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A novel approach to neutrosophic sets in UP-algebras

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Abstract

The notion of neutrosophic sets in UP-algebras was introduced by Songsaeng and Iampan [M. Songsaeng, A. Iampan, Eur. J. Pure Appl. Math., **12** (2019), 1382–1409]. In this paper, the notions of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras are introduced, and several properties are investigated. Conditions for neutrosophic sets to be special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras are provided. Relations between special neutrosophic UP-subalgebras (resp., special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, special neutrosophic strong UP-ideals) and their level subsets are considered.

Keywords: UP-algebra, special neutrosophic UP-subalgebra, special neutrosophic near UP-filter, special neutrosophic UP-filter, special neutrosophic UP-ideal, special neutrosophic strong UP-ideal.

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1. Introduction and preliminaries

Smarandache [19] introduced the notion of neutrosophic sets in 1999 which is a more general platform that extends the notions of classic sets, (intuitionistic) fuzzy sets and interval valued (intuitionistic) fuzzy sets (see [19, 20]). Neutrosophic set theory is applied to various part which is referred to the site

http://fs.gallup.unm.edu/neutrosophy.htm

The above-mentioned part has been derived from [25]. Wang et al. [28] introduced the notion of interval neutrosophic sets in 2005. Khan et al. [13] introduced the notion of neutrosophic $\mathcal{N}$-structures and their applications in semigroups in 2017. Jun et al. [8, 9] studied neutrosophic $\mathcal{N}$-structures to BCK/BCI-algebras and neutrosophic subalgebras of several types in BCK/BCI-algebras in 2017. Jun et al. [10] studied neutrosophic positive implicative $\mathcal{N}$-ideals in BCK-algebras in 2018. Kim et al. [14] studied generalizations of neutrosophic subalgebras in BCK/BCI-algebras based on neutrosophic points in 2018.

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Songsaeng and Iampan [23] introduced the notions of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strong UP-ideals of UP-algebras in UP-algebras.

In this paper, the notions of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras are introduced, and several properties are investigated. Conditions for neutrosophic sets to be special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras are provided. Relations between special neutrosophic UP-subalgebras (resp., special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, special neutrosophic strong UP-ideals) and their level subsets are considered.

Before we begin our study, we will give the definition and useful properties of UP-algebras.

An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of X (i.e., a nullary operation) if it satisfies the following axioms:

(UP-1) $(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$;

(UP-2) $(\forall x \in X)(0 \cdot x = x)$;

(UP-3) $(\forall x \in X)(x \cdot 0 = 0)$; and

(UP-4) $(\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

The branch of the logical algebra, UP-algebras were introduced by Iampan [5], and he proved that the notion of UP-algebras is a generalization of KU-algebras (see [16]). Later Somjanta et al. [21] studied fuzzy UP-subalgebras, fuzzy UP-ideals and fuzzy UP-filters of UP-algebras. Guntasow et al. [4] studied fuzzy translations of a fuzzy set in UP-algebras. Kesorn et al. [12] studied intuitionistic fuzzy sets in UP-algebras. Kaijae et al. [11] studied anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras. Tanamoon et al. [26] studied Q-fuzzy sets in UP-algebras. Sripaeng et al. [24] studied anti Q-fuzzy UP-ideals and anti Q-fuzzy UP-subalgebras of UP-algebras. Dokkhamdang et al. [3] studied Generalized fuzzy sets in UP-algebras. Songsaeng and Iampan [22] studied $\mathcal{N}$-fuzzy UP-algebras and its level subsets, etc.

Example 1.1 ([18])**.** Let X be a universal set and let $\Omega \in \mathcal{P}(X)$, where $\mathcal{P}(X)$ means the power set of X. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$, where A^C means the complement of a subset A. Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 1 with respect to* Ω. Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^\Omega(X)$ by putting $A * B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), *, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 2 with respect to* Ω. In particular, $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type 1*, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type 2*.

Example 1.2 ([3])**.** Let $\mathbb{N}$ be the set of all natural numbers with two binary operations $\circ$ and $\bullet$ defined by

$$(\forall x, y \in \mathbb{N})\left(x \circ y = \begin{cases} y, & \text{if } x < y, \\ 0, & \text{otherwise,} \end{cases}\right)$$

and

$$(\forall x, y \in \mathbb{N})\left(x \bullet y = \begin{cases} y, & \text{if } x > y \text{ or } x = 0, \\ 0, & \text{otherwise,} \end{cases}\right).$$

Then $(\mathbb{N}, \circ, 0)$ and $(\mathbb{N}, \bullet, 0)$ are UP-algebras.

Example 1.3 ([15])**.** Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	0	0	2	3	2	5
2	0	1	0	3	1	5
3	0	1	2	0	4	5
4	0	0	0	3	0	5
5	0	0	2	0	2	0

Then $(X, \cdot, 0)$ is a UP-algebra.

For more examples of UP-algebras, see [1, 2, 6, 17, 18].

In a UP-algebra $X = (X, \cdot, 0)$, the following assertions are valid (see [5, 6]).

$$(\forall x \in X)(x \cdot x = 0), \tag{1.1}$$
$$(\forall x, y, z \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \tag{1.2}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{1.3}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \tag{1.4}$$
$$(\forall x, y \in X)(x \cdot (y \cdot x) = 0), \tag{1.5}$$
$$(\forall x, y \in X)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \tag{1.6}$$
$$(\forall x, y \in X)(x \cdot (y \cdot y) = 0), \tag{1.7}$$
$$(\forall a, x, y, z \in X)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0), \tag{1.8}$$
$$(\forall a, x, y, z \in X)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \tag{1.9}$$
$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0), \tag{1.10}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \tag{1.11}$$
$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \text{ and} \tag{1.12}$$
$$(\forall a, x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0). \tag{1.13}$$

From [5], the binary relation $\leqslant$ on a UP-algebra $X = (X, \cdot, 0)$ defined as follows:

$$(\forall x, y \in X)(x \leqslant y \Leftrightarrow x \cdot y = 0).$$

Definition 1.4 ([4, 5, 7, 21])**.** A nonempty subset S of a UP-algebra $(X, \cdot, 0)$ is called

(1) a *UP-subalgebra* of X if $(\forall x, y \in S)(x \cdot y \in S)$;

(2) a *near UP-filter* of X if

  (i) the constant 0 of X is in S, and

  (ii) $(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S)$;

(3) a *UP-filter* of X if

  (i) the constant 0 of X is in S, and

  (ii) $(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S)$;

(4) a *UP-ideal* of X if

  (i) the constant 0 of X is in S, and

  (ii) $(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$;

(5) a *strong UP-ideal* (renamed from a strongly UP-ideal) of X if

(i) the constant 0 of X is in S, and

(ii) $(\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

Guntasow et al. [4] and Iampan [7] proved that the notion of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals, and UP-ideals is a generalization of strong UP-ideals. Moreover, they also proved that the only strong UP-ideal of a UP-algebra X is X.

2. NSs of special types in UP-algebras

The notion of a fuzzy set in a nonempty set was first considered by Zadeh [29] in 1965. A *fuzzy set* (briefly, FS) in a nonempty set X (or a fuzzy subset of X) is an arbitrary function $f : X \to [0, 1]$, where $[0, 1]$ is the unit segment of the real line, and the fuzzy set $\overline{f}$ defined by $\overline{f}(x) = 1 - f(x)$ for all $x \in X$ is said to be the *complement* of f in X. In 1999, Smarandache [19] introduced the notion of neutrosophic sets as the following definition. A *neutrosophic set* (briefly, NS) in a nonempty set X is a structure of the form:

$$\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}, \tag{2.1}$$

where $\lambda_T : X \to [0, 1]$ is a *truth membership function*, $\lambda_I : X \to [0, 1]$ is an *indeterminate membership function*, and $\lambda_F : X \to [0, 1]$ is a *false membership function*. For our convenience, we will denote a NS as $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F) = (X, \lambda_{T,I,F}) = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}$.

Definition 2.1 ([19])**.** Let Λ be a NS in a nonempty set X. The NS $\overline{\Lambda} = (X, \overline{\lambda}_{T,I,F})$ in X defined by

$$(\forall x \in X)\begin{pmatrix}\overline{\lambda}_T(x) = 1 - \lambda_T(x)\\ \overline{\lambda}_I(x) = 1 - \lambda_I(x)\\ \overline{\lambda}_F(x) = 1 - \lambda_F(x)\end{pmatrix} \tag{2.2}$$

is called the *complement* of Λ in X.

Remark 2.2 ([23]). For all NS Λ in a nonempty set X, we have $\Lambda = \overline{\overline{\Lambda}}$.

Lemma 2.3 ([27])**.** *Let* $a, b, c \in \mathbb{R}$*. Then the following statements hold:*

(1) $a - \min\{b, c\} = \max\{a - b, a - c\}$*, and*

(2) $a - \max\{b, c\} = \min\{a - b, a - c\}$.

Lemma 2.4 ([23])**.** *Let* f *be a fuzzy set in a nonempty set* X*. Then the following statements hold:*

(1) $(\forall x, y, z \in X)(\overline{f}(x) \geqslant \min\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \leqslant \max\{f(y), f(z)\})$;

(2) $(\forall x, y, z \in X)(\overline{f}(x) \leqslant \min\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \geqslant \max\{f(y), f(z)\})$;

(3) $(\forall x, y, z \in X)(\overline{f}(x) \geqslant \max\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \leqslant \min\{f(y), f(z)\})$; *and*

(4) $(\forall x, y, z \in X)(\overline{f}(x) \leqslant \max\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \geqslant \min\{f(y), f(z)\})$.

In what follows, let X denote a UP-algebra $(X, \cdot, 0)$ unless otherwise specified.

Definition 2.5 ([23])**.** A NS Λ in X is called a *neutrosophic UP-subalgebra* of X if it satisfies the following conditions:

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \geqslant \min\{\lambda_T(x), \lambda_T(y)\}), \tag{2.3}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \leqslant \max\{\lambda_I(x), \lambda_I(y)\}), \tag{2.4}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \geqslant \min\{\lambda_F(x), \lambda_F(y)\}). \tag{2.5}$$

Definition 2.6 ([23]). A NS Λ in X is called a *neutrosophic near UP-filter* of X if it satisfies the following conditions:

$$(\forall x \in X)(\lambda_T(0) \geqslant \lambda_T(x)), \tag{2.6}$$
$$(\forall x \in X)(\lambda_I(0) \leqslant \lambda_I(x)), \tag{2.7}$$
$$(\forall x \in X)(\lambda_F(0) \geqslant \lambda_F(x)), \tag{2.8}$$
$$(\forall x, y \in X)(\lambda_T(x \cdot y) \geqslant \lambda_T(y)), \tag{2.9}$$
$$(\forall x, y \in X)(\lambda_I(x \cdot y) \leqslant \lambda_I(y)), \tag{2.10}$$
$$(\forall x, y \in X)(\lambda_F(x \cdot y) \geqslant \lambda_F(y)). \tag{2.11}$$

Definition 2.7 ([23]). A NS Λ in X is called a *neutrosophic UP-filter* of X if it satisfies the following conditions: (2.6), (2.7), (2.8), and

$$(\forall x, y \in X)(\lambda_T(y) \geqslant \min\{\lambda_T(x \cdot y), \lambda_T(x)\}), \tag{2.12}$$
$$(\forall x, y \in X)(\lambda_I(y) \leqslant \max\{\lambda_I(x \cdot y), \lambda_I(x)\}), \tag{2.13}$$
$$(\forall x, y \in X)(\lambda_F(y) \geqslant \min\{\lambda_F(x \cdot y), \lambda_F(x)\}). \tag{2.14}$$

Definition 2.8 ([23]). A NS Λ in X is called a *neutrosophic UP-ideal* of X if it satisfies the following conditions: (2.6), (2.7), (2.8), and

$$(\forall x, y, z \in X)(\lambda_T(x \cdot z) \geqslant \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}), \tag{2.15}$$
$$(\forall x, y, z \in X)(\lambda_I(x \cdot z) \leqslant \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}), \tag{2.16}$$
$$(\forall x, y, z \in X)(\lambda_F(x \cdot z) \geqslant \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}). \tag{2.17}$$

Definition 2.9 ([23]). A NS Λ in X is called a *neutrosophic strong UP-ideal* (renamed from a neutrosophic strongly UP-ideal) of X if it satisfies the following conditions: (2.6), (2.7), (2.8), and

$$(\forall x, y, z \in X)(\lambda_T(x) \geqslant \min\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\}), \tag{2.18}$$
$$(\forall x, y, z \in X)(\lambda_I(x) \leqslant \max\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\}), \tag{2.19}$$
$$(\forall x, y, z \in X)(\lambda_F(x) \geqslant \min\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\}). \tag{2.20}$$

Definition 2.10 ([23]). A NS Λ in X is said to be *constant* if Λ is a constant function from X to $[0, 1]^3$. That is, λ_T, λ_I, and λ_F are constant functions from X to $[0, 1]$.

Theorem 2.11 ([23]). *A NS Λ in X is constant if and only if it is a neutrosophic strong UP-ideal of X.*

Songsaeng and Iampan [23] proved that the notion of neutrosophic UP-subalgebras is a generalization of neutrosophic near UP-filters, neutrosophic near UP-filters is a generalization of neutrosophic UP-filters, neutrosophic UP-filters is a generalization of neutrosophic UP-ideals, and neutrosophic UP-ideals is a generalization of neutrosophic strong UP-ideals.

Now, we introduce the notions of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

Definition 2.12. A NS Λ in X is called an *special neutrosophic UP-subalgebra* of X if it satisfies the following conditions:

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \leqslant \max\{\lambda_T(x), \lambda_T(y)\}), \tag{2.21}$$
$$(\forall x, y \in X)(\lambda_I(x \cdot y) \geqslant \min\{\lambda_I(x), \lambda_I(y)\}), \tag{2.22}$$
$$(\forall x, y \in X)(\lambda_F(x \cdot y) \leqslant \max\{\lambda_F(x), \lambda_F(y)\}). \tag{2.23}$$

Example 2.13. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	0	4
2	0	0	0	0	4
3	0	1	1	0	4
4	0	3	3	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.2 & 0.3 & 0.5 & 0.7 & 0.8 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.7 & 0.6 & 0.5 & 0.2 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.4 & 0.6 & 0.7 & 0.9 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic UP-subalgebra of X.

Definition 2.14. A NS Λ in X is called an *special neutrosophic near UP-filter* of X if it satisfies the following conditions:

$$(\forall x \in X)(\lambda_T(0) \leqslant \lambda_T(x)), \tag{2.24}$$
$$(\forall x \in X)(\lambda_I(0) \geqslant \lambda_I(x)), \tag{2.25}$$
$$(\forall x \in X)(\lambda_F(0) \leqslant \lambda_F(x)), \tag{2.26}$$
$$(\forall x, y \in X)(\lambda_T(x \cdot y) \leqslant \lambda_T(y)), \tag{2.27}$$
$$(\forall x, y \in X)(\lambda_I(x \cdot y) \geqslant \lambda_I(y)), \tag{2.28}$$
$$(\forall x, y \in X)(\lambda_F(x \cdot y) \leqslant \lambda_F(y)). \tag{2.29}$$

Example 2.15. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	1	0	1	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.3 & 0.5 & 0.6 & 0.2 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.8 & 0.7 & 0.3 & 0.4 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.2 & 0.6 & 0.7 & 0.5 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic near UP-filter of X.

Definition 2.16. A NS Λ in X is called an *special neutrosophic UP-filter* of X if it satisfies the following conditions: (2.24), (2.25), (2.26), and

$$(\forall x, y \in X)(\lambda_T(y) \leqslant \max\{\lambda_T(x \cdot y), \lambda_T(x)\}), \tag{2.30}$$
$$(\forall x, y \in X)(\lambda_I(y) \geqslant \min\{\lambda_I(x \cdot y), \lambda_I(x)\}), \tag{2.31}$$
$$(\forall x, y \in X)(\lambda_F(y) \leqslant \max\{\lambda_F(x \cdot y), \lambda_F(x)\}). \tag{2.32}$$

Example 2.17. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	1	0	1	4
3	0	0	0	0	4
4	0	1	1	1	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.5 & 0.4 & 0.5 & 0.8 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.8 & 0.3 & 0.5 & 0.3 & 0.4 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.2 & 0.6 & 0.4 & 0.6 & 0.3 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic UP-filter of X.

Definition 2.18. A NS Λ in X is called an *special neutrosophic UP-ideal* of X if it satisfies the following conditions: (2.24), (2.25), (2.26), and

$$(\forall x, y, z \in X)(\lambda_T(x \cdot z) \leqslant \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}), \tag{2.33}$$
$$(\forall x, y, z \in X)(\lambda_I(x \cdot z) \geqslant \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}), \tag{2.34}$$
$$(\forall x, y, z \in X)(\lambda_F(x \cdot z) \leqslant \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}). \tag{2.35}$$

Example 2.19. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	0	4
3	0	0	2	0	4
4	0	0	0	0	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.3 & 0.5 & 0.4 & 0.6 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.7 & 0.4 & 0.7 & 0.3 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.2 & 0.7 & 0.3 & 0.9 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic UP-ideal of X.

Definition 2.20. A NS Λ in X is called an *special neutrosophic strong UP-ideal* of X if it satisfies the following conditions: (2.24), (2.25), (2.26), and

$$(\forall x, y, z \in X)(\lambda_T(x) \leqslant \max\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\}), \tag{2.36}$$
$$(\forall x, y, z \in X)(\lambda_I(x) \geqslant \min\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\}), \tag{2.37}$$
$$(\forall x, y, z \in X)(\lambda_F(x) \leqslant \max\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\}). \tag{2.38}$$

Example 2.21. Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	0
2	0	1	0	3	4
3	0	1	0	0	4
4	0	1	0	3	0

We define a NS Λ in X as follows:

$$(\forall x \in X)\begin{pmatrix} \lambda_T(x) = 0.5 \\ \lambda_I(x) = 0.4 \\ \lambda_F(x) = 0.7 \end{pmatrix}.$$

Hence, Λ is a special neutrosophic strong UP-ideal X.

Theorem 2.22. *Every special neutrosophic UP-subalgebra of* X *satisfies the conditions* (2.24), (2.25), *and* (2.26).

Proof. Assume that Λ is a special neutrosophic UP-subalgebra of X. Then for all $x \in X$,

$$\begin{aligned} \lambda_T(0) &= \lambda_T(x \cdot x) \leqslant \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x) \text{ by (1.1) and (2.21)} \\ \lambda_I(0) &= \lambda_I(x \cdot x) \geqslant \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x) \text{ by (1.1) and (2.22)} \\ \lambda_F(0) &= \lambda_F(x \cdot x) \leqslant \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x) \text{ by (1.1) and (2.23).} \end{aligned}$$

Hence, Λ satisfies the conditions (2.24), (2.25), and (2.26). □

By Lemma 2.4 (1) and (4), we have the following five theorems.

Theorem 2.23. *A NS* Λ *in* X *is a neutrosophic UP-subalgebra of* X *if and only if* $\overline{\Lambda}$ *is a special neutrosophic UP-subalgebra of* X.

Theorem 2.24. *A NS* Λ *in* X *is a neutrosophic near UP-filter of* X *if and only if* $\overline{\Lambda}$ *is a special neutrosophic near UP-filter of* X.

Theorem 2.25. *A NS* Λ *in* X *is a neutrosophic UP-filter of* X *if and only if* $\overline{\Lambda}$ *is a special neutrosophic UP-filter of* X.

Theorem 2.26. *A NS* Λ *in* X *is a neutrosophic UP-ideal of* X *if and only if* $\overline{\Lambda}$ *is a special neutrosophic UP-ideal of* X.

Theorem 2.27. *A NS* Λ *in* X *is a neutrosophic strong UP-ideal of* X *if and only if* $\overline{\Lambda}$ *is a special neutrosophic strong UP-ideal of* X.

Theorem 2.28. *A NS* Λ *in* X *is constant if and only if it is a special neutrosophic strong UP-ideal of* X.

Proof. It is straightforward by Remark 2.2 and Theorems 2.11 and 2.27. □

By Remark 2.2 and Theorems 2.23, 2.24, 2.25, 2.26, and 2.27, we have that the notion of special neutrosophic UP-subalgebras is a generalization of special neutrosophic near UP-filters, special neutrosophic near UP-filters is a generalization of special neutrosophic UP-filters, special neutrosophic UP-filters is a generalization of special neutrosophic UP-ideals, and special neutrosophic UP-ideals is a generalization of special neutrosophic strong UP-ideals. Moreover, by Theorem 2.28, we obtain that special neutrosophic strong UP-ideals and constant neutrosophic set coincide.

Theorem 2.29. *If* Λ *is a special neutrosophic UP-subalgebra of* X *satisfying the following condition:*

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} \lambda_T(x) \leqslant \lambda_T(y) \\ \lambda_I(x) \geqslant \lambda_I(y) \\ \lambda_F(x) \leqslant \lambda_F(y) \end{cases} \right), \tag{2.39}$$

then Λ *is a special neutrosophic near UP-filter of* X.

Proof. Assume that Λ is a special neutrosophic UP-subalgebra of X satisfying the condition (2.39). By Theorem 2.22, we have Λ satisfies the conditions (2.24), (2.25), and (2.26). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$\begin{aligned} \lambda_T(x \cdot y) &= \lambda_T(0) \leqslant \lambda_T(y) \text{ by (2.24)} \\ \lambda_I(x \cdot y) &= \lambda_I(0) \geqslant \lambda_I(y) \text{ by (2.25)} \\ \lambda_F(x \cdot y) &= \lambda_F(0) \leqslant \lambda_F(y) \text{ by (2.26).} \end{aligned}$$

Case 2: $x \cdot y \neq 0$. Then

$$\begin{aligned}\lambda_T(x \cdot y) &\leqslant \max\{\lambda_T(x), \lambda_T(y)\} = \lambda_T(y), \quad (2.21) \text{ and } (2.39) \text{ for } \lambda_T,\\ \lambda_I(x \cdot y) &\geqslant \min\{\lambda_I(x), \lambda_I(y)\} = \lambda_I(y), \quad (2.22) \text{ and } (2.39) \text{ for } \lambda_I,\\ \lambda_F(x \cdot y) &\leqslant \max\{\lambda_F(x), \lambda_F(y)\} = \lambda_F(y). \quad (2.23) \text{ and } (2.39) \text{ for } \lambda_F.\end{aligned}$$

Hence, Λ is a special neutrosophic near UP-filter of X. □

Theorem 2.30. *If Λ is a special neutrosophic near UP-filter of X satisfying the following condition:*

$$\lambda_T = \lambda_I = \lambda_F, \tag{2.40}$$

then Λ is a special neutrosophic UP-filter of X.

Proof. Assume that Λ is a special neutrosophic near UP-filter of X satisfying the condition (2.40). Then Λ satisfies the conditions (2.24), (2.25), and (2.26). Next, let $x, y, z \in X$. Then

$$\begin{aligned}\max\{\lambda_T(x \cdot y), \lambda_T(x)\} &= \max\{\lambda_I(x \cdot y), \lambda_T(x)\} \quad \text{by } (2.40)\\ &\geqslant \max\{\lambda_I(y), \lambda_T(x)\} \quad \text{by } (2.28)\\ &= \max\{\lambda_T(y), \lambda_T(x)\} \quad \text{by } (2.40)\\ &\geqslant \lambda_T(y),\\ \min\{\lambda_I(x \cdot y), \lambda_I(x)\} &= \min\{\lambda_T(x \cdot y), \lambda_I(x)\} \quad \text{by } (2.40)\\ &\leqslant \min\{\lambda_T(y), \lambda_I(x)\} \quad \text{by } (2.27)\\ &= \min\{\lambda_I(y), \lambda_I(x)\} \quad \text{by } (2.40)\\ &\leqslant \lambda_I(y),\\ \max\{\lambda_F(x \cdot y), \lambda_F(x)\} &= \max\{\lambda_I(x \cdot y), \lambda_F(x)\} \quad \text{by } (2.40)\\ &\geqslant \max\{\lambda_I(y), \lambda_F(x)\} \quad \text{by } (2.28)\\ &= \max\{\lambda_F(y), \lambda_F(x)\} \quad \text{by } (2.40)\\ &\geqslant \lambda_F(y).\end{aligned}$$

Hence, Λ is a special neutrosophic UP-filter of X. □

Theorem 2.31. *If Λ is a special neutrosophic UP-filter of X satisfying the following condition:*

$$(\forall x, y, z \in X) \begin{pmatrix} \lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z)) \\ \lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z)) \\ \lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z)) \end{pmatrix}, \tag{2.41}$$

then Λ is a special neutrosophic UP-ideal of X.

Proof. Assume that Λ is a special neutrosophic UP-filter of X satisfying the condition (2.41). Then Λ satisfies the conditions (2.24), (2.25), and (2.26). Next, let $x, y, z \in X$. Then

$$\begin{aligned}\lambda_T(x \cdot z) &\leqslant \max\{\lambda_T(y \cdot (x \cdot z)), \lambda_T(y)\} \quad \text{by } (2.30)\\ &= \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\} \quad \text{by } (2.41) \text{ for } \lambda_T\\ \lambda_I(x \cdot z) &\geqslant \min\{\lambda_I(y \cdot (x \cdot z)), \lambda_I(y)\} \quad \text{by } (2.31)\\ &= \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\} \quad \text{by } (2.41) \text{ for } \lambda_I\\ \lambda_F(x \cdot z) &\leqslant \max\{\lambda_F(y \cdot (x \cdot z)), \lambda_F(y)\} \quad \text{by } (2.32)\\ &= \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\} \quad \text{by } (2.41) \text{ for } \lambda_F.\end{aligned}$$

Hence, Λ is a special neutrosophic UP-ideal of X. □

Theorem 2.32. *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leqslant x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \leqslant \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(z) \geqslant \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(z) \leqslant \max\{\lambda_F(x), \lambda_F(y)\} \end{cases}\right), \tag{2.42}$$

then Λ is a special neutrosophic UP-subalgebra of X.

Proof. Assume that Λ is a NS in X satisfying the condition (2.42). Let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \geqslant x \cdot y$. It follows from (2.42) that

$$\lambda_T(x \cdot y) \leqslant \max\{\lambda_T(x), \lambda_T(y)\}, \quad \lambda_I(x \cdot y) \geqslant \min\{\lambda_I(x), \lambda_I(y)\}, \quad \lambda_F(x \cdot y) \leqslant \max\{\lambda_F(x), \lambda_F(y)\}.$$

Hence, Λ is a special neutrosophic UP-subalgebra of X. □

Theorem 2.33. *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leqslant x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \leqslant \lambda_T(y) \\ \lambda_I(z) \geqslant \lambda_I(y) \\ \lambda_F(z) \leqslant \lambda_F(y) \end{cases}\right), \tag{2.43}$$

then Λ is a special neutrosophic near UP-filter of X.

Proof. Assume that Λ is a NS in X satisfying the condition (2.43). Let $x \in X$. By (UP-2) and (1.1), we have $0 \cdot (x \cdot x) = 0$, that is, $0 \leqslant x \cdot x$. It follows from (2.43) that $\lambda_T(0) \leqslant \lambda_T(x), \lambda_I(0) \geqslant \lambda_I(x)$, and $\lambda_F(0) \leqslant \lambda_F(x)$. Next, let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \geqslant x \cdot y$. It follows from (2.43) that $\lambda_T(x \cdot y) \leqslant \lambda_T(y), \lambda_I(x \cdot y) \geqslant \lambda_I(y)$, and $\lambda_F(x \cdot y) \leqslant \lambda_F(y)$. Hence, Λ is a special neutrosophic near UP-filter of X. □

Theorem 2.34. *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leqslant x \cdot y \Rightarrow \begin{cases} \lambda_T(y) \leqslant \max\{\lambda_T(z), \lambda_T(x)\} \\ \lambda_I(y) \geqslant \min\{\lambda_I(z), \lambda_I(x)\} \\ \lambda_F(y) \leqslant \max\{\lambda_F(z), \lambda_F(x)\} \end{cases}\right), \tag{2.44}$$

then Λ is a special neutrosophic UP-filter of X.

Proof. Assume that Λ is a NS in X satisfying the condition (2.44). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leqslant x \cdot 0$. It follows from (2.44) that

$$\lambda_T(0) \leqslant \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \ \lambda_I(0) \geqslant \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \ \lambda_F(0) \leqslant \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x).$$

Next, let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \geqslant x \cdot y$. It follows from (2.44) that

$$\lambda_T(y) \leqslant \max\{\lambda_T(x \cdot y), \lambda_T(x)\}, \quad \lambda_I(y) \geqslant \min\{\lambda_I(x \cdot y), \lambda_I(x)\}, \quad \lambda_F(y) \leqslant \max\{\lambda_F(x \cdot y), \lambda_F(x)\}.$$

Hence, Λ is a special neutrosophic UP-filter of X. □

Theorem 2.35. *If Λ is a NS in X satisfying the following condition:*

$$(\forall a, x, y, z \in X)\left(a \leqslant x \cdot (y \cdot z) \Rightarrow \begin{cases} \lambda_T(x \cdot z) \leqslant \max\{\lambda_T(a), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \geqslant \min\{\lambda_I(a), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \leqslant \max\{\lambda_F(a), \lambda_F(y)\} \end{cases}\right), \tag{2.45}$$

then Λ is a special neutrosophic UP-ideal of X.

Proof. Assume that Λ is a NS in X satisfying the condition (2.45). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leqslant 0 \cdot (x \cdot 0)$. It follows from (2.45) that

$$\lambda_T(0) = \lambda_T(0 \cdot 0) \leqslant \max\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \quad \text{(UP-2)}$$
$$\lambda_I(0) = \lambda_I(0 \cdot 0) \geqslant \min\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \quad \text{(UP-2)}$$
$$\lambda_F(0) = \lambda_F(0 \cdot 0) \leqslant \max\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \quad \text{(UP-2)}$$

Next, let $x, y, z \in X$. By (1.1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \geqslant x \cdot (y \cdot z)$. It follows from (2.45) that

$$\lambda_T(x \cdot z) \leqslant \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\},$$
$$\lambda_I(x \cdot z) \geqslant \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\},$$
$$\lambda_F(x \cdot z) \leqslant \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.$$

Hence, Λ is a special neutrosophic UP-ideal of X. □

For any fixed numbers $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0,1]$ such that $\alpha^+ < \alpha^-, \beta^+ < \beta^-, \gamma^+ < \gamma^-$ and a nonempty subset G of X, a NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}] = (X, \lambda_T^G[^{\alpha^+}_{\alpha^-}], \lambda_I^G[^{\beta^-}_{\beta^+}], \lambda_F^G[^{\gamma^+}_{\gamma^-}])$ in X, where $\lambda_T^G[^{\alpha^+}_{\alpha^-}], \lambda_I^G[^{\beta^-}_{\beta^+}]$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}]$ are functions on X which are given as follows:

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x) = \begin{cases} \alpha^+, & \text{if } x \in G, \\ \alpha^-, & \text{otherwise,} \end{cases} \quad \lambda_I^G[^{\beta^-}_{\beta^+}](x) = \begin{cases} \beta^-, & \text{if } x \in G, \\ \beta^+, & \text{otherwise,} \end{cases} \quad \lambda_F^G[^{\gamma^+}_{\gamma^-}](x) = \begin{cases} \gamma^+, & \text{if } x \in G, \\ \gamma^-, & \text{otherwise.} \end{cases}$$

A NS ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}] = (X, {}^G\lambda_T[^{\alpha^-}_{\alpha^+}], {}^G\lambda_I[^{\beta^+}_{\beta^-}], {}^G\lambda_F[^{\gamma^-}_{\gamma^+}])$ in X, where ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}]$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}]$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}]$ are functions on X which are given as follows:

$${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \begin{cases} \alpha^-, & \text{if } x \in G, \\ \alpha^+, & \text{otherwise,} \end{cases} \quad {}^G\lambda_I[^{\beta^+}_{\beta^-}](x) = \begin{cases} \beta^+, & \text{if } x \in G, \\ \beta^-, & \text{otherwise,} \end{cases} \quad {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \begin{cases} \gamma^-, & \text{if } x \in G, \\ \gamma^+, & \text{otherwise.} \end{cases}$$

Lemma 2.36. *Let $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0,1]$. Then the following statements hold:*

(1) $\overline{\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]} = {}^G\Lambda[^{1-\alpha^+,1-\beta^-,1-\gamma^+}_{1-\alpha^-,1-\beta^+,1-\gamma^-}]$, *and*

(2) $\overline{{}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]} = \Lambda^G[^{1-\alpha^-,1-\beta^+,1-\gamma^-}_{1-\alpha^+,1-\beta^-,1-\gamma^+}]$.

Proof.

(1). Let $\overline{\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]}$ be a NS in X. Then $\overline{\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]} = (X, \overline{\lambda_T^G[^{\alpha^+}_{\alpha^-}]}, \overline{\lambda_I^G[^{\beta^-}_{\beta^+}]}, \overline{\lambda_F^G[^{\gamma^+}_{\gamma^-}]})$. Since

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x) = \begin{cases} \alpha^+, & \text{if } x \in G, \\ \alpha^-, & \text{otherwise,} \end{cases} \quad \lambda_I^G[^{\beta^-}_{\beta^+}](x) = \begin{cases} \beta^-, & \text{if } x \in G, \\ \beta^+, & \text{otherwise,} \end{cases} \quad \lambda_F^G[^{\gamma^+}_{\gamma^-}](x) = \begin{cases} \gamma^+, & \text{if } x \in G, \\ \gamma^-, & \text{otherwise.} \end{cases}$$

Thus

$$\overline{\lambda_T^G[^{\alpha^+}_{\alpha^-}]}(x) = \begin{cases} 1-\alpha^+, & \text{if } x \in G, \\ 1-\alpha^-, & \text{otherwise} \end{cases} = {}^G\lambda_T[^{1-\alpha^+}_{1-\alpha^-}](x),$$
$$\overline{\lambda_I^G[^{\beta^-}_{\beta^+}]}(x) = \begin{cases} 1-\beta^-, & \text{if } x \in G, \\ 1-\beta^+, & \text{otherwise} \end{cases} = {}^G\lambda_I[^{1-\beta^-}_{1-\beta^+}](x),$$
$$\overline{\lambda_F^G[^{\gamma^+}_{\gamma^-}]}(x) = \begin{cases} 1-\gamma^+, & \text{if } x \in G, \\ 1-\gamma^-, & \text{otherwise} \end{cases} = {}^G\lambda_F[^{1-\gamma^+}_{1-\gamma^-}](x).$$

Hence, $(X, {}^G\lambda_T[^{1-\alpha^+}_{1-\alpha^-}], {}^G\lambda_I[^{1-\beta^-}_{1-\beta^+}], {}^G\lambda_F[^{1-\gamma^+}_{1-\gamma^-}]) = {}^G\Lambda[^{1-\alpha^+,1-\beta^-,1-\gamma^+}_{1-\alpha^-,1-\beta^+,1-\gamma^-}]$.

(2). Let $\overline{{}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]}$ be a NS in X. Then $\overline{{}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]} = (X, \overline{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]}, \overline{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}]}, \overline{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}]})$. Since

$$ {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \begin{cases} \alpha^-, & \text{if } x \in G, \\ \alpha^+, & \text{otherwise,} \end{cases} \quad {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) = \begin{cases} \beta^+, & \text{if } x \in G, \\ \beta^-, & \text{otherwise,} \end{cases} \quad {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \begin{cases} \gamma^-, & \text{if } x \in G, \\ \gamma^+, & \text{otherwise.} \end{cases} $$

Thus

$$ \overline{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]}(x) = \begin{cases} 1-\alpha^-, & \text{if } x \in G, \\ 1-\alpha^+, & \text{otherwise} \end{cases} = \lambda_T^G[^{1-\alpha^-}_{1-\alpha^+}](x), $$
$$ \overline{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}]}(x) = \begin{cases} 1-\beta^+, & \text{if } x \in G, \\ 1-\beta^-, & \text{otherwise} \end{cases} = \lambda_I^G[^{1-\beta^+}_{1-\beta^-}](x), $$
$$ \overline{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}]}(x) = \begin{cases} 1-\gamma^-, & \text{if } x \in G, \\ 1-\gamma^+, & \text{otherwise} \end{cases} = \lambda_F^G[^{1-\gamma^-}_{1-\gamma^+}](x). $$

Hence, $(X, \lambda_T^G[^{1-\alpha^-}_{1-\alpha^+}], \lambda_I^G[^{1-\beta^+}_{1-\beta^-}], \lambda_F^G[^{1-\gamma^-}_{1-\gamma^+}]) = \Lambda^G[^{1-\alpha^-,1-\beta^+,1-\gamma^-}_{1-\alpha^+,1-\beta^-,1-\gamma^+}]$. □

Lemma 2.37. *If the constant* 0 *of* X *is in a nonempty subset* G *of* X*, then a NS* ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *satisfies the conditions* (2.24)*,* (2.25)*, and* (2.26)*.*

Proof. If $0 \in G$, then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) = \alpha^-$, ${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](0) = \beta^+$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](0) = \gamma^-$. Thus

$$ (\forall x \in X) \begin{pmatrix} {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) = \alpha^- \leqslant {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) \\ {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](0) = \beta^- \geqslant {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) \\ {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](0) = \gamma^- \leqslant {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) \end{pmatrix}. $$

Hence, ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (2.24), (2.25), and (2.26). □

Lemma 2.38. *If a NS* ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *satisfies the condition* (2.24) *(resp.,* (2.25)*,* (2.26)*), then the constant* 0 *of* X *is in a nonempty subset* G *of* X*.*

Proof. Assume that a NS ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X satisfies the condition (2.24). Then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) \leqslant {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x)$ for all $x \in X$. Since G is nonempty, there exists $g \in G$. Thus ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](g) = \alpha^-$, so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) \leqslant {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](g) = \alpha^-$, that is, ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](0) = \alpha^-$. Hence, $0 \in G$. □

Theorem 2.39. *A NS* ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ *in* X *is a special neutrosophic UP-subalgebra of* X *if and only if a nonempty subset* G *of* X *is a UP-subalgebra of* X*.*

Proof. Assume that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-subalgebra of X. Let $x, y \in G$. Then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^- = {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)$. Thus

$$ {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) \leqslant \max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} = \alpha^- \leqslant {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) \text{ by (2.21)} $$

and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^-$. Thus $x \cdot y \in G$. Hence, G is a UP-subalgebra of X.

Conversely, assume that G is a UP-subalgebra of X. Let $x, y \in X$.

Case 1: $x, y \in G$. Then

$$^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^- = {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) = \beta^+ = {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y), \quad {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \gamma^- = {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y).$$

Thus

$$\max\{^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} = \alpha^-,$$
$$\min\{^{G}\lambda_I[^{\beta^+}_{\beta^-}](x), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\} = \beta^+,$$
$$\max\{^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\} = \gamma^-.$$

Since G is a UP-subalgebra of X, we have $x \cdot y \in G$ and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^-$, ${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) = \beta^+$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) = \gamma^-$. Hence,

$$^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^- \leqslant \alpha^- = \max\{^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\},$$
$$^{G}\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) = \beta^+ \geqslant \beta^+ = \min\{^{G}\lambda_I[^{\beta^+}_{\beta^-}](x), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\},$$
$$^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) = \gamma^- \leqslant \gamma^- = \max\{^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\}.$$

Case 2: $x \notin G$ or $y \notin G$. Then

$$^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](x) = \alpha^- \text{ or } {}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](y) = \alpha^-,$$
$$^{G}\lambda_I[^{\beta^-}_{\beta^+}](x) = \beta^+ \text{ or } {}^{G}\lambda_I[^{\beta^-}_{\beta^+}](y) = \beta^+,$$
$$^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](x) = \gamma^- \text{ or } {}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](y) = \gamma^-.$$

Thus

$$\max\{^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](x), {}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^-,$$
$$\min\{^{G}\lambda_I[^{\beta^-}_{\beta^+}](x), {}^{G}\lambda_I[^{\beta^-}_{\beta^+}](y)\} = \beta^+,$$
$$\max\{^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](x), {}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^-.$$

Therefore,

$$^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](x \cdot y) \geqslant \alpha^- = \max\{^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](x), {}^{G}\lambda_T[^{\alpha^+}_{\alpha^-}](y)\},$$
$$^{G}\lambda_I[^{\beta^-}_{\beta^+}](x \cdot y) \leqslant \beta^+ = \min\{^{G}\lambda_I[^{\beta^-}_{\beta^+}](x), {}^{G}\lambda_I[^{\beta^-}_{\beta^+}](y)\},$$
$$^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](x \cdot y) \geqslant \gamma^- = \max\{^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](x), {}^{G}\lambda_F[^{\gamma^+}_{\gamma^-}](y)\}.$$

Hence, ${}^{G}\Lambda[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a special neutrosophic UP-subalgebra of X. □

Theorem 2.40. *A NS ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X is a special neutrosophic near UP-filter of X if and only if a nonempty subset G of X is a near UP-filter of X.*

Proof. Assume that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic near UP-filter of X. Since ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (2.24), it follows from Lemma 2.38 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^-$. Thus

$$^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) \leqslant {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^- \leqslant {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) \text{ by (2.27)}$$

and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^-$. Thus $x \cdot y \in G$. Hence, G is a near UP-filter of X.

Conversely, assume that G is a near UP-filter of X. Since $0 \in G$, it follows from Lemma 2.37 that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (2.24), (2.25), and (2.26). Next, let $x, y \in X$.

Case 1: $y \in G$. Then ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^-$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}](y) = \beta^+$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y) = \gamma^-$. Since G is a near UP-filter of X, we have $x \cdot y \in G$ and so ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^-$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) = \beta^+$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) = \gamma^-$. Thus

$$\begin{aligned} {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) &= \alpha^- \leqslant \alpha^- = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y), \\ {}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) &= \beta^+ \geqslant \beta^+ = {}^G\lambda_I[^{\beta^+}_{\beta^-}](y), \\ {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) &= \gamma^- \leqslant \gamma^- = {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y). \end{aligned}$$

Case 2: $y \notin G$. Then ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^+$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}](y) = \beta^-$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y) = \gamma^+$. Thus

$$\begin{aligned} {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) &\leqslant \alpha^+ = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y), \\ {}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) &\geqslant \beta^- = {}^G\lambda_I[^{\beta^+}_{\beta^-}](y), \\ {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) &\leqslant \gamma^+ = {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y). \end{aligned}$$

Hence, ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic near UP-filter of X. □

Theorem 2.41. *A NS ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X is a special neutrosophic UP-filter of X if and only if a nonempty subset G of X is a UP-filter of X.*

Proof. Assume that ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-filter of X. Since ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (2.24), it follows from Lemma 2.38 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) = \alpha^- = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x)$. Thus

$${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) \leqslant \max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x)\} = \alpha^- \leqslant {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) \text{ by (2.30)}$$

and so ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^-$. Thus $y \in G$. Hence, G is a UP-filter of X.

Conversely, assume that G is a UP-filter of X. Since $0 \in G$, it follows from Lemma 2.37 that ${}^G\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (2.24), (2.25), and (2.26). Next, let $x, y \in X$.

Case 1: $x \cdot y \in G$ and $x \in G$. Then

$$\begin{aligned} {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) &= \alpha^- = {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x), \\ {}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) &= \beta^+ = {}^G\lambda_I[^{\beta^+}_{\beta^-}](x), \\ {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) &= \gamma^- = {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x). \end{aligned}$$

Since G is a UP-filter of X, we have $y \in G$ and so ${}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^-$, ${}^G\lambda_I[^{\beta^+}_{\beta^-}](y) = \beta^+$, and ${}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y) = \gamma^-$. Thus

$$\begin{aligned} {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](y) &= \alpha^- \leqslant \alpha^- = \max\{{}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y), {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x)\}, \\ {}^G\lambda_I[^{\beta^+}_{\beta^-}](y) &= \beta^+ \geqslant \beta^+ = \min\{{}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y), {}^G\lambda_I[^{\beta^+}_{\beta^-}](x)\}, \\ {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](y) &= \gamma^- \leqslant \gamma^+ = \max\{{}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y), {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x)\}. \end{aligned}$$

Case 2: $x \cdot y \notin G$ or $x \notin G$. Then

$$\begin{aligned} {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot y) &= \alpha^+ \text{ or } {}^G\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^+, \\ {}^G\lambda_I[^{\beta^+}_{\beta^-}](x \cdot y) &= \beta^- \text{ or } {}^G\lambda_I[^{\beta^+}_{\beta^-}](x) = \beta^-, \\ {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot y) &= \gamma^+ \text{ or } {}^G\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \gamma^+. \end{aligned}$$

Thus

$$\max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot y), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x)\} = \alpha^+,$$
$$\min\{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot y), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x)\} = \beta^-,$$
$$\max\{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot y), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x)\} = \gamma^+.$$

Therefore,

$${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) \leqslant \alpha^+ = \max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot y), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x)\},$$
$${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) \geqslant \beta^- = \min\{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot y), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x)\},$$
$${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) \leqslant \gamma^+ = \max\{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot y), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x)\}.$$

Hence, ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-filter of X. □

Theorem 2.42. *A NS ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X is a special neutrosophic UP-ideal of X if and only if a nonempty subset G of X is a UP-ideal of X.*

Proof. Assume that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-ideal of X. Since ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the condition (2.24), it follows from Lemma 2.38 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x\cdot(y\cdot z) \in G$ and $y \in G$. Then ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)) = \alpha^- = {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)$. Thus

$${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot z) \leqslant \max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} = \alpha^- \leqslant {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot z) \text{ by (2.33)}$$

and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot z) = \alpha^-$. Thus $x\cdot z \in G$. Hence, G is a UP-ideal of X.

Conversely, assume that G is a UP-ideal of X. Since $0 \in G$, it follows from Lemma 2.37 that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ satisfies the conditions (2.24), (2.25), and (2.26). Next, let $x, y, z \in X$.

Case 1: $x\cdot(y\cdot z) \in G$ and $y \in G$. Then

$${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)) = \alpha^- = {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y),$$
$${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)) = \beta^+ = {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y),$$
$${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)) = \gamma- = {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y).$$

Thus

$$\max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} = \alpha^-,$$
$$\min\{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\} = \beta^+,$$
$$\max\{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\} = \gamma^-.$$

Since G is a UP-ideal of X, we have $x\cdot z \in G$ and so ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot z) = \alpha^-$, ${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot z) = \beta^+$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot z) = \gamma^-$. Thus

$${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot z) = \alpha^- \leqslant \alpha^- = \max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x\cdot(y\cdot z)), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\},$$
$${}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot z) = \beta^+ \geqslant \beta^+ = \min\{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x\cdot(y\cdot z)), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\},$$
$${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot z) = \gamma^- \leqslant \gamma^- = \max\{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x\cdot(y\cdot z)), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\}.$$

Case 2: $x \cdot (y \cdot z) \notin G$ or $y \notin G$. Then

$$\begin{aligned}
{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot (y \cdot z)) = \alpha^+ \text{ or } {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y) = \alpha^+,\\
{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x \cdot (y \cdot z)) = \beta^- \text{ or } {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y) = \beta^-,\\
{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot (y \cdot z)) = \gamma^+ \text{ or } {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y) = \gamma^+.
\end{aligned}$$

Thus

$$\begin{aligned}
\max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot (y \cdot z)), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\} = \alpha^+,\\
\min\{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x \cdot (y \cdot z)), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\} = \beta^-,\\
\max\{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot (y \cdot z)), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\} = \gamma^+.
\end{aligned}$$

Therefore,

$$\begin{aligned}
{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot z) \leqslant \alpha^+ = \max\{{}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x \cdot (y \cdot z)), {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](y)\},\\
{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x \cdot z) \geqslant \beta^- = \min\{{}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x \cdot (y \cdot z)), {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](y)\},\\
{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot z) \leqslant \gamma^+ = \max\{{}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x \cdot (y \cdot z)), {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](y)\}.
\end{aligned}$$

Hence, ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic UP-ideal of X. □

Theorem 2.43. *A NS ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ in X is a special neutrosophic strong UP-ideal of X if and only if a nonempty subset G of X is a strong UP-ideal of X.*

Proof. Assume that ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic strong UP-ideal of X. By Theorem 2.28, we have ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]$ is constant, that is, ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]$ is constant. Since G is nonempty, we have ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^-$ for all $x \in X$. Thus $G = X$. Hence, G is a strong UP-ideal of X.

Conversely, assume that G is a strong UP-ideal of X. Then $G = X$, so

$$(\forall x \in X)\begin{pmatrix} {}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}](x) = \alpha^- \\ {}^{G}\lambda_I[^{\beta^+}_{\beta^-}](x) = \beta^+ \\ {}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}](x) = \gamma^- \end{pmatrix}.$$

Thus ${}^{G}\lambda_T[^{\alpha^-}_{\alpha^+}]$, ${}^{G}\lambda_I[^{\beta^+}_{\beta^-}]$, and ${}^{G}\lambda_F[^{\gamma^-}_{\gamma^+}]$ are constant, that is, ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is constant. By Theorem 2.28, we have ${}^{G}\Lambda[^{\alpha^-,\beta^+,\gamma^-}_{\alpha^+,\beta^-,\gamma^+}]$ is a special neutrosophic strong UP-ideal of X. □

3. Level subsets of a NS of special types

In this paper, we discuss the relationships among special neutrosophic UP-subalgebras (resp., special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, special neutrosophic strong UP-ideals) of UP-algebras and their level subsets.

Definition 3.1 ([21])**.** Let f be a fuzzy set in A. For any $t \in [0, 1]$, the sets

$$U(f; t) = \{x \in X \mid f(x) \geqslant t\}, \qquad L(f; t) = \{x \in X \mid f(x) \leqslant t\}, \qquad E(f; t) = \{x \in X \mid f(x) = t\}$$

are called an *upper t-level subset*, a *lower t-level subset*, and an *equal t-level subset* of f, respectively.

Theorem 3.2. *A NS Λ in X is a special neutrosophic UP-subalgebra of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $L(\lambda_T; \alpha)$, $U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-subalgebras of X if $L(\lambda_T; \alpha)$, $U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.*

Proof. Assume that Λ is a special neutrosophic UP-subalgebra of X. Let $\alpha, \beta, \gamma \in [0, 1]$ be such that $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x, y \in L(\lambda_T; \alpha)$. Then $\lambda_T(x) \leqslant \alpha$ and $\lambda_T(y) \leqslant \alpha$, so α is a upper bound of $\{\lambda_T(x), \lambda_T(y)\}$. By (2.21), we have $\lambda_T(x \cdot y) \leqslant \max\{\lambda_T(x), \lambda_T(y)\} \leqslant \alpha$. Thus $x \cdot y \in L(\lambda_T; \alpha)$.

Let $x, y \in U(\lambda_I; \beta)$. Then $\lambda_I(x) \geqslant \beta$ and $\lambda_I(y) \geqslant \beta$, so β is an lower bound of $\{\lambda_I(x), \lambda_I(y)\}$. By (2.22), we have $\lambda_I(x \cdot y) \geqslant \min\{\lambda_I(x), \lambda_I(y)\} \geqslant \beta$. Thus $x \cdot y \in U(\lambda_I; \beta)$.

Let $x, y \in L(\lambda_F; \gamma)$. Then $\lambda_F(x) \leqslant \gamma$ and $\lambda_F(y) \leqslant \gamma$, so γ is a upper bound of $\{\lambda_F(x), \lambda_F(y)\}$. By (2.23), we have $\lambda_F(x \cdot y) \leqslant \max\{\lambda_F(x), \lambda_F(y)\} \leqslant \gamma$. Thus $x \cdot y \in L(\lambda_F; \gamma)$.

Hence, $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-subalgebras of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the set $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are UP-subalgebras if $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x, y \in X$. Then $\lambda_T(x), \lambda_T(y) \in [0, 1]$. Choose $\alpha = \max\{\lambda_T(x), \lambda_T(y)\}$. Thus $\lambda_T(x) \leqslant \alpha$ and $\lambda_T(y) \leqslant \alpha$, so $x, y \in L(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T; \alpha)$ is a UP-subalgebra of X and so $x, y \in L(\lambda_T; \alpha)$. Thus $\lambda_T(x \cdot y) \leqslant \alpha = \max\{\lambda_T(x), \lambda_T(y)\}$.

Let $x, y \in X$. Then $\lambda_I(x), \lambda_I(y) \in [0, 1]$. Choose $\beta = \min\{\lambda_I(x), \lambda_I(y)\}$. Thus $\lambda_I(x) \geqslant \beta$ and $\lambda_I(y) \geqslant \beta$, so $x, y \in U(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $U(\lambda_I; \beta)$ is a UP-subalgebra of X and so $x, y \in U(\lambda_I; \beta)$. Thus $\lambda_I(x \cdot y) \geqslant \beta = \min\{\lambda_I(x), \lambda_I(y)\}$.

Let $x, y \in X$. Then $\lambda_F(x), \lambda_F(y) \in [0, 1]$. Choose $\gamma = \max\{\lambda_F(x), \lambda_F(y)\}$. Thus $\lambda_F(x) \leqslant \gamma$ and $\lambda_F(y) \leqslant \gamma$, so $x, y \in L(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F; \gamma)$ is a UP-subalgebra of X and so $x, y \in L(\lambda_F; \gamma)$. Thus $\lambda_F(x \cdot y) \leqslant \gamma = \max\{\lambda_F(x), \lambda_F(y)\}$.

Therefore, Λ is a special neutrosophic UP-subalgebra of X. □

Theorem 3.3. *A NS Λ in X is a special neutrosophic near UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are near UP-filters of X if $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.*

Proof. Assume that Λ is a special neutrosophic near UP-filter of X. Let $\alpha, \beta, \gamma \in [0, 1]$ be such that $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x \in L(\lambda_T; \alpha)$. Then $\lambda_T(x) \leqslant \alpha$. By (2.24), we have $\lambda_T(0) \leqslant \lambda_T(x) \leqslant \alpha$. Thus $0 \in L(\lambda_T; \alpha)$. Next, let $y \in L(\lambda_T; \alpha)$. Then $\lambda_T(y) \leqslant \alpha$. By (2.27), we have $\lambda_T(x \cdot y) \leqslant \lambda_T(y) \leqslant \alpha$. Thus $x \cdot y \in L(\lambda_T; \alpha)$.

Let $x \in U(\lambda_I; \beta)$. Then $\lambda_I(x) \geqslant \beta$. By (2.25), we have $\lambda_I(0) \geqslant \lambda_I(x) \geqslant \beta$. Thus $0 \in U(\lambda_I; \beta)$. Next, let $y \in U(\lambda_I; \beta)$. Then $\lambda_I(y) \geqslant \beta$. By (2.28), we have $\lambda_I(x \cdot y) \geqslant \lambda_I(y) \geqslant \beta$. Thus $x \cdot y \in U(\lambda_I; \beta)$.

Let $x \in L(\lambda_F; \gamma)$. Then $\lambda_F(x) \leqslant \gamma$. By (2.26), we have $\lambda_F(0) \leqslant \lambda_F(x) \leqslant \gamma$. Thus $0 \in L(\lambda_F; \gamma)$. Next, $y \in L(\lambda_F; \gamma)$. Then $\lambda_F(y) \leqslant \gamma$. By (2.28), we have $\lambda_F(x \cdot y) \leqslant \lambda_F(y) \leqslant \gamma$. Thus $x \cdot y \in L(\lambda_F; \gamma)$.

Hence, $L(\lambda_T; \alpha)$, $U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are near UP-filters of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the set $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are near UP-filters if $L(\lambda_T; \alpha), U(\lambda_I; \beta)$, and $L(\lambda_F; \gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(0) \in [0, 1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \leqslant \alpha$, so $x \in L(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T; \alpha)$ is a near UP-filter of X and so $0 \in L(\lambda_T; \alpha)$. Thus $\lambda_T(0) \leqslant \alpha = \lambda_T(x)$. Next, let $y \in X$. Then $\lambda_T(y) \in [0, 1]$. Choose $\alpha = \lambda_T(y)$. Thus $\lambda_T(y) \leqslant \alpha$, so $y \in L(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T; \alpha)$ is a near UP-filter of X, and so $x \cdot y \in L(\lambda_T; \alpha)$. Thus $\lambda_T(x \cdot y) \leqslant \alpha = \lambda_T(y)$.

Let $x \in X$. Then $\lambda_I(0) \in [0, 1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \geqslant \beta$, so $x \in U(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $U(\lambda_I; \beta)$ is a near UP-filter of X and so $0 \in U(\lambda_I; \beta)$. Thus $\lambda_I(0) \geqslant \beta = \lambda_I(x)$. Next, let $y \in X$. Then $\lambda_I(y) \in [0, 1]$. Choose $\beta = \lambda_I(y)$. Thus $\lambda_I(y) \geqslant \beta$, so $y \in U(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $U(\lambda_I; \beta)$ is a near UP-filter of X, and so $x \cdot y \in U(\lambda_I; \beta)$. Thus $\lambda_I(x \cdot y) \geqslant \beta = \lambda_I(y)$.

Let $x \in X$. Then $\lambda_F(0) \in [0, 1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \leqslant \gamma$, so $x \in L(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F; \gamma)$ is a near UP-filter of X and so $0 \in L(\lambda_F; \gamma)$. Thus $\lambda_F(0) \leqslant \gamma = \lambda_F(x)$. Next, let $y \in X$. Then $\lambda_F(y) \in [0, 1]$. Choose $\gamma = \lambda_F(y)$. Thus $\lambda_F(y) \leqslant \gamma$, so $y \in L(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F; \gamma)$ is a near UP-filter of X, and so $x \cdot y \in L(\lambda_F; \gamma)$. Thus $\lambda_F(x \cdot y) \leqslant \gamma = \lambda_F(y)$.

Therefore, Λ is a special neutrosophic near UP-filter of X. □

Theorem 3.4. *A NS Λ in X is a special neutrosophic UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-filters of X if $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.*

Proof. Assume that Λ is a special neutrosophic UP-filter of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $L(\lambda_T;\alpha)$, $U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x \in L(\lambda_T;\alpha)$. Then $\lambda_T(x) \leqslant \alpha$. By (2.24), we have $\lambda_T(0) \leqslant \lambda_T(x) \leqslant \alpha$. Thus $0 \in L(\lambda_T;\alpha)$. Next, let $x \cdot y \in L(\lambda_T;\alpha)$ and $x \in L(\lambda_T;\alpha)$. Then $\lambda_T(x \cdot y) \leqslant \alpha$ and $\lambda_T(x) \leqslant \alpha$, so α is a upper bound of $\{\lambda_T(x \cdot y), \lambda_T(x)\}$. By (2.30), we have $\lambda_T(y) \leqslant \max\{\lambda_T(x \cdot y), \lambda_T(x)\} \leqslant \alpha$. Thus $y \in L(\lambda_T;\alpha)$.

Let $x \in U(\lambda_I;\beta)$. Then $\lambda_I(x) \geqslant \beta$. By (2.25), we have $\lambda_I(0) \geqslant \lambda_I(x) \geqslant \beta$. Thus $0 \in U(\lambda_I;\beta)$. Next, let $x \cdot y \in U(\lambda_I;\beta)$ and $x \in U(\lambda_I;\beta)$. Then $\lambda_I(x \cdot y) \geqslant \beta$ and $\lambda_I(x) \geqslant \beta$, so β is an lower bound of $\{\lambda_I(x \cdot y), \lambda_I(x)\}$. By (2.31), we have $\lambda_I(y) \geqslant \min\{\lambda_I(x \cdot y), \lambda_I(x)\} \geqslant \beta$. Thus $y \in U(\lambda_I;\beta)$.

Let $x \in L(\lambda_F;\gamma)$. Then $\lambda_F(x) \leqslant \gamma$. By (2.26), we have $\lambda_F(0) \leqslant \lambda_F(x) \leqslant \gamma$. Thus $0 \in L(\lambda_F;\gamma)$. Next, let $x \cdot y \in L(\lambda_F;\gamma)$ and $x \in L(\lambda_F;\gamma)$. Then $\lambda_F(x \cdot y) \leqslant \gamma$ and $\lambda_F(x) \leqslant \gamma$, so γ is a upper bound of $\{\lambda_F(x \cdot y), \lambda_F(x)\}$. By (2.32), we have $\lambda_F(y) \leqslant \max\{\lambda_F(x \cdot y), \lambda_F(x)\} \leqslant \gamma$. Thus $y \in L(\lambda_F;\gamma)$.

Hence, $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-filters of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the set $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-filters if $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0,1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \leqslant \alpha$, so $x \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a UP-filter of X and so $0 \in L(\lambda_T;\alpha)$. Thus $\lambda_T(0) \leqslant \alpha = \lambda_T(x)$. Next, let $x, y \in X$. Then $\lambda_T(x \cdot y), \lambda_T(x) \in [0,1]$. Choose $\alpha = \max\{\lambda_T(x \cdot y), \lambda_T(x)\}$. Thus $\lambda_T(x \cdot y) \leqslant \alpha$ and $\lambda_T(x) \leqslant \alpha$, so $x \cdot y, x \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a UP-filter of X and so $y \in L(\lambda_T;\alpha)$. Thus $\lambda_T(y) \leqslant \alpha = \max\{\lambda_T(x \cdot y), \lambda_T(x)\}$.

Let $x \in X$. Then $\lambda_I(x) \in [0,1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \geqslant \beta$, so $x \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a UP-filter of X and so $0 \in U(\lambda_I;\beta)$. Thus $\lambda_I(0) \geqslant \beta = \lambda_I(x)$. Next, let $x, y \in X$. Then $\lambda_I(x \cdot y), \lambda_I(x) \in [0,1]$. Choose $\beta = \min\{\lambda_I(x \cdot y), \lambda_I(x)\}$. Thus $\lambda_I(x \cdot y) \geqslant \beta$ and $\lambda_I(x) \geqslant \beta$, so $x \cdot y, x \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a UP-filter of X and so $y \in U(\lambda_I;\beta)$. Thus $\lambda_I(y) \geqslant \beta = \min\{\lambda_I(x \cdot y), \lambda_I(x)\}$.

Let $x \in X$. Then $\lambda_F(x) \in [0,1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \leqslant \gamma$, so $x \in L(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F;\gamma)$ is a UP-filter of X and so $0 \in L(\lambda_F;\gamma)$. Thus $\lambda_F(0) \leqslant \gamma = \lambda_F(x)$. Next, let $x, y \in X$. Then $\lambda_F(x \cdot y), \lambda_F(x) \in [0,1]$. Choose $\gamma = \max\{\lambda_F(x \cdot y), \lambda_F(x)\}$. Thus $\lambda_F(x \cdot y) \leqslant \gamma$ and $\lambda_F(x) \leqslant \gamma$, so $x \cdot y, x \in L(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F;\gamma)$ is a UP-filter of X and so $y \in L(\lambda_F;\gamma)$. Thus $\lambda_F(y) \leqslant \gamma = \max\{\lambda_F(x \cdot y), \lambda_F(x)\}$.

Therefore, Λ is a special neutrosophic UP-filter of X. □

Theorem 3.5. *A NS Λ in X is a special neutrosophic UP-ideals of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-ideals of X if $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.*

Proof. Assume that Λ is a special neutrosophic UP-ideal of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $L(\lambda_T;\alpha)$, $U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x \in L(\lambda_T;\alpha)$. Then $\lambda_T(x) \leqslant \alpha$. By (2.24), we have $\lambda_T(0) \leqslant \lambda_T(x) \leqslant \alpha$. Thus $0 \in L(\lambda_T;\alpha)$. Next, let $x \cdot (y \cdot z) \in L(\lambda_T;\alpha)$ and $y \in L(\lambda_T;\alpha)$. Then $\lambda_T(x \cdot (y \cdot z)) \leqslant \alpha$ and $\lambda_T(y) \leqslant \alpha$, so α is a upper bound of $\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$. By (2.33), we have $\lambda_T(x \cdot z) \leqslant \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\} \leqslant \alpha$. Thus $x \cdot z \in L(\lambda_T;\alpha)$.

Let $x \in U(\lambda_I;\beta)$. Then $\lambda_I(x) \geqslant \beta$. By (2.25), we have $\lambda_I(0) \geqslant \lambda_I(x) \geqslant \beta$. Thus $0 \in U(\lambda_I;\beta)$. Next, let $x \cdot (y \cdot z) \in U(\lambda_I;\beta)$ and $y \in U(\lambda_I;\beta)$. Then $\lambda_I(x \cdot (y \cdot z)) \geqslant \beta$ and $\lambda_I(y) \geqslant \beta$, so β is an lower bound of $\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$. By (2.34), we have $\lambda_I(x \cdot z) \geqslant \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\} \geqslant \beta$. Thus $x \cdot z \in U(\lambda_I;\beta)$.

Let $x \in L(\lambda_F;\gamma)$. Then $\lambda_F(x) \leqslant \gamma$. By (2.26), we have $\lambda_F(0) \leqslant \lambda_F(x) \leqslant \gamma$. Thus $0 \in L(\lambda_F;\gamma)$. Next, let $x \cdot (y \cdot z) \in L(\lambda_F;\gamma)$ and $y \in L(\lambda_F;\gamma)$. Then $\lambda_F(x \cdot (y \cdot z)) \leqslant \gamma$ and $\lambda_F(y) \leqslant \gamma$, so γ is a upper bound of $\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$. By (2.35), we have $\lambda_F(x \cdot z) \leqslant \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\} \leqslant \gamma$. Thus $x \cdot z \in L(\lambda_F;\gamma)$.

Hence, $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-ideals of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the set $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are UP-ideals if $L(\lambda_T;\alpha), U(\lambda_I;\beta)$, and $L(\lambda_F;\gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0,1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \leqslant \alpha$, so $x \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a UP-ideal of X and so $0 \in L(\lambda_T;\alpha)$. Thus $\lambda_T(0) \leqslant \alpha = \lambda_T(x)$. Next, let $x, y, z \in X$. Then $\lambda_T(x \cdot (y \cdot z)), \lambda_T(y) \in [0,1]$. Choose $\alpha = \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$. Thus $\lambda_T(x \cdot (y \cdot z)) \leqslant \alpha$ and $\lambda_T(y) \leqslant \alpha$, so $x \cdot (y \cdot z), y \in L(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $L(\lambda_T;\alpha)$ is a UP-ideal of X and so $x \cdot z \in L(\lambda_T;\alpha)$. Thus $\lambda_T(x \cdot z) \leqslant \alpha = \max\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$.

Let $x \in X$. Then $\lambda_I(x) \in [0,1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \geqslant \beta$, so $x \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a UP-ideal of X and so $0 \in U(\lambda_I;\beta)$. Thus $\lambda_I(0) \geqslant \beta = \lambda_I(x)$. Next, let $x, y, z \in X$. Then $\lambda_I(x \cdot (y \cdot z)), \lambda_I(y) \in [0,1]$. Choose $\beta = \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$. Thus $\lambda_I(x \cdot (y \cdot z)) \geqslant \beta$ and $\lambda_I(y) \geqslant \beta$, so $x \cdot (y \cdot z), y \in U(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $U(\lambda_I;\beta)$ is a UP-ideal of X and so $x \cdot z \in U(\lambda_I;\beta)$. Thus $\lambda_I(x \cdot z) \geqslant \beta = \min\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$.

Let $x \in X$. Then $\lambda_F(x) \in [0,1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \leqslant \gamma$, so $x \in L(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F;\gamma)$ is a UP-ideal of X and so $0 \in L(\lambda_F;\gamma)$. Thus $\lambda_F(0) \leqslant \gamma = \lambda_F(x)$. Next, let $x, y, z \in X$. Then $\lambda_F(x \cdot (y \cdot z)), \lambda_F(y) \in [0,1]$. Choose $\gamma = \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$. Thus $\lambda_F(x \cdot (y \cdot z)) \leqslant \gamma$ and $\lambda_F(y) \leqslant \gamma$, so $x \cdot (y \cdot z), y \in L(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F;\gamma)$ is a UP-ideal of X and so $x \cdot z \in L(\lambda_F;\gamma)$. Thus $\lambda_F(x \cdot z) \leqslant \gamma = \max\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$.

Therefore, Λ is a special neutrosophic UP-ideal of X. □

Definition 3.6 ([23]). Let Λ be a NS in X. For $\alpha, \beta, \gamma \in [0,1]$, the sets

$$\begin{aligned} ULU_\Lambda(\alpha,\beta,\gamma) &= \{x \in X \mid \lambda_T \geqslant \alpha, \lambda_I \leqslant \beta, \lambda_F \geqslant \gamma\}, \\ LUL_\Lambda(\alpha,\beta,\gamma) &= \{x \in X \mid \lambda_T \leqslant \alpha, \lambda_I \geqslant \beta, \lambda_F \leqslant \gamma\}, \\ E_\Lambda(\alpha,\beta,\gamma) &= \{x \in X \mid \lambda_T = \alpha, \lambda_I = \beta, \lambda_F = \gamma\} \end{aligned}$$

are called a ULU-(α,β,γ)-*level subset*, an LUL-(α,β,γ)-*level subset*, and an E-(α,β,γ)-*level subset* of Λ, respectively. Then we see that

$$\begin{aligned} ULU_\Lambda(\alpha,\beta,\gamma) &= U(\lambda_T;\alpha) \cap L(\lambda_I;\beta) \cap U(\lambda_F;\gamma), \\ LUL_\Lambda(\alpha,\beta,\gamma) &= L(\lambda_T;\alpha) \cap U(\lambda_I;\beta) \cap L(\lambda_F;\gamma), \\ E_\Lambda(\alpha,\beta,\gamma) &= E(\lambda_T;\alpha) \cap E(\lambda_I;\beta) \cap E(\lambda_F;\gamma). \end{aligned}$$

Corollary 3.7. *A NS Λ in X is a special neutrosophic UP-subalgebra of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, $LUL_\Lambda(\alpha,\beta,\gamma)$ is a UP-subalgebra of X, where $LUL_\Lambda(\alpha,\beta,\gamma)$ is nonempty.*

Proof. It is straightforward by Theorem 3.2. □

Corollary 3.8. *A NS Λ in X is a special neutrosophic near UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, $LUL_\Lambda(\alpha,\beta,\gamma)$ is a near UP-filter of X, where $LUL_\Lambda(\alpha,\beta,\gamma)$ is nonempty.*

Proof. It is straightforward by Theorem 3.3. □

Corollary 3.9. *A NS Λ in X is a special neutrosophic UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, $LUL_\Lambda(\alpha,\beta,\gamma)$ is a UP-filter of X, where $LUL_\Lambda(\alpha,\beta,\gamma)$ is nonempty.*

Proof. It is straightforward by Theorem 3.4. □

Corollary 3.10. *A NS Λ in X is a special neutrosophic UP-ideal of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, $LUL_\Lambda(\alpha,\beta,\gamma)$ is a UP-ideal of X, where $LUL_\Lambda(\alpha,\beta,\gamma)$ is nonempty.*

Proof. It is straightforward by Theorem 3.5. □

4. Conclusions

In this paper, we have introduced the notions of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals of UP-algebras and investigated some of their important properties. Then, we have the diagram of generalization of NSs of special types in UP-algebras as shown in Figure 1.

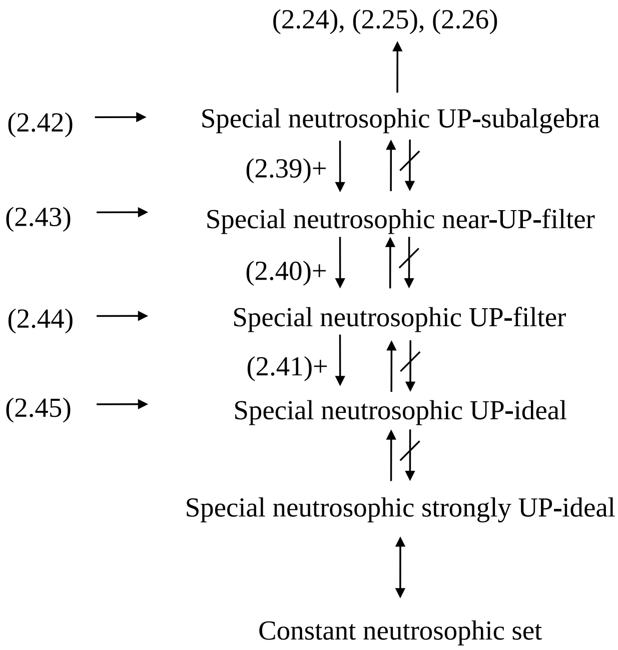

Figure 1: NSs of special types in UP-algebras.

In our future study, we will apply this notion/results to other type of NSs in UP-algebras. Also, we will study the soft set theory/cubic set theory of special neutrosophic UP-subalgebras, special neutrosophic near UP-filters, special neutrosophic UP-filters, special neutrosophic UP-ideals, and special neutrosophic strong UP-ideals.

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