Available online at www.isr-publications.com/jmcs
J. Math. Computer Sci., 21 (2020), 23–30
Research Article

Online: ISSN 2008-949X
Journal of Mathematics and Computer Science
Journal Homepage: www.isr-publications.com/jmcs

Conformally transformed vector fields on special (α, β)-metric

N. Natesh, S. K. Narasimhamurthy, M. K. Roopa*

Department of Mathematics, Kuvempu University, Shankaraghatta–577 451, Shimoga, Karnataka, INDIA.

Abstract

In this paper, we characterize the PDE's of conformal vector fields on Finsler space with special (α, β)-metrics. Further, we prove that conformally transformed vector field related by F and also corresponding conformal factors c and $\tilde{c}$.

Keywords: Conformal vector field, special (α, β)-metrics.

2020 MSC: 53B40, 53C60.

1. Introduction

The first to treat the conformal theory of Finsler metrics generally was M. S. Knebelman. He defined two metric functions F and $\bar{F}$ as conformal if the length of an arbitrary vector in the one is proportional to the length in the other, that is if $g_{ij} = \phi g_{ij}$. The length of vector ϵ means here the fact that ϕg_{ij} , as well as g_{ij} , must be Finsler metric tensor, he showed that ϕ falls into a point function and also proved that if (M, F) be an n-dimensional Finsler manifold and ϕ a transformation on M, then ϕ is called the conformal transformation, if it preserves the angles. Let X be a vector field on M and ϕ_t be the local one-parameter group of local transformations on M generated by X. Then X is called a conformal vector field on M if each ϕ_t is a local conformal transformation of M.

Conformal vector fields play an important role in Finsler geometry. Some problems on (α, β)-metrics can be solved by constructing a conformal vector field on a Riemannian metric with certain curvature features. For two conformally related Finsler metrics on a manifold, their conformal vector field coincide [6].

In [4], Shen and Xia study the conformal vector fields on Randers spaces under certain curvature conditions. In [1], Huang and Mo shows that a conformal vector field of a Randers space of isotropic S-curvature must be homothetic. In [2], Kang characterizes the conformal vector fields of an (α, β)-metrics

*Corresponding author
Email addresses: nateshmaths@gmail.com (N. Natesh), nmurthysk@gmail.com (S. K. Narasimhamurthy), roopamk1002@gmail.com (M. K. Roopa)

doi: 10.22436/jmcs.021.01.03

Received: 2019-07-17 Revised: 2019-08-06 Accepted: 2019-08-21

by some PDE's in a special case $\phi^1 \neq 0$. Recently, Natesh et al. studied the conformal vector fields on Finsler space with special (α, β)-metrics [3].

As above reviews, this work characterize the PDE's of conformal vector fields on Finsler space with special (α, β)-metric. Then we proved the conformally transformed vector fields of Finsler metric $\tilde{F}$ related by F and also with conformal factor related to each other (see Proposition 3.3, 4.2).

2. Preliminaries of conformal vector fields

Let F be a Finsler metric on a manifold M and V be a vector field on M. Let ϕ_t be the flow generated by V. $\tilde{\phi_t} : TM \to TM$ by $\tilde{\phi_t} = (\phi_t(x), \phi_t * (y))$. V is said to be conformal if

$$\phi_t^* \tilde{F} = e^{-2\sigma_t} F, \tag{2.1}$$

where σ_t is a function M for every t. Differentiating (2.1) by t at $t = 0$. We obtain

$$X_v(F) = -2cF,$$

where we define

$$X_v = V^i \frac{\partial}{\partial x^i} + y^i \frac{\partial V^j}{\partial x^i} \frac{\partial}{\partial y^j}, \quad c = \frac{d}{dt|_t = 0} \sigma_t. \tag{2.2}$$

In (2.2), the function c is called the conformal factor.

Remark 2.1. A vector field V is conformal satisfying (2.1) if and only if (2.2) holds for some scalar function c. In this case, c and σ_t are related by

$$\sigma_t = \int_0^t c(\phi_s) ds, \quad c = \frac{d}{dt|_t = 0} \sigma_t. \tag{2.3}$$

Remark 2.2. By (2.3) we easily see that $\tilde{\phi}_t^* F = e^{-2\sigma_t} F$ for a scalar function c if and only if c is constant along every integral curve of V.

Lemma 2.3 ([5])**.** *Let $\beta = b_i(x) y^i$ be a 1-form and V be a vector field on a Riemanian manifold $(M, \dot{\alpha})$ with $\alpha = \sqrt{a_{ij} y^i y^j}$. Then we have*

$$X_v(\alpha^2) = 2V_{0;0}, \quad X_v(\beta) = (V^j \frac{\partial b_i}{\partial x^i}) y^i = (V^j b_{i;j} + b^j V_{j;i}) y^i, \tag{2.4}$$

where $V_i = a_{ij} V^j$ and $b^i = a^{ij} b_j$, and the covariant derivative is taken with respect to the Levi-Civita connection of α.

3. Conformal vector fields on special (α, β)-metric, $F = \alpha + \frac{\beta^2}{\alpha}$

In this, section, we study the conformally transformed vector field on Finsler space with special (α, β)-metric, $F = \alpha + \frac{\beta^2}{\alpha}$. For this, first we prove the following lemma.

Lemma 3.1. *A vector field V on a Finsler space with special (α, β) metric $F = \alpha + \frac{\beta^2}{\alpha}$ is conformal with conformal factor c if and only if it satisfies*

$$X_v(F^2) = -4 \left(cF^2 + F^1 \frac{\partial V_i}{\partial y^k} y^i \right),$$

or

$$V_{0|0} = -2 \left(cF^2 + F^1 \frac{\partial V_i}{\partial y^k} y^i \right),$$

where

$$F^1 = 2\alpha k s_0^i - m s_0 r_{00} b^i.$$

Proof. By Remark 2.1, it is sufficient to show that $X_v(F^2) = 2V_{0|0} + F^1 \frac{\partial V_i}{\partial y^k} y^i$. Let us consider the spray coefficients G^i as

$$G^i = G^i_\alpha + Py^i + Q^i,$$

where

$$\begin{aligned}
P &= \alpha^{-1}\theta(-2\alpha Q s_0 + r_{00}),\\
Q^i &= \alpha Q s^i_0 + \psi(-2\alpha Q s_0 + r_{00}) b^i,\\
\theta &= \frac{\phi\phi^{'} - s(\phi\phi^{''} + \phi^{'} * \phi^{'})}{2\phi\left((\phi - s\phi^{'}) + (b^2 - s^2)\phi^{''}\right)},\\
Q &= \frac{\phi^{'}}{\phi - s\phi^{'}},\\
\psi &= \frac{1}{2}\frac{\phi^{''}}{\phi - s\phi^{'} + (b^2 - s^2)\phi^{''}}.
\end{aligned}$$

By calculation we get,

$$\begin{aligned}
\theta &= \frac{4s^3}{(2+2s^2)(1-3s^2+2b^2)},\\
Q &= \frac{1+s^2}{1-s^2},\\
\psi &= \frac{1}{1-s^2+2b^2},\\
P &= \frac{4s^3}{\alpha(1-s^2)(2+2s^2)}\left\{\frac{-2(\alpha+s^2)s - 0 + r_{00}(1-s^2)}{(1-3s^2+2b^2)}\right\},\\
Q^{'} &= \alpha\left(\frac{1+s^2}{1-s^2}\right)s^i_0 - \left\{\frac{2\alpha s_0 - r_{00}}{1-s^2+2b^2}\right\}b^i.
\end{aligned}$$

Let

$$\begin{aligned}
G^i &= g^{im}G_m = \frac{1}{4}\left\{[F^2]_{x^k y^k}Y^k - [F^2]_{x^i}\right\},\\
G^k_k &= \frac{\partial G^k}{\partial y^i},\\
G^k_{ij} &= \frac{\partial^2 G^k}{\partial y^i \partial y^j}.
\end{aligned}$$

Then the spray coefficients of class of Finsler metric $F = \alpha + \frac{\beta^2}{\alpha}$ is

$$G^k = \overset{\alpha}{G^k} - (Ls_0 + Mr_{00})\, y^i + \alpha k s^i_0 - R s_0 r_{00} b^i,$$

where

$$\begin{aligned}
L &= \frac{s^3(2\alpha - 2s^2)}{(2\alpha + 2\alpha s^4)(1-3s^2+2b^2)},\\
M &= \frac{4s^2(1-s^2)}{2\alpha(1+s^4)(1-3s^2+2b^2)},\\
K &= \frac{1+s^2}{1-s^2},
\end{aligned}$$

$$R = \frac{2\alpha}{1 - s^2 + 2b}.$$

From [6] by computation shows that

$$X_v(F^2) = V^i (F^2)_{x^i} + 2\frac{\partial V^i}{\partial x^j} y^i y^j = V^i (F^2)_{x^i} + 2\frac{\partial V^i}{\partial x^j} y^i y^j - \frac{\partial y^i}{\partial x^j} V^i y^j, \tag{3.1}$$

$$\frac{\partial V^i}{\partial x^j} y^i y^j = V_{0|0} + 2(\alpha k s_0^i - M s_0 r_{00} b^i)\frac{\partial V^i}{\partial y^k} y^i + 2V_k G^k, \tag{3.2}$$

$$4V_k G^k = 2V_i \frac{\partial y^i}{\partial x^k} y^k - V^i \left\{ (\frac{1}{2\alpha} + \beta^4)\frac{\partial a_{ij}}{\partial x^i} y^i y^j + 4\beta(1 - 2\alpha^3\beta^2)\frac{\partial b_i}{\partial x^i} y^i \right\}. \tag{3.3}$$

Sunstitute (3.2) and (3.3) in (3.1) then we have

$$X_F(F^2) = 2V_{0|0} + 2(\alpha k s_0^i - M s_0 r_{00} b^i)\frac{\partial V^i}{\partial y^k} y^i. \tag{3.4}$$

Therefore, it desired the claim. □

Remark 3.2. From (3.4), suppose to be $\frac{\partial V_i}{\partial y^k} y^i = 0$. Then Lemma 3.2 becomes as, if V be vector field and F be special (α, β)-metric, then it is conformal with conformal factor c if and only if it satisfies $X_v(F^2) = -4cF^2$ or $V_{0|0} = -2cF^2$.

Proposition 3.3. *Let $(M, \tilde{F} = (\tilde{\alpha} + \frac{\tilde{\beta}^2}{\tilde{\alpha}^2}))$ be conformally transformation of special (α, β) metric $F(= \alpha + \frac{\beta^2}{\alpha})$ related with $\tilde{F} = e^{\frac{\sigma}{2}} F$ for a scalar function σ. Then V is a conformal vector field of (M, F) if and only if V is a conformal vector field of $(M, \tilde{F})$. Further, their conformal factor c and $\tilde{c}$ are related by, $\tilde{c} = c - \frac{1}{4}V(\sigma)$.*

Proof. Assume that V is conformal vector field of F with the conformal factor c. Then by Remark 3.2 we have

$$X_v(F^2) = -4cF^2.$$

Then by conformally related with $\tilde{F} = e^{\frac{\sigma}{2}} F$, we have

$$X_v(F^2) = X_v(e^\sigma)F^2 + e^\sigma X_v(F^2),$$

$$X_v(F^2) = (V(\sigma) - 4c)\,\tilde{F^2}. \tag{3.5}$$

Again by Remark 3.2 implies that V is also a conformal vector field of $\tilde{F}$ and the conformal factor $\tilde{c}$. Therefore, from (3.5) we get

$$\tilde{c} = c - \frac{1}{4}V(\sigma).$$ □

Theorem 3.4. *Let $F = \alpha + \frac{\beta^2}{\alpha}$ be Finsler metric and $V = V^i(x)\frac{\partial}{\partial x^i}$ be a vector field. Then V is a conformal vector field of F with the conformal factor c if and only if it satisfies $V_{i;j} + V_{j;i} = -4ca_{ij}$ and $V^j b_{i;j} + b^j V_{j;i} = -2cb_i$, where c is a scalar function.*

Proof. Since we says that V is a conformal vector field of F if and only if $X_v(F^2) = -4cF^2$. From [3] by computation shows that

$$X_v(F^2) = (1 - s^4)X_v(\alpha^2) + 4\alpha s(1 + s^2)X_v(\beta).$$

Now, plugging (3.1), we see that

$$X_v(F^2) = -4cF^2,$$

is written as

$$V_{0;0} + \alpha\left(\frac{2s}{1-s^2}\right)\left(V^i b_{j;i} + b^i V_{i;j}\right) y^j = \frac{4sc}{1-s^2}\alpha^2, \tag{3.6}$$

where ";" represents the covariant derivative with respect to α.

In order to simplify (3.6). We choose a special coordinate system (s, y^a) at a fixed point on a manifold as usually used. Fix an arbitrary point $x \in M$ and take an orthogonal basis e_i at x such that

$$\alpha = \sqrt{\sum_{i=1}^{n}(y_i)^2}, \quad \beta = by^i.$$

It follows that $\beta = s\alpha$ such that

$$y^1 = \frac{s}{\sqrt{b^2-s^2}}\tilde{\alpha}\left(\tilde{\alpha} = \sqrt{\sum_{a=2}^{n}(y^a)^2}\right).$$

Then if we change coordinate (y^i) to (s, y^a) we get

$$\alpha = \frac{b}{\sqrt{b^2-s^2}}\tilde{\alpha}, \quad \beta = \frac{bs}{\sqrt{b^2-s^2}}\tilde{\alpha}.$$

Let,

$$\overline{V}_{0;0} = V_{a;b}y^a y^b, \quad \overline{V}_{1;0} = V_{1;a}y^a,$$

$$\overline{V}_{0;1} = V_{a;1}y^a, \quad \overline{b}_{0;i} = b_{a;i}y^a.$$

Note, that under the coordinate (s, y^a), we have $b_i = b$, $\overline{b}_0 = 0$, but generally $\overline{b}_{0;i} \neq 0$.

Under these coordinate, Equation (3.6) is equivalent to

$$0 = b\left(b\overline{V}_{1;0} + V^i\overline{b}_{0;i}\right)\frac{2s}{1-s^2} + \left(\overline{V}_{1;0} + \overline{V}_{0;1}\right)s, \tag{3.7}$$

$$0 = b\left[b(2bc + V^i b_{1|i} + bV_{1|1})\frac{2s^2}{1-s^2} + 2b^2c + V_{1|1}s^2\right]\alpha^2 + (b^2-s^2)\overline{V}_{0;0}. \tag{3.8}$$

For (3.7), we will prove

$$b\overline{V}_{1;0} + V^i\overline{b}_{0;i} = 0, \quad \overline{V}_{1;0} + \overline{V}_{0;1} = 0. \tag{3.9}$$

If $b\overline{V}_{1;0} + V^i\overline{b}_{0;i} \neq 0$ at a point, then we prove $Q = \frac{2s}{1-s^2} = ks$, for some constant k. Solving $Q = ks$ with $\phi(0) = 1$, we get $\phi(s) = \sqrt{1+ks}$ and hence F is Riemannian. So we must have the first equation in (3.9).

Again by (3.8), we have the second equation in (3.9). For (3.8) we will prove

$$\overline{V}_{0;0} = -2c\overline{\alpha}^2, \quad V^i b_{1|i} + bV_{1|1} = -2bc, \quad V_{1|1} = -2cF. \tag{3.10}$$

Putting $s = 0$ in (3.8) we have

$$\tilde{V}0;0 = -2c\tilde{\alpha}^2.$$

By this, (3.8) is reduced to

$$b\left(2bc + V^i b_{1|i} + bV_{1|1})\right)\frac{2s}{1-s^2} + \left(V_{1|1} + 2c\right)s = 0.$$

Similarly, repeating this procedure of (3.7) we obtain second and third values of (3.10). Therefore, it follows from (3.9) and (3.10) then we get the claim. □

4. Conformal vector fields on special (α, β)-metric, $F = \alpha + \beta + \frac{\beta^2}{\alpha}$

In this, section, we study the conformally transformed vector field on Finsler space with special (α, β)-metric. Now we prove the following lemma.

Lemma 4.1. *A vector field* V *on a Finsler space with special* (α, β) *metric is conformal with conformal factor* c*, if and only if it satisfies* $X_v(F^2) = -4cF^2$ *or* $V_{0|0} = -2cF^2$.

Proof. By Remark 2.1, it is suffycies to show $X_v(F^2) = 2V_{0|0}$. Let

$$G^i = g^{im}G_m = \frac{1}{4}\left\{[F^2]_{x^k y^k}Y^k - [F^2]_{x^i}\right\}, \quad G^k_k = \frac{\partial G^k}{\partial y^i}, G^k_{ij}\frac{\partial^2 G^k}{\partial y^i \partial y^j}.$$

In the view of section third the spray coefficients G^i is on the Finsler metric $F = \alpha + \beta + \frac{\beta^2}{\alpha}$, we get,

$$G^k = \overset{\alpha}{G^k} + A_1 A_2 y^i + A_3 s^i_0 - \{A_4 s_0 + r_{00}\} b^i,$$

where

$$\begin{aligned}
A_1 &= \frac{r_{00}(1-s^2) - 2\alpha(1+2s)s_0}{2 - 2s - 4b^2 - 4s^2b^2 - 4sb^2 + 2s^3 + 6s^4}, \\
A_2 &= \frac{1 - 3s^2 - 4s^3}{\alpha(1-s^2)}, \\
A_3 &= \frac{\alpha(1+2s)}{1-s^2}, \\
A_4 &= \frac{2\alpha(1+2s)}{(1-s^2)(1-3s^2-2b^2)} s_0.
\end{aligned}$$

We know that

$$X_v(F^2) = V^i(F^2)_{x^i} + 2\frac{\partial V^i}{\partial x^j}y^i y^j = V^i(F^2)_{x^i} + 2\frac{\partial V^i}{\partial x^j}y^i y^j - \frac{\partial y^i}{\partial x^j}V^i y^j, \tag{4.1}$$

$$\frac{\partial V^i}{\partial x^j}y^i y^j = V_{0|0} + 2(A_3 s^i_0 - A_4 s_0 r_{00} b^i)\frac{\partial V^i}{\partial y^k}y^i + 2V_k G^k, \tag{4.2}$$

$$\begin{aligned}
4V_k G^k = 2V^i \frac{\partial y_i}{\partial x^k}y^k - \frac{\partial a_{ij}}{\partial x^i}y^i y^j \left(\frac{1}{2\alpha} + \frac{\beta^4}{2\alpha^3}\right) \\
+ \frac{\partial b_i}{\partial x^i}\left(6\beta - 4\beta^3 + 2\alpha + \frac{6\beta^2}{\alpha}\right) + \frac{\partial \alpha}{\partial x^i}\left(2\beta - \frac{2\beta^3}{\alpha^2}\right).
\end{aligned} \tag{4.3}$$

Substitute (4.2) and (4.3) in (4.1) then we have

$$X_v(F^2) = 2V_{0|0} + 2(A_3 s^i_0 - A_4 s_0 r_{00} b^i)\frac{\partial V^i}{\partial y^k}y^i.$$

Therefore, it is desired. □

Proposition 4.2. *Let* $(M, \tilde{F} = (\tilde{\alpha} + \tilde{\beta} + \frac{\tilde{\beta}^2}{\tilde{\alpha}^2}))$ *be conformally transformation of special* (α, β) *metric* $F = \alpha + \beta + \frac{\beta^2}{\alpha}$ *related with* $\tilde{F} = e^{\frac{\sigma}{2}}F$ *for a scalar function. Then* V *is a conformal vector field of* (M, F) *if and only if* V *is a conformal vector field of* $(M, \tilde{F})$*. Further, their conformal factor* c *and* $\tilde{c}$ *are related by,* $\tilde{c} = c - \frac{1}{4}V(\sigma)$.

Proof. Assume that V is conformal vector field of F with the conformal factor c. Then by Remark 3.2 we have

$$X_v(F^2) = -4cF^2.$$

Then by conformally related with $\tilde{F} = e^{\frac{\sigma}{2}}F$, we have

$$X_v(F^2) = X_v(e^\sigma)F^2 + e^\sigma X_v(F^2),$$

$$X_v(F^2) = (V(\sigma) - 4c)\,\tilde{F}^2. \tag{4.4}$$

Again, by Remark 3.2 implies that V is also a conformal vector field of $\tilde{F}$ and the conformal factor $\tilde{c}$. Therefore, from (4.4) we get

$$\tilde{c} = c - \frac{1}{4}V(\sigma).$$

□

Theorem 4.3. *Let $F = \alpha + \beta + \frac{\beta^2}{\alpha}$ be Finsler metric and $V = V^i(x)\frac{\partial}{\partial x^i}$ be a vector field. Then V is a conformal vector field of F with the conformal factor c, if and only if it satisfies*

$$V_{i;j} + V_{j;i} = -4ca_{ij},$$

and

$$V^j b_{i;j} + b^j V_{j;i} = -2cb_i,$$

where c is a scalar function.

Proof. Since we say that V is a conformal vector field of F if and only if $X_v(F^2) = -4cF^2$.

From [3] by computation shows that

$$X_v(F^2) = (1 - s^4)X_v(\alpha^2) + 4\alpha s(1 + s^2)X_v(\beta).$$

Now, plugging Equation (2.4), we see that

$$X_v(F^2) = -4cF^2,$$

is written as

$$V_{0;0} + \alpha\left(\frac{2s}{1-s^2}\right)\left(V^i b_{j;i} + b^i V_{i;j}\right)y^j = \frac{4sc}{1-s^2}\alpha^2, \tag{4.5}$$

where ";" represents the covariant derivative with respect to α.

In order to simplify (4.5), we choose a special coordinate system (s, y^a) at a fixed point on a manifold as usually used. Fix an arbitrary point $x \in M$ and take an orthogonal basis e_i at x such that

$$\alpha = \sqrt{\sum_{i=1}^{n}(y_i)^2}, \quad \beta = by^i.$$

It follows that $\beta = s\alpha$ such that

$$y^1 = \frac{s}{\sqrt{b^2 - s^2}}\tilde{\alpha}\left(\tilde{\alpha} = \sqrt{\sum_{a=2}^{n}(y^a)^2}\right).$$

Then if we change coordinate (y^i) to (s, y^a) we get

$$\alpha = \frac{b}{\sqrt{b^2 - s^2}}\tilde{\alpha}, \quad \beta = \frac{bs}{\sqrt{b^2 - s^2}}\tilde{\alpha}.$$

Let,

$$\overline{V}_{0;0} = V_{a;b}y^a y^b, \quad \overline{V}_{1;0} = V_{1;a}y^a,$$

$$\overline{V}_{0;1} = V_{a;1}y^a, \quad \overline{b}_{0;i} = b_{a;i}y^a.$$

Note that under the coordinate (s, y^a), we have $b_i = b$, $\overline{b}_0 = 0$, but generally $\overline{b}_{0;i} \neq 0$. Under these coordinate, (4.5) is equivalent to

$$0 = b\left(b\overline{V}_{1;0} + V^i\overline{b}_{0;i}\right)\frac{2s}{1 - s^2} + \left(\overline{V}_{1;0} + \overline{V}_{0;1}\right)s, \tag{4.6}$$

$$0 = b\left[b(2bc + V^i b_{1|i} + bV_{1|1})\frac{2s^2}{1 - s^2} + 2b^2c + V_{1|1}s^2\right]\alpha^2 + (b^2 - s^2)\overline{V}_{0;0}. \tag{4.7}$$

For (4.6), we will prove

$$b\overline{V}_{1;0} + V^i\overline{b}_{0;i} = 0, \quad \overline{V}_{1;0} + \overline{V}_{0;1} = 0. \tag{4.8}$$

If $b\overline{V}_{1;0} + V^i\overline{b}_{0;i} \neq 0$ at a point, then we prove $Q = \frac{2s}{1-s^2} = ks$ for some constant k. Solving $Q = ks$ with $\phi(0) = 1$. We get $\phi(s) = \sqrt{1 + ks}$ and hence F is Riemannian. So we must have the first equation in (4.8). Again by (4.7), we have the second equation in (4.8).

For (4.7) we will prove

$$\overline{V}_{0;0} = -2c\overline{\alpha}^2, \quad V^i b_{1|i} + bV_{1|1} = -2bc, \quad V_{1|1} = -2cF. \tag{4.9}$$

Putting $s = 0$ in (4.7) we have

$$\tilde{V}0;0 = -2c\tilde{\alpha}^2.$$

By this, (4.7) is reduced to

$$b\left(2bc + V^i b_{1|i} + bV_{1|1})\right)\frac{2s}{1 - s^2} + \left(V_{1|1} + 2c\right)s = 0.$$

Similarly, repeating this procedure of (4.7) we obtain the second and third values of (4.9). Therefore, it follows from (4.8) and (4.9) then we get the cliam. □

Remark 4.4. Let $F = \alpha\phi(\frac{\beta}{\alpha})$ be an (α, β)-metric with $\phi(0) = 1$. For this, holds for all class of (α, β)-metrics.

References

[1] L. B. Huang, X. H. Mo, *On conformal fields of a Randers metric with isotropic S-curvature*, Illinois J. Math., **57** (2013), 685–696. 1

[2] L. Kang, *On conformal vector fields of (α, β)- spaces*, preprint. 1

[3] N. Natesh, S. K. Narasimhamurthy, Roopa M.K., *Conformal vector fields on Finsler space with special (α, β)-metric*, J. Adv. Math. Comput. Sci., **31** (2019), 1–8. 1, 3, 4

[4] Z. M. Shen, Q. L. Xia, *On conformal vector fields on Randers manifolds*, Sci. China Math., **55** (2012), 1869–1882. 1

[5] G. J. Yang, *On a class of Finsler metrics of scalar flag curvature*, arXiv, **2013** (2013), 22 pages. 2.3

[6] G. J. Yang, *Conformal vector fields on projectively flat (α, β) Finsler space*, arXiv, 2014 (2014), 12 pages. 1, 3