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C_m-supermagic labeling of polygonal snake graphs

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Abstract

An H-supermagic labeling of a graph G admitting an H-covering was defined by Guitérrez and Lladó [A. Guitérrez, A. Lladó, J. Combin. Math. Combin. Comput., **55** (2005), 43–56]. In this work, we shall show that polygonal snake graphs admit C_m-supermagic labeling.

Keywords: H-magic labeling, H-supermagic labeling, triangular snake, m-polygonal snake.

2010 MSC: 05C78.

1. Introduction and preliminaries

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Graph labeling was first introduced by Rosa [12] in 1966. Since then there are various types of labeling that have been studied and developed (see [1]).

A finite simple graph $G(V, E)$ admits an H-covering if every edge of G belongs to a subgraph of G isomorphic to H. Guitérrez and Lladó [2] introduced the notion of an H-magic labeling as follows: Let $G = (V, E)$ be a finite simple graph that admits H-covering. A bijection function $\lambda : V \cup G \to \{1, 2, 3, \ldots, |V| + |E|\}$ is called H-magic labeling of G if for every subgraph $H' = (V', E')$ of G isomorphic to H, $\sum_{v\in V'} \lambda(v) + \sum_{e\in E'} \lambda(e) = m(\lambda)$ is constant. Here $m(\lambda)$ is called as magic sum. The graph G is called H-supermagic if $\lambda(V) = \{1, 2, 3, \ldots, |V|\}$.

Llado and Moragas [9] studied some C_n-supermagic graphs. In [10] Maryati, Baskoro and Salaman studied path-supermagic labeling. Supermagicness of book graphs was given by Jeyanthi and Selvagopal [4]. C_4-supermagicness of the book with n tetragonal pages was given by Ngurah et al. [11]. Roswitha et al. [13] investigated H-supermagicness of some classes of graphs such as a Jahangir graph, a wheel graph for even n, and a complete bipartie graph $K_{m,n}$ for $m = 2$. C_4-supermagic labelings of the Cartesian product of paths and graphs was given by Kojima [8]. Kathiresan et al. [7] showed that generalized

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book is supermagic. C_k-supermagic strength of k-polygonal snake graphs studied in [3]. In [5, 6], C_m-supermagicness of some graphs was also investigated. Selvagopal and Jeyanthi [14] showed that polygonal snake graph has C_m-supermagic labeling. In this work, we shall show that polygonal snake graphs admit C_m-supermagic labeling.

2. Results

Theorem 2.1. *Triangular snake graph $\Delta_n, n \geqslant 2$ admits a C_3-supermagic labeling.*

Proof. Triangular snake graph Δ_n has $2n+1$ vertices and $3n$ edges. The vertices and edges of Δ_n are given as

$$\begin{aligned} V &= \{v_{b_i} : i = 1, \dots, n+1\} \cup \{v_i : i = 1, \dots, n\}, \\ E &= \{e_{1i} : e_{1i} = v_{b_i} v_i : i = 1, \dots, n\} \\ &\quad \cup \left\{e_{2i} : e_{2i} = v_i v_{b_{i+1}} : i = 1, \dots, n\right\} \cup \left\{e_{3i} : e_{3i} = v_{b_i} v_{b_{i+1}} : i = 1, \dots, n\right\}, \end{aligned}$$

where v_{b_i} are the base vertices and e_{3i} are the base edges. An example for the triangular snake graph $\Delta_n; n \geqslant 2$ is shown in Figure 1.

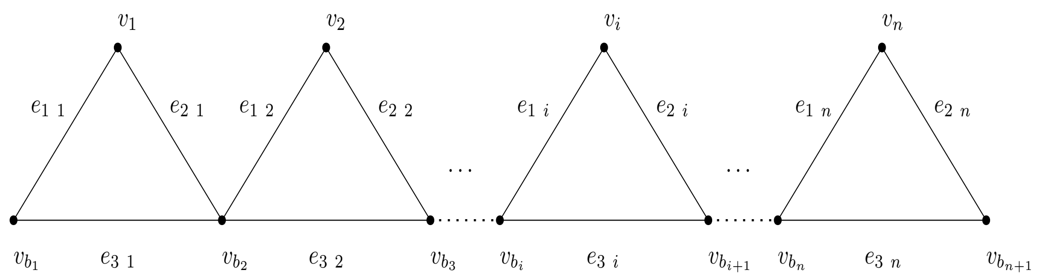

Figure 1: Triangular snake graph $\Delta_n; n \geqslant 2$.

We construct a labeling $\lambda : V \cup E \to \{1, 2, 3, \dots, |V| + |E|\}$ as follows:

$$\begin{aligned} \lambda(v_{b_i}) &= i, \ i = 1, 2, 3, ..., n+1, \\ \lambda(v_i) &= 2n + 2 - i, \ i = 1, 2, 3, ..., n, \\ \lambda(e_{1i}) &= 2n + 1 + i, \ i = 1, 2, 3, ..., n, \\ \lambda(e_{2i}) &= 5n + 2 - i, \ i = 1, 2, 3, ..., n, \\ \lambda(e_{3i}) &= 4n + 2 - i, \ i = 1, 2, 3, ..., n. \end{aligned}$$

Here, for all $v \in V$, we have $\lambda(v) \in \{1, 2, 3, \dots, 2n+1\}$ and for any subgraph $H' = (V', E')$ isomorphic to C_3, we have

$$\begin{aligned} \sum_{v \in V'} \lambda(v) &= \lambda(v_{b_i}) + \lambda(v_{b_{i+1}}) + \lambda(v_i) = 2n + 3 + i, \\ \sum_{e \in E'} \lambda(e) &= \lambda(e_{1i}) + \lambda(e_{2i}) + \lambda(e_{3i}) = 11n + 5 - i, \end{aligned}$$

and the magic constant is

$$m(\lambda) = \sum_{v \in V'} \lambda(v) + \sum_{e \in E'} \lambda(e) = 13n + 8.$$

Therefore triangular snake *graph* $\Delta_n, n \geqslant 2$, admits a C_3-supermagic labeling. □

Theorem 2.2. *m-polygonal snake graph $\Delta_n^m, n \geqslant 2, m \geqslant 4$, admits a C_m-supermagic labeling.*

Proof. m-polygonal snake graph Δ_n^m has $(m-1)n+1$ vertices and mn edges. The vertices and edges of Δ_n^m are given below:

$$\begin{aligned} V =& \{v_{b_i} : i = 1, \dots, n+1\} \cup \{v_{ji} : j = 1, \dots, m-2, i = 1, \dots, n\}, \\ E =& \{e_{1i} : e_{1i} = v_{b_i} v_{1i} : i = 1, \dots, n\} \\ & \cup \{e_{ji} : e_{ji} = v_{j-1i} v_{ji} : j = 2, \dots, m-2, i = 1, \dots, n\} \\ & \cup \{e_{m-1i} : e_{m-1i} = v_{m-2i} v_{b_{i+1}} : i = 1, \dots, n\} \\ & \cup \{e_{mi} : e_{mi} = v_{b_i} v_{b_{i+1}} : i = 1, \dots, n\}, \end{aligned}$$

where v_{b_i} are the base vertices and e_{mi} are the base edges. An example for the m-polygonal snake graph $\Delta_n^m, n \geqslant 2, m \geqslant 4$ is shown in Figure 2.

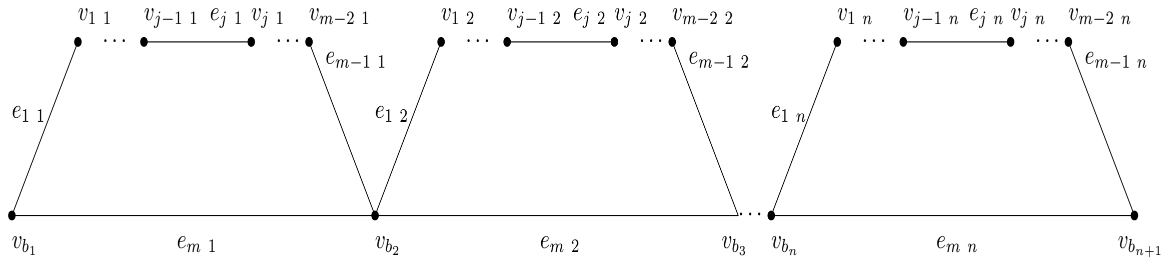

Figure 2: m-polygonal snake graph $\Delta_n^m; n \geqslant 2, m \geqslant 4$.

We consider the following two cases.

Case 1: m is even. We construct a labeling $\lambda : V \cup E \to \{1, 2, 3, \dots, |V| + |E|\}$ as follows:

$$\begin{aligned} &\lambda(v_{b_i}) = i,\ i = 1, 2, 3, \dots, n+1, \\ &\lambda(v_{1i}) = 2n + 2 - i,\ i = 1, 2, 3, \dots, n, \\ &\lambda(v_{2i}) = 3n + 2 - i,\ i = 1, 2, 3, \dots, n, \\ &\lambda(v_{ji}) = \begin{cases} jn + 1 + i, & j = 3, 5, 7, \dots, m-3, \quad i = 1, 2, 3, \dots, n, \\ n + jn + 2 - i, & j = 4, 6, 8, \dots, m-2, \quad i = 1, 2, 3, \dots, n, \end{cases} \\ &\lambda(e_{ji}) = \begin{cases} jn - 2n + mn + 1 + i, & j = 1, 3, 5, \dots, m-1, \quad i = 1, 2, 3, \dots, n, \\ jn - n + mn + 2 - i, & j = 2, 4, 6, \dots, m, \qquad i = 1, 2, 3, \dots, n. \end{cases} \end{aligned}$$

Here, for all $v \in V$, we have $\lambda(v) \in \{1, 2, 3, \dots, (m-1)n+1\}$ and for any subgraph $H' = (V', E')$ isomorphic to C_m, we have

$$\begin{aligned} &\sum_{v \in V'} \lambda(v) = \lambda(v_{b_i}) + \lambda(v_{b_{i+1}}) + \lambda(v_{1i}) + \lambda(v_{2i}) + \sum_{j=3}^{m-2} \lambda(v_{ji}) = \frac{3}{2}m + n + \frac{1}{2}m^2 n - mn - 1, \\ &\sum_{e \in E'} \lambda(e) = \sum_{j=1}^{m} \lambda(e_{ji}) = \frac{1}{2}m(3mn - 2n + 3), \end{aligned}$$

and the magic constant is

$$m(\lambda) = \sum_{v \in V'} \lambda(v) + \sum_{e \in E'} \lambda(e) = 3m + n + 2m^2 n - 2mn - 1.$$

Case 2: m is odd. We construct a labeling $\lambda : V \cup E \to \{1, 2, 3, \dots, |V| + |E|\}$ as follows:

$$\lambda(v_{b_i}) = i,\ i = 1, 2, 3, \dots, n+1,$$

$$\lambda(v_{1i}) = 2n+2-i,\ i=1,2,3,\ldots,n,$$
$$\lambda(v_{ji}) = \begin{cases} jn+1+i, & j=2,4,6,\ldots,m-3, \quad i=1,2,3,\ldots,n,\\ n+jn+2-i, & j=3,5,7,\ldots,m-2, \quad i=1,2,3,\ldots,n,\end{cases}$$
$$\lambda(e_{ji}) = \begin{cases} jn-2n+mn+1+i, & j=1,3,5,\ldots,m-2, \quad i=1,2,3,\ldots,n,\\ jn-n+mn+2-i, & j=2,4,6,\ldots,m-1, \quad i=1,2,3,\ldots,n,\end{cases}$$
$$\lambda(e_{mi}) = 2mn-n+2-i,\ i=1,2,3,\ldots,n.$$

Here, for all $v \in V$, we have $\lambda(v) \in \{1,2,3,\ldots,(m-1)n+1\}$ and for any subgraph $H' = (V', E')$ isomorphic to C_m, we have

$$\sum_{v\in V'} \lambda(v) = \lambda(v_{b_i}) + \lambda(v_{b_{i+1}}) + \lambda(v_{1i}) + \sum_{j=2}^{m-2} \lambda(v_{ji}) = \frac{3}{2}m + \frac{1}{2}n + \frac{1}{2}m^2n - mn - \frac{3}{2} + i,$$
$$\sum_{e\in E'} \lambda(e) = \sum_{j=1}^{m-1} \lambda(e_{ji}) + \lambda(e_{mi}) = \frac{3}{2}m + \frac{1}{2}n + \frac{3}{2}m^2n - mn + \frac{1}{2} - i,$$

and the magic constant is

$$m(\lambda) = \sum_{v\in V'} \lambda(v) + \sum_{e\in E'} \lambda(e) = 3m + n + 2m^2n - 2mn - 1.$$

Hence $\Delta_n^m; n \geqslant 2, m \geqslant 4$, admits a C_m-supermagic labeling. □

Theorem 2.3. *Isomorphic copies of triangular snake graph* $k\Delta_n; n,k \geqslant 2$, *admits a* C_3*-supermagic labeling.*

Proof. k copies of triangular snake graph $k\Delta_n$ has $(2n+1)k$ vertices and $3nk$ edges. The vertices and edges of $k\Delta_n$ are given below:

$$V = \left\{v_{b_i}^s : i=1,\ldots,n+1,\ s=1,\ldots,k\right\} \cup \{v_i^s : i=1,\ldots,n\ s=1,\ldots,k\},$$
$$E = \left\{e_{1i}^s : e_{1i}^s = v_{b_i}^s v_i^s : i=1,\ldots,n,\ s=1,\ldots,k\right\}$$
$$\cup \left\{e_{2i}^s : e_{2i}^s = v_i^s v_{b_{i+1}}^s : i=1,\ldots,n,\ s=1,\ldots,k\right\}$$
$$\cup \left\{e_{3i}^s : e_{3i}^s = v_{b_i}^s v_{b_{i+1}}^s : i=1,\ldots,n\ s=1,\ldots,k\right\},$$

where $v_{b_i}^s$ are the base vertices and e_{3i}^s are the base edges. An example for the k isomorphic copies of triangular snake graph $k\Delta_n; n,k \geqslant 2$ is shown in Figure 3.

We construct a labeling $\lambda : V \cup E \to \{1,2,3,\ldots,|V|+|E|\}$ as follows:

$$\lambda\left(v_{b_i}^s\right) = (i-1)k + s,\ i=1,2,3,\ldots,n+1,$$
$$\lambda(v_i^s) = (2-i)k - s + 2kn + 1,\ i=1,2,3,\ldots,n,$$
$$\lambda(e_{1i}^s) = k - n + 2kn + ns + i,\ i=1,2,3,\ldots,n,$$
$$\lambda(e_{2i}^s) = k + n + 5kn - ns + 1 - i,\ i=1,2,3,\ldots,n,$$
$$\lambda(e_{3i}^s) = (2-i)k - s + 4kn + 1,\ i=1,2,3,\ldots,n.$$

where $s = 1,\ldots,k$. Here, for all $v \in V$, we have $\lambda(v) \in \{1,2,3,\ldots,(2n+1)k\}$ and for any subgraph $H' = (V', E')$ isomorphic to C_3, we have

$$\sum_{v\in V'} \lambda(v) = \lambda(v_{b_i}^s) + \lambda(v_{b_{i+1}}^s) + \lambda(v_i^s) = (1+i)k + s + 2kn + 1,$$
$$\sum_{e\in E'} \lambda(e) = \lambda(e_c^s) + \lambda(e_{1i}^s) + \lambda(e_{2i}^s) = (4-i)k - s + 11kn + 2,$$

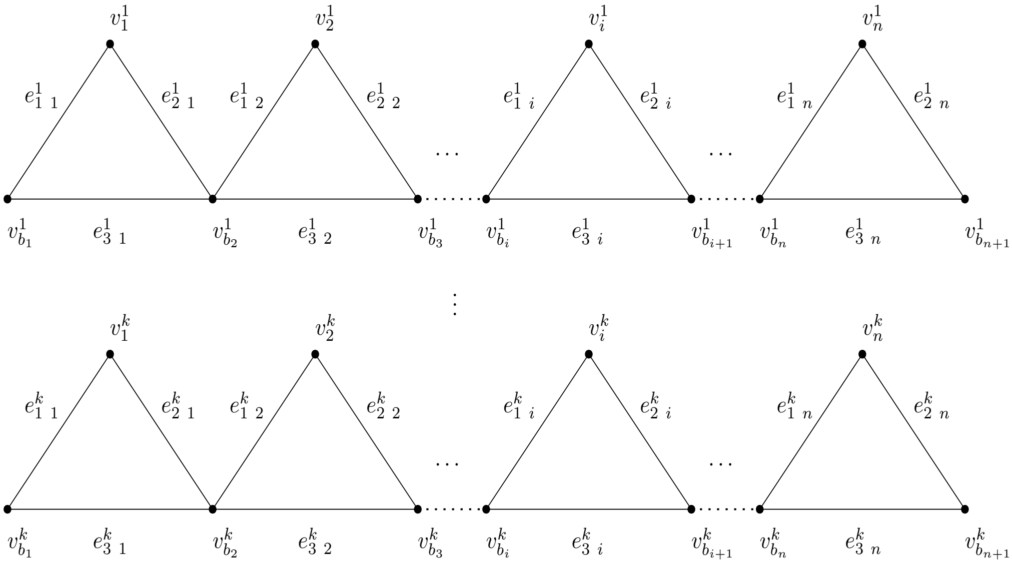

Figure 3: k isomorphic copies of triangular snake graph $k\Delta_n; n, k \geqslant 2$.

and the magic constant is

$$m(\lambda) = \sum_{v \in V'} \lambda(v) + \sum_{e \in E'} \lambda(e) = 5k + 13kn + 3,$$

that implies *isomorphic copies of triangular snake graph* $k\Delta_n; n, k \geqslant 2$ admits a C_3-supermagic labeling. □

Theorem 2.4. *Isomorphic copies of* m*-polygonal snake graph* $k\Delta_n^m; n, k \geqslant 2, m \geqslant 4$*, admits a* C_m*-supermagic labeling.*

Proof. k copies of $k\Delta_n^m$ has $((m-1)n+1)k$ vertices and mnk edges. The vertices and edges of $k\Delta_n^m$ are given below:

$$\begin{aligned} V &= \left\{v_{b_i}^s : i = 1, \ldots, n+1,\ s = 1, \ldots, k\right\} \cup \left\{v_{ji}^s : j = 1, \ldots m-2, i = 1, \ldots, n\ s = 1, \ldots, k\right\}, \\ E &= \left\{e_{1i}^s : e_{1i}^s = v_{b_i}^s v_{1i}^s : i = 1, \ldots, n,\ s = 1, \ldots, k\right\} \\ &\cup \left\{e_{ji}^s : e_{ji}^s = v_{j-1i}^s v_{ji}^s : j = 2, \ldots m-2, i = 1, \ldots, n,\ s = 1, \ldots, k\right\} \\ &\cup \left\{e_{m-1i}^s : e_{m-1i}^s = v_{m-2i}^s v_{b_{i+1}}^s : i = 1, \ldots, n,\ s = 1, \ldots, k\right\} \\ &\cup \left\{e_{mi}^s : e_{mi}^s = v_{b_i}^s v_{b_{i+1}}^s : i = 1, \ldots, n\ s = 1, \ldots, k\right\}, \end{aligned}$$

where $v_{b_i}^s$ are the base vertices and e_{mi}^s are the base edges. An example for the k isomorphic copies of m-polygonal snake graph $k\Delta_n^m; n, k \geqslant 2, m \geqslant 4$ is shown in Figure 4.

We consider the following two cases.

Case 1: m is even. We construct a labeling $\lambda : V \cup E \to \{1, 2, 3, \ldots, |V| + |E|\}$ as follows:

$$\begin{aligned} &\lambda\left(v_{b_i}^s\right) = s + k(i-1),\ i = 1, 2, 3, \ldots, n+1, \\ &\lambda(v_{1i}^s) = (2-i)k - s + 2kn + 1,\ i = 1, 2, 3, \ldots, n, \\ &\lambda(v_{2i}^s) = (2-i)k - s + 3kn + 1,\ i = 1, 2, 3, \ldots, n, \\ &\lambda(v_{ji}^s) = \begin{cases} ik + s + jkn, & j = 3, 5, 7, \ldots, m-3, \quad i = 1, 2, 3, \ldots, n, \\ (2-i)k - s + kn + jkn + 1, & j = 4, 6, 8, \ldots, m-2, \quad i = 1, 2, 3, \ldots, n, \end{cases} \\ &\lambda(e_{ji}^s) = \begin{cases} ik + s - 2kn + jkn + kmn, & j = 1, 3, 5, \ldots, m-1, \quad i = 1, 2, 3, \ldots, n, \\ (2-i)k - s - kn + jkn + kmn + 1, & j = 2, 4, 6, \ldots, m, \qquad i = 1, 2, 3, \ldots, n, \end{cases} \end{aligned}$$

where $s = 1, \dots, k$. Here, for all $v \in V$, we have $\lambda(v) \in \{1, 2, 3, \dots, ((m-1)n+1)k\}$ and for any subgraph $H' = (V', E')$ isomorphic to C_m, we have

$$\sum_{v \in V'} \lambda(v) = \lambda(v^s_{b_i}) + \lambda(v^s_{b_{i+1}}) + \lambda(v^s_{1i}) + \lambda(v^s_{2i}) + \sum_{j=3}^{m-2} \lambda(v^s_{ji})$$
$$= \frac{1}{2}m - k + km + kn + \frac{1}{2}km^2n - kmn,$$
$$\sum_{e \in E'} \lambda(e) = \sum_{j=1}^{m} \lambda(e^s_{ji}) = \frac{1}{2}m\,(2k - 2kn + 3kmn + 1)\,,$$

and the magic constant is

$$m\,(\lambda) = \sum_{v \in V'} \lambda(v) + \sum_{e \in E'} \lambda(e) = m - k + 2km + kn + 2km^2n - 2kmn.$$

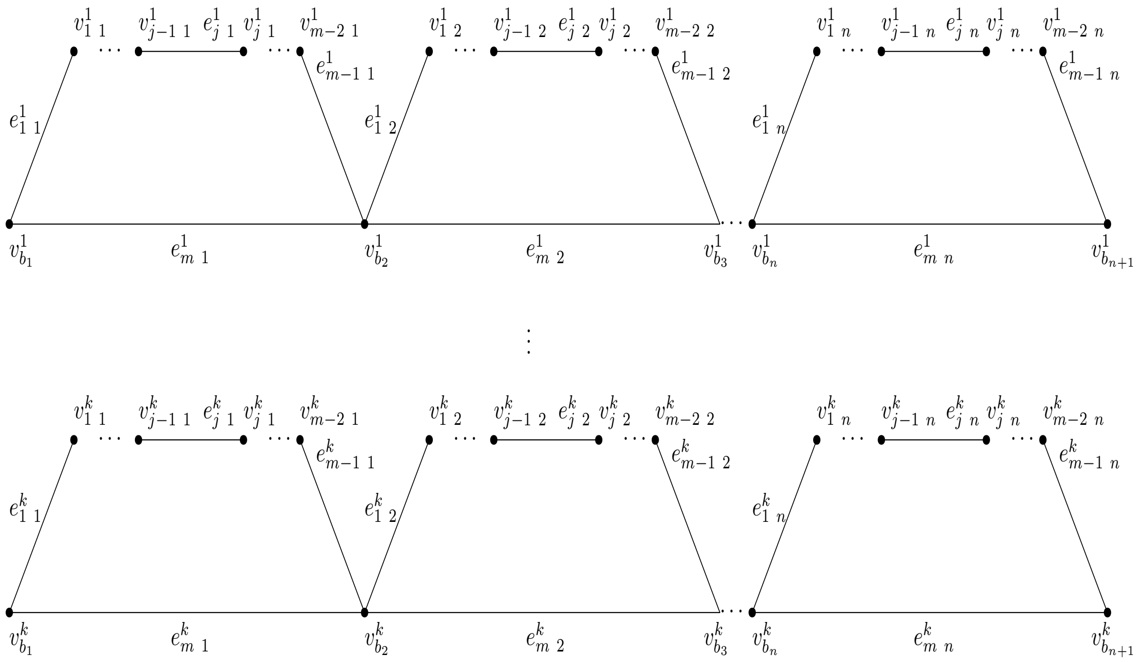

Figure 4: k isomorphic copies of m-polygonal snake graph $k\Delta^m_n; n, k \geqslant 2, m \geqslant 4$.

Case 2: m is odd. We construct a labeling $\lambda : V \cup E \to \{1, 2, 3, \dots, |V| + |E|\}$ as follows:

$$\lambda\left(v^s_{b_i}\right) = s + k\,(i-1)\,,\ i = 1, 2, 3, \dots, n+1,$$
$$\lambda(v^s_{1i}) = (2-i)\,k - s + 2kn + 1,\ i = 1, 2, 3, \dots, n,$$
$$\lambda(v^s_{ji}) = \begin{cases} ik + s + jkn, & j = 2, 4, 5, \dots, m-3, \quad i = 1, 2, 3, \dots, n, \\ (2-i)\,k - s + kn + jkn + 1, & j = 3, 5, 7, \dots, m-2, \quad i = 1, 2, 3, \dots, n, \end{cases}$$
$$\lambda(e^s_{ji}) = \begin{cases} ik + s - 2kn + jkn + kmn, & j = 1, 3, 5, \dots, m-2, \quad i = 1, 2, 3, \dots, n, \\ (2-i)\,k - s - kn + jkn + kmn + 1, & j = 2, 4, 6, \dots, m-1, \quad i = 1, 2, 3, \dots, n, \end{cases}$$
$$\lambda(e^s_{mi}) = (2-i)\,k - s - kn + 2kmn + 1,\ i = 1, 2, 3, \dots, n,$$

where $s = 1, \dots, k$. Here, for all $v \in V$, we have $\lambda(v) \in \{1, 2, 3, \dots, ((m-1)n+1)k\}$ and for any subgraph $H' = (V', E')$ isomorphic to C_m, we have

$$\sum_{v \in V'} \lambda(v) = \lambda(v^s_{b_i}) + \lambda(v^s_{b_{i+1}}) + \lambda(v^s_{1i}) + \sum_{j=2}^{m-2} \lambda(v^s_{ji})$$

$$= \frac{1}{2}m - (2-i)k + s + km + \frac{1}{2}kn + \frac{1}{2}km^2n - kmn - \frac{1}{2},$$

$$\sum_{e \in E'} \lambda(e) = \sum_{j=1}^{m-1} \lambda(e^s_{ji}) + \lambda(e^s_{mi}) = (1-i)k + \frac{1}{2}m - s + km + \frac{1}{2}kn + \frac{3}{2}km^2n - kmn + \frac{1}{2},$$

and the magic constant is

$$m(\lambda) = \sum_{v \in V'} \lambda(v) + \sum_{e \in E'} \lambda(e) = m - k + 2km + kn + 2km^2n - 2kmn.$$

Hence $k\Delta_n^m; n, k \geqslant 2, m \geqslant 4$, admits a C_m-supermagic labeling. □

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