial^2 \vartheta_2}{\partial x^2} + \vartheta_2 - 6\vartheta_0^2 \vartheta_1^2 - 4\vartheta_0^3 \vartheta_2 \right] ds = \frac{343 e^{\frac{3x}{\sqrt{10}}} \left(9 + 4(e^{\frac{3x}{\sqrt{10}}})^2 - 27 e^{\frac{3x}{\sqrt{10}}}\right)}{3000 \left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{11}{3}}} t^3,$$

$$\vdots$$

The series solutions are therefore given by

$$\vartheta(x,t) = \vartheta_0 + \vartheta_1 + \vartheta_2 + \vartheta_3 + \cdots,$$

$$\vartheta(x,t) = \frac{1}{\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} + \frac{7e^{\frac{3x}{\sqrt{10}}}}{5\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}} + \frac{49e^{\frac{3x}{\sqrt{10}}}\left(-3 + 2e^{\frac{3x}{\sqrt{10}}}\right)}{100\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{8}{3}}}t^2 + \frac{343e^{\frac{3x}{\sqrt{10}}}\left(9 + 4(e^{\frac{3x}{\sqrt{10}}})^2 - 27e^{\frac{3x}{\sqrt{10}}}\right)}{3000\left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{11}{3}}}t^3 + \cdots$$

and hence the exact solution becomes

$$\vartheta(x,t) = \left[\frac{1}{2}\tanh\left\{-\frac{3}{2\sqrt{10}}\left(x - \frac{7}{\sqrt{10}}t\right)\right\} + \frac{1}{2}\right].$$

6. Conclusion

In the current work, it is elucidated that VIM with He's polynomials was successfully employed for the approximate solution of NWSE with initial conditions. The values of Lagrange multiplier were investigated by using the variational theory in correction functional. This strategy is utilized in a straight way dealing without adomain polynomials, linearization, or constrictive hypotheses and yields an exact solution after a few iterations. We employed HPM in such a way that a small parameter, (say, p) is considered to 1 and obtained the series of so called He's polynomials. It is worth mentioning that VIM with He's polynomials is capable of reducing the computational work and such treatment is suitable for other nonlinear problems, especially the fractal calculus or the fractional calculus. For future work, we hope that this modification of VIM with He's polynomials will shed more light on the variation theory.

Acknowledgment

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