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Filteristic soft BCK-algebras

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Abstract

We study the soft sets applied on the structure of filters of BCK-algebras. A connection between soft BCK-algebras and filteristic soft BCK-algebras is given also. Finally, important operations such as intersection, union, “AND”, “OR”, and subset operations of soft filters and filteristic soft BCK-algebras are investigated.

1. Introduction

To solve complicated problem in economics, engineering, and environment, we can not Successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theory have their own difficulties. Uncertainties cannot be handle using traditional mathematical tools but may be deal with using a wide rang of existing theory such as probability theory, theory of fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theory have own difficulties which are pointed out in [8]. Maji et al. [7] and Molodtsov [8] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [8] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodstov pointed out several directions for the applications of soft sets. At present, works on the soft set

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theory are progressing rapidly. Maji et at [7] described the application of soft set theory are to a decision making problem. Maji et at [7] also studied several operations of soft sets. Chen et at. [2] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. Aktas and Cagman [1] studied the basic concepts to clarify their differences. They also discussed the notion of soft groups.

2. Preliminaries

An algebra (X, *, 0) of type (2, 0) is called a BCI-algebra if it satisfies the following conditions:

(1) $(\forall x, y, z \in X)(((x * y) * (x * z)) * (z * y)=0)$,
(2) $(\forall x, y \in X)((x * (x * y)) * y=0)$,
(3) $(\forall x \in X)$ $(x * x=0)$,
(4) $(x, y \in X)$ $(x * y = 0, y * x = 0 => x = y)$.

If a BCI-algebra X satisfies the following condition:

(5) $(\forall x \in X)$ $(0 * x = 0)$,
Then X is called a BCK-algebra. Any BCK-algebra X satisfies the following axioms:

(a1) $(\forall x \in X)$ $(x * 0 = x)$,
(a2) $(\forall x, y, z \in X)$ $(x \leq y => x * z \leq y * z, z * y \leq z * x)$,
(a3) $(\forall x, y, z \in X)$ $\big((x * y) * z = (x * z) * y\big)$,
(a4) $(\forall x, y, z \in X)$ $\big((x * z) * (y * z) \leq x * y\big)$,

Where $x \leq y$ if and only if $x * y = 0$. A *BCK*-algebra *X* is said to be *commutative* if $x \wedge y = y \wedge x$ for all $x, y \in X$ where $x \wedge y = y * (y * x)$. A nonempty subset *S* of a *BCK*-algebra *X* is called a *subalgebra* of *X* if $x, y \in S$ for all $x, y \in S$.

Definition 2.1. [3] A *filter* of X is a nonempty subset *F* such that:
(i) $x \in F$ and $y \in F$ imply $x \wedge y \in F$,
(ii) $x \in F$ and $x \leq y$ imply $y \in F$.

In this case we write:

$F \prec_f X$.

Definition 2.2. [6] Let *(F, A)* and *(G,B)* be two soft sets over a common universe *U*. The *intersection* of (*F,A*) and (*G,B*) is defined to be the soft set (*H,C*) satisfying the following conditions:

(i) $C = A \cap B$,
(ii) (For all $e \in C$) ($H(e) = F(e)$ or $G(e)$), (as both are same set)).

In this case we write:

$$(F,A) \tilde{\cap} (G,B) = (H,C).$$

Definition 2.3. [6]Let *(F, A)* and *(G, B)* be two soft sets over a common universe U. The *union* of *(F,A)* and (*G,B*) is defined to be the soft set (*H,C*) satisfying the following conditions:

(i) $C = A \cup B$,

(ii) For all $e \in C$,

$$H(e)=\begin{cases} F(e) & \text{if } e \in A\backslash B \\ G(e) & \text{if } e \in B\backslash A \\ G(e) \cup H(e) & \text{if } e \in A \cap B \end{cases}$$

In this case we write :

$$(F,A)\ \tilde{\cup}\ (G,B) = (H,C).$$

Definition 2.4. [6] If *(F, A)* and *(G,B)* are two soft sets over a common universe U, " *(F,A)* AND (G,B)" denoted by

$$(F,A)\ \tilde{\Lambda}\ (G,B),$$

is defined by

$$(F,A)\ \tilde{\Lambda}\ (G,B) = (H,A\times B).$$

Where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for all $(\alpha, \beta) \in A\times B$.

3. Soft filters BCK-algebras

Definition 3.1. [4] Let *(F, A)* be a soft set over *X*. Then *(F, A)* is called a *soft BCK-algebra* over *X* if *F(x)* is a subalgebra of *X* for all $x \in A$.

Definition 3.2. Let *(F, A)* be a soft *BCK*-algebras over *X*. A soft set (*G,I*) over *X* is called a *soft filter* of (*F,A*), denoted by:

$$(G,I) \ \tilde{\prec}_f \ (F, X),$$

if it satisfies:

(i) $I \subset A$

(ii) $(\forall x \in 1)\ (G(x) \ \prec_f \ F(x))$.

Theorem 3.3. Let *(F, A)* be a soft *BCK*-algebra aver *X*. For any soft sets (G_1, I_1) and (G_2, I_2) over *X* where I and J are not disjoint, we have:

$$(G_1, I_1) \tilde{\prec}_f (F, A),\ (G_2, I_2) \tilde{\prec}_f (F, A) \Rightarrow (G_1, I_1) \tilde{\cap} (G_2, I_2) \tilde{\prec}_f (F, A).$$

Proof: Using Definition 2.2 we can write

$$(G_1, I_1) \tilde{\cap} (G_2, I_2) = (G, I)$$

Where I = $I_1 \cap I_2$ and G(x) = G_1 (x) or G_2 (x) for all $x \in$ I. Obviously I subset A and G:I→ P(x) is a mapping hence (G,I) is a soft set over X. Since $(G_1, I_1) \tilde{\prec}_f (F, A)$, and $(G_2, I_2) \tilde{\prec}_f (F, A)$, we know that G(x) = G_1 (x) $\prec_f$ F(x) and G(x) = G_2 (x) $\prec_f$ F(x) for all $x \in$ I. Hence

$$(G_1, I_1) \tilde{\cap} (G_2, I_2) = (G, I) \tilde{\prec}_f (F, A).$$

Corollary 3.4. Let *(F, A)* be a soft *BCK*-algebra aver *X*. For any soft sets (G,I) and (H,I) over *X* we have:

$$(G,I) \ \tilde{\prec}_f \ (F, X),\ (H,I) \ \tilde{\prec}_f \ (F, X) \Rightarrow (G,I) \tilde{\cap} (H.I) \tilde{\prec}_f (F, A).$$

Theorem 3.5. Let *(F, A)* be a soft *BCK*-algebra aver *X*. For any soft sets *(G,I)* and *(H,J)* over X in which *I* and J are disjoint, we have:

$$(G,I) \tilde{\prec}_f (F, A),\ (H,J) \tilde{\prec}_f (F, A) \Rightarrow (G,I) \tilde{\cup} (H,J) \tilde{\prec}_f (F, A).$$

Proof: Assume that (G,I) $\tilde{\prec}_f$ (F,A) and (H,J) $\tilde{\prec}_f$ (F,A). By means of Definition 2.3, we can write (G,I) $\tilde{\cup}$ (H,J) = (K,U) where U=I ∪ J and for every $x \in U$,

$$K(x) = \begin{cases} G(x) & \text{if } x \in I \backslash J \\ H(x) & \text{if } x \in J \backslash I \\ G(x) \cup H(x) & \text{if } x \in I \cap J \end{cases}$$

Since I and j are disjoint, either $x \in I \backslash J$ or $x \in J \backslash I$ for all $x \in U$. If $x \in I \backslash J$, then K(x) = G(x) $\prec_f$ F(x) since (G,I) $\tilde{\prec}_f$ (F ,A). If $x \in J \backslash I$, then K(x) = H(x) $\prec_f$ F(x) since

(H,J) $\tilde{\prec}_f$ (F ,A). Thus K(x) $\prec_f$ F(x) for $x \in U$, and so

$$(G,I) \tilde{\cup} (H,J) = (K,U) \tilde{\prec}_f (F,A).$$

4. Soft filteristic BCK-algebras

Definition 4.1. Let *(F, A)* be a soft set over *X*. Then *(F,A)* is called a *filteristic* soft *BCK*-algebra over *X* if *F(x)* is a filter of *X* for all $x \in A$.

Theorem 4.2. Let (*F, A*) and *(G, B)* be two filteristic soft BCK-algebra over *X*. If $A \cap B \neq \phi$, then the *intersection* $(F,A) \tilde{\cap} (G,B)$ is a filteristic soft BCK-algebra over *X*.

Proof: Using Definition 2.3, we can write $(F,A) \tilde{\cap} (G,B)$ =(H,C), where C = A ∩ B and H(x) = F(x) or G(x) for all $x \in C$. Note that H: C→P(x) is a mapping, and therefore (H,C) is a soft set over X. Since (F,A) and (G,B) are filteristic soft BCK-algebras over X, it follows that H(x) = F(x) is a filter of X, or H(x) = G(x) is a filter of X for all $x \in C$. Hence (H,C) = (F,A) $\tilde{\cap}$ (G,B) is a filteristic soft BCK-algebras over X.

Corollary 4.3. Let (*F, A*) and *(G, A)* be two filteristic soft BCK-algebra over *X*. then their *intersection (F,A)* $\tilde{\cap}$ *(G,B)* is a filteristic soft BCK-algebra over *X*.

Theorem 4.4. Let *(F, A)* and *(G, B)* be two filteristic soft *BCK*-algebra over *X*. If *A* and *B* are disjoint, then the *union (F,A)* $\tilde{\cup}$ *(G,B)* is a filteristic soft *BCK*-algebra over *X.*

Proof: Using Definition 2.3, we can write (F,A) $\tilde{\cup}$ (G,B) =(H,C), where C = A ∪ B and for every e∈ *C,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A\backslash B \\ G(e) & \text{if } e \in B\backslash A \\ G(e) \cup H(e) & \text{if } e \in A\cap B \end{cases}$$

Since A and B are disjoint, either e ∈ A\B or e ∈ B\A for all e∈ *C.* If e ∈ A\B, then H(x) = F(x) is a filter of X, Since (F,A) is a filteristic soft BCK-algebra over X. If e ∈ B\A, then H(x) = G(x) is a filter of X, since (G,B) is a filteristic soft BCK-algebra over X. Hence (H,C) = (F,A) $\tilde{\cup}$ (G,B) is a filteristic soft BCK-algebra over X.

Theorem 4.5. If *(F, A)* and *(G, B)* are filteristic soft *BCK*-algebra over *X*, then *(F, A)* $\tilde{\Lambda}$ *(G,B)*
is a filteristic soft *BCK*-algebra over *X*.

Proof: By means of Definition 2.4 we know that (F,A) $\tilde{\Lambda}$ (G,B) = (H, A×B) where H(x,y) = F(x) ∩ G(y) for all (x,y) ∈ A×B. Since F(x) and G(y) are filter of X, the intersection F(x) ∩G(y) is also a filter of X. Hence H(x,y) is a filter of X for all (x,y) ∈ A×B, and therefore (F,A) $\tilde{\Lambda}$ (G,B) = (H, A×B) is a filteristic soft BCK-algebra over X.

Definition 4.6. A filteristic soft BCK-algebra (F,A) over X is said to be whole if F(x) =X for all *x* ∈ *A.*

Lemma 4.7 Let *f: X → Y* be an onto homomorphism of *BCK*-algebras. If *(F, A)* is a filteristic soft *BCK*-algebra over *X*. then *(f (F), A)* is a filteristic soft *BCK*-algebra over *Y.*

Proof: For every *x* ∈ *A* we have f(F)(x) = f(F(x)) is a filter of Y since F(x) is a filter of X and its onto homomorphic image is also a filter of Y. Hence (f(F),A) is a filteristic soft BCK-algebra over Y.

Theorem 4.8. Let *f: X → Y* is a homomorphism of *BCK*-algebras and let *(F, A)* be a filteristic soft *BCK*-algebra over *X*. If f is onto and *(F, A)* is whole, then *(f (F), A)* is a whole filteristic soft *BCK*-algebras over *Y.*

Proof: Suppose that f is onto and (F,A) is whole. Then F(x) = X for all *x* ∈ A, and so f(F)(x) = f(F(x)) = f(X) =Y for all *x* ∈ A. It follows from Lemma 4.7 and Definition 4.6 that (f(F),A) is a whole filteristic soft BCK-algebra over Y.

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