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Note on the stability property of the boundary equilibrium of a May cooperative system with strong and weak cooperative partners

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Abstract

In this paper, we revisit the stability property of the boundary equilibrium of a May cooperative system with strong and weak cooperative partners. Our result essentially improves the corresponding result of Zhao et al. [L. Zhao, B. Qin, F. D. Chen, Adv. Difference Equ., **2018** (2018), 13 pages].

Keywords: Cooperative system, boundary equilibrium, global stability.

2010 MSC: 34C25, 92D25, 34D20, 34D40.

1. Introduction

During the last decade, many scholars ([2–14, 16–25]) investigated the dynamic behaviors of the mutualism and commensalism model. Some substantial progress has been made on the stability, permanence and extinction of the mutualism model. For example, under some very simple assumptions, Xie et al. [16] showed that unique positive equilibrium of a cooperative system incorporating harvesting is globally attractive; Xie et al. [17] showed that the unique positive equilibrium of an integrodifferential model of mutualism is globally attractive. Recently, stimulated by the idea of Mohammadi and Mahzoon [15], Zhao et al. [25] proposed the following May cooperative system with strong and weak cooperative partners

$$\begin{aligned} \frac{dH_1}{dt} &= r_1H_1\left(1-\frac{H_1}{a_1+b_1P}-c_1H_1-\frac{\alpha H_2}{r_1}\right),\\ \frac{dH_2}{dt} &= H_2\big(\alpha H_1+d-eH_2\big),\\ \frac{dP}{dt} &= r_2P\left(1-\frac{P}{a_2+b_2H_1}-c_2P\right), \end{aligned} \tag{1.1}$$

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where $r_i, a_i, b_i, c_i, d, i = 1, 2$ are positive constants. We will consider system (1.1) together with the initial condition $H_i(0) > 0$, $i = 1, 2, P(0) > 0$ in system (1.1). Obviously, any solution of system (1.1) remains positive for all $t \geqslant 0$. One could refer to [25] for more detail about the construction of the model.

The system (1.1) always admits a boundary equilibrium $E_2(0, H_{2*}, P_*)$ (see Theorem 3.2 in [25]), indeed, by simple computation, one could see that $H_{2*} = \frac{d}{e}, P_* = \frac{a_2}{1 + a_2 c_2}$. Concerned with the stability property of this equilibrium, by constructing some suitable Lyapunv function, the authors obtained the following results (see Theorem 4.2 in [25] for more details).

Theorem 1.1. *If the assumption* (B_3) *and* (B_5) *hold, where*

(B_3) $M = 1 - \frac{\alpha d}{r_1 e} < 0$;

(B_5) $\alpha^2 < r_1 c_1 e$,

then the equilibrium point $E_2(0, H_{2*}, P_*)$ *system is globally asymptotically stable.*

Now let's consider the following example.

Example 1.2.

$$\begin{aligned}
\frac{dH_1}{dt} &= 3H_1\left(1 - \frac{H_1}{2+2P} - H_1 - \frac{3.5H_2}{3}\right),\\
\frac{dH_2}{dt} &= H_2(3.5H_1 + 2 - 2H_2),\\
\frac{dP}{dt} &= 2P\left(1 - \frac{P}{2+0.8H_1} - 1.5P\right).
\end{aligned} \tag{1.2}$$

Here, corresponding to system (1.1), we take $r_1 = 3, a_1 = 2, b_1 = 2, c_1 = 1, \alpha = 3.5, r_1 = 3$, $d = 2, e = 2, r_2 = 2, a_2 = 2, b_2 = 0.8, c_2 = 1.5$. By simple computation, one could easily see that

$$M = 1 - \frac{\alpha d}{r_1 e} = -\frac{0.5}{3} = -\frac{1}{6} < 0 \quad \text{and} \quad \alpha^2 = 12.25 > 6 = r_1 c_1 e.$$

That is, the coefficients of the system (1.2) satisfy the condition (B_3), however, they do not satisfy the condition (B_5), however, numeric simulations (Figs 1–3) show that in this case, the boundary equilibrium $(0, 1, \frac{2}{3})$ is globally attractive.

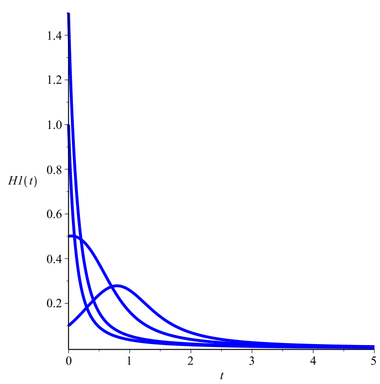

Figure 1: Numeric simulations of the first component system (1.2), the initial conditions $(x(0), y(0)) = (1, 2, 0.7), (1.5, 1, 0.3), (0.5, 0.2, 0.1)$ and $(0.1, 0.1, 2)$, respectively.

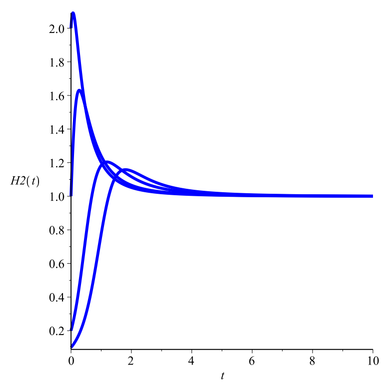

Figure 2: Numeric simulations of the second component system (1.2), the initial conditions $(x(0), y(0)) = (1, 2, 0.7), (1.5, 1, 0.3), (0.5, 0.2, 0.1)$ and $(0.1, 0.1, 2)$, respectively.

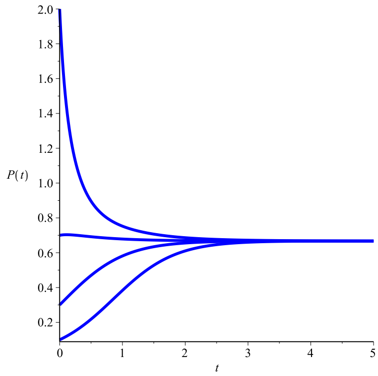

Figure 3: Numeric simulations of the third component system (1.2), the initial conditions $(x(0), y(0)) = (1, 2, 0.7), (1.5, 1, 0.3), (0.5, 0.2, 0.1)$ and $(0.1, 0.1, 2)$, respectively.

Above numeric simulations show that there still have room to improve the main result ([25, Theorem 4.2]). The aim of this paper is try to obtain a new set of sufficient conditions to ensure the global attractivity of the positive equilibrium, more precisely, we will obtain the following result.

Theorem 1.3. *If the assumption* (B_3) *holds, where*

$$(B_3) \quad M = 1 - \frac{\alpha d}{r_1 e} < 0,$$

then the equilibrium point $E_2(0, H_{2*}, P_*)$ *system is globally attractive.*

Remark 1.4. Compared with Theorem 1.1 and Theorem 1.3, One could easily see that condition (B_5) is redundantly and unnecessary.

We will prove Theorem 1.3 in the next section and end this paper by a briefly discussion.

2. Proof of the main result

We need the following lemma to prove the main result.

Lemma 2.1 ([1])**.** *Let* $a > 0, b > 0$.

(I) *If* $\frac{dx}{dt} \geqslant x(b - ax)$, *then* $\liminf\limits_{t\to+\infty} x(t) \geqslant \frac{b}{a}$ *for* $t \geqslant 0$ *and* $x(0) > 0$;

(II) *If* $\frac{dx}{dt} \leqslant x(b - ax)$, *then* $\limsup\limits_{t\to+\infty} x(t) \leqslant \frac{b}{a}$ *for* $t \geqslant 0$ *and* $x(0) > 0$.

Now we are in the position of proving the Theorem 1.3, we mention here that Zhao et al. [25] had already proved part of the results of Theorem 1.3, however, for the sake of completeness, here we give the detail proof of the Theorem 1.3.

Proof of Theorem 1.3. From the second equation of system (1.1), one has

$$\frac{dH_2}{dt} \geqslant H_2(d - eH_2), \tag{2.1}$$

applying Lemma 2.1 to (2.1) leads to

$$\liminf_{t\to+\infty} H_2(t) \geqslant \frac{d}{e}. \tag{2.2}$$

Condition (B_3) implies that there exists a enough small $\varepsilon > 0$ such that

$$1 - \frac{\alpha}{r_1}\left(\frac{d}{e} - \varepsilon\right) < 0. \tag{2.3}$$

Indeed, for all ε which satisfies $0 < \varepsilon < \frac{\frac{\alpha d}{r_1 e} - 1}{\frac{\alpha}{r_1}}$, inequality (2.3) holds. For this ε, from (2.2) there exists an enough large T_1 such that

$$H_2(t) > \frac{d}{e} - \varepsilon \quad \text{for all} \quad t \geqslant T_1. \tag{2.4}$$

From (2.4) and the first equation of system (1.1), for $t > T_1$, one has

$$\frac{dH_1}{dt} < r_1 H_1 \left(1 - \frac{H_1}{a_1 + b_1 P} - c_1 H_1 - \frac{\alpha}{r_1}\left(\frac{d}{e} - \varepsilon\right)\right) < r_1 H_1 \left(1 - \frac{\alpha}{r_1}\left(\frac{d}{e} - \varepsilon\right)\right).$$

Thus

$$H_1(t) < H_1(T_1) \exp\left\{ r_1 \left(1 - \frac{\alpha}{r_1}\left(\frac{d}{e} - \varepsilon\right)\right)(t - T_1)\right\}.$$

It then follows from (2.3) that

$$\lim_{t\to+\infty} H_1(t) = 0. \tag{2.5}$$

For any $\varepsilon_1 > 0$ enough small, from (2.5), there exists an enough large $T_2 > T_1$ such that

$$H_1(t) < \varepsilon_1 \quad \text{for all} \quad t > T_2. \tag{2.6}$$

For $t > T_2$, from the second equation of system (1.1) and (2.6), it follows that

$$\frac{dH_2}{dt} < H_2(\alpha\varepsilon_1 + d - eH_2). \tag{2.7}$$

By applying Lemma 2.1 to (2.7), it immediately follows that

$$\limsup_{t\to+\infty} H_2(t) < \frac{\alpha\varepsilon_1 + d}{e}. \tag{2.8}$$

Since ε_1 is any enough small positive constant, setting $\varepsilon_1 \to 0$ in (2.8) leads to

$$\limsup_{t\to+\infty} H_2(t) \leqslant \frac{d}{e}. \tag{2.9}$$

(2.2) together with (2.9) shows that

$$\frac{d}{e} \leqslant \liminf_{t\to+\infty} H_2(t) \leqslant \limsup_{t\to+\infty} H_2(t) \leqslant \frac{d}{e}.$$

Hence

$$\lim_{t\to+\infty} H_2(t) = \frac{d}{e}. \tag{2.10}$$

For $t > T_2$, from the third equation of system (1.1) and (2.6), it follows that

$$\frac{dP}{dt} < r_2 P\left(1 - \frac{P}{a_2 + b_2\varepsilon_1} - c_2 P\right). \tag{2.11}$$

By applying Lemma 2.1 to (2.11), it immediately follows that

$$\limsup_{t\to+\infty} P(t) < \frac{1}{\frac{1}{a_2 + b_2\varepsilon_1} + c_2}. \tag{2.12}$$

Since ε_1 is any enough small positive constant, setting $\varepsilon_1 \to 0$ in (2.12) leads to

$$\limsup_{t\to+\infty} P(t) \leqslant \frac{1}{\frac{1}{a_2} + c_2} = \frac{a_2}{1 + a_2 c_2}. \tag{2.13}$$

On the other hand, from the third equation of system (1.1), we also have

$$\frac{dP}{dt} > r_2 P\left(1 - \frac{P}{a_2} - c_2 P\right). \tag{2.14}$$

By applying Lemma 2.1 to (2.14), it immediately follows that

$$\liminf_{t\to+\infty} P(t) > \frac{1}{\frac{1}{a_2} + c_2} = \frac{a_2}{1 + a_2 c_2}. \tag{2.15}$$

(2.13) together with (2.15) shows that

$$\frac{a_2}{1 + a_2 c_2} \leqslant \liminf_{t\to+\infty} P(t) \leqslant \limsup_{t\to+\infty} P(t) \leqslant \frac{a_2}{1 + a_2 c_2}.$$

Hence

$$\lim_{t\to+\infty} P(t) = \frac{a_2}{1 + a_2 c_2}. \tag{2.16}$$

(2.5), (2.10), and (2.16) show that the equilibrium point $E_2(0, H_{2*}, P_*)$ is globally attractive. This ends the proof of Theorem 1.3. □

3. Discussion

In [25], Zhao et al. proposed the system (1.1), by constructing some suitable Lyapunov function, they obtained a set of sufficient conditions which ensure the global asymptotically stable of the boundary equilibrium $E_2(0, H_{2*}, P_*)$, by carefully checking the conditions, we found that (B_3) requires α enough large, that is, the translate rate of strong partners to weak partners is enough large, however, condition (B_5) requires α enough small, that is, conditions (B_3) and (B_5) seem to contradict to each other, this motivated us to revisit the globally attractivity of the boundary equilibrium E_2, with the aim of finding some set of conditions which may seems no contract. By applying the theory of differential inequality, we finally show that condition (B_5) is not needed and could be dropped. Our result (Theorem 1.3) essentially improve one of the main results of [25] (Theorem 4.2).

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