Available online at www.isr-publications.com/jmcs
J. Math. Computer Sci., 21 (2020), 127–135
Research Article

Online: ISSN 2008-949X

Journal of Mathematics and Computer Science

Journal Homepage: www.isr-publications.com/jmcs

Modified Laplace transform and its properties

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Abstract

In this paper we propose a new definition of the modified Laplace transform $\mathcal{L}_a(f(t))$ for a piece-wise continuous function of exponential order which further reduces to simple Laplace transform for $a = e$ where $a \neq 1$ and $a > 0$. Also we prove some basic results of this modified Laplace transform and connection with different functions.

Keywords: Laplace transform, convolution, double Laplace transform.

2020 MSC: 44A10, 44A35, 44A30.

1. Introduction and Definitions

Named after Pierre-Simon Laplace, Laplace transform was first introduced in 1782 during the study of probability theory, and is one of the important tools for solving linear constant coefficient, ordinary or partial differential equations under suitable initial and boundary value problem. It is basically a linear operator of a function $f(t)$ with $\mathfrak{R}(\arg(t)) \geqslant 0$ that transforms it into a function $f(s)$ with a complex argument s. The Laplace transform, in the analysis of linear time-invariant systems (harmonic oscillators, electrical circuits, optical devices and mechanical systems), acts as a transformation from time domain to the frequency domain. It is much similar to Fourier transform though the difference actually lies in that the Fourier transform expresses a function or signal as a series of modes of vibration, whereas the Laplace transform settle a function into its moments. It finds very wide application in various area of physics, electrical engineering, control engineering, optics, mathematics and signal processing. For further details (see the references [1, 2, 6–11, 13, 14]).

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doi: 10.22436/jmcs.021.02.04

Received: 2019-09-04 *Revised:* 2019-10-14 *Accepted:* 2020-03-03

Definition 1.1 (Laplace transform)**.** The Laplace transform of a function $f(t)$ is defined as:

$$\mathcal{L}(f(t)) = \mathcal{F}(s) = \int_0^\infty e^{-st} f(t)\, dt, \quad (\Re(s) > 0),$$

where, the function $f(t)$ is piece-wise continuous and of exponential order.

The inverse Laplace transform is defined as:

$$f(t) = \mathcal{L}^{-1}(\mathcal{F}(s)) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} f(t)\, dt, \quad (\Re(s) > 0,\ c > 0),$$

where $\mathcal{L}^{-1}$ is notation of inverse Laplace transform.

Definition 1.2 (Double Laplace transform)**.** Lately, Lokenath Debnath [3] came up with an interested work in which he defined the double Laplace transform in following manner

$$L(f(x,y)) = L[Lf(x;y); x \rightarrowtail p; y \rightarrowtail q] = \int_0^\infty \int_0^\infty e^{-(px+qy)} f(x,y) dx\, dy. \ (\Re(p), \Re(q) > 0)$$

and studied its various properties to solve partial differential equations and integral equations.

Definition 1.3 (Gamma function)**.** The gamma function is defined by

$$\Gamma(m) = \int_0^\infty e^{-t} t^{m-1} dt, \quad (\Re(m) > 0).$$

Definition 1.4 (Pochhamer symbol)**.** The Pochhamer symbol is defined as

$$(a)_n = a(a+1)(a+2)(a+3)\cdots(a+n-1),$$

where $n \in \mathbb{N}$, a is neither zero nor a negative integer,

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}, \quad (a)_{n-k} = \frac{(-1)^k (a)_n}{(1-a-n)_k}, \ (0 \leqslant k \leqslant n).$$

Definition 1.5 (Hypergeometric function)**.** The hypergeometric function has the series representation as:

$${}_2F_1\left[\begin{matrix} a, & b; & \\ & c; & \end{matrix}\ t\right] = \sum_{n=0}^\infty \frac{(a)_n (b)_n}{(c)_n} \frac{t^n}{n!}. \tag{1.1}$$

Definition 1.6 (Confluent hypergeometric function)**.** The confluent hypergeometric function defined by

$${}_1F_1[a; b; t] = \sum_{n=0}^\infty \frac{(a)_n}{(b)_n} \frac{t^n}{n!}. \tag{1.2}$$

Definition 1.7 (The generalized hypergeometric function)**.** The Generalized hypergeometric function ${}_pF_q$ with p numerator parameter and q denominator parameter is defined by

$${}_pF_q[a_1, a_2 \ldots, a_p; b_1, b_2 \ldots, b_q; t] = \sum_{n=0}^\infty \frac{(a_1)_n, (a_2)_n, \ldots, (a_p)_n}{(b_1)_n, (b_2)_n, \ldots, (b_q)_n} \frac{t^n}{n!}, \tag{1.3}$$

where$a_i \in \mathbb{C}\ (i = 1, 2, \ldots, p)$, $b_i \in \mathbb{C} \setminus \mathbb{Z}_0^-$, $(i = 1, 2, \ldots, q)$, where $\mathbb{Z}_0^- = \ldots -2, -1, 0$.

Definition 1.8 (Laguerre polynomial)**.** The Laguerre polynomial of one variable t is defined by by the following relation

$$L_n(t) = \sum_{k=0}^n \frac{(-1)^k n!\, t^k}{(k!)^2 (n-k)!}. \tag{1.4}$$

Definition 1.9 (Bessel function)**.** The Bessel function of order zero is defined by the following relation

$$J_0(t) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{2^{2k}\,\Gamma(1+k)}. \tag{1.5}$$

Definition 1.10 (Legendre polynomials)**.** The Legendre polynomial is defined by

$$P_n(t) = \sum_{k=0}^{[\frac{n}{2}]} \frac{(-1)^k (\frac{1}{2})_{n-k} (2t)^{n-2k}}{k!(n-2k)}. \tag{1.6}$$

2. Definition of modified Laplace transform

The modified Laplace transform for a function $f(t)$ which is piece-wise continuous and of exponential order is defined by the integral

$$\mathcal{L}_a(f(t)) = \mathcal{F}(s;a) = \int_0^{\infty} a^{-st} f(t)\,dt, \quad (\Re(s) > 0,\ a \in (0,\infty)\setminus\{1\}), \tag{2.1}$$

provided that the integral in (2.1) exists.

For $a = e$, the modified Laplace transform becomes simple Laplace transform given by

$$\mathcal{L}_e(f(t)) = \mathcal{F}(s;e) = \mathcal{L}(f(t)) = \int_0^{\infty} e^{-st} f(t)\,dt.$$

2.1. Linear property of modified Laplace transform

If $f(t)$ and $g(t)$ are two functions whose modified Laplace transform exists, then for any constant α and β, we have

$$\mathcal{L}_a\left(\alpha(f(t)) + \beta(g(t))\right) = \alpha\left(\mathcal{L}_a f(t) + \beta \mathcal{L}_a g(t)\right).$$

Proof. We can easily write

$$\mathcal{L}_a\left(\alpha(f(t)) + \beta(g(t))\right) = \int_0^{\infty} a^{-st} \mathcal{L}_a\left(\alpha(f(t)) + \beta(g(t))\right) dt = \alpha\left(\int_0^{\infty} a^{-st} f(t) dt\right) + \beta\left(\int_0^{\infty} a^{-st} g(t) dt\right),$$

which, on using (2.1), we get desired result. □

Proposition 2.1. *The following properties are valid for modified Laplace transform.*

(1) *If* $f(t) = 1$, *then*

$$\mathcal{L}_a(1) = \frac{1}{s.\log\, a}, \quad (s > 0).$$

(2) *If* $f(t) = t$, *then*

$$\mathcal{L}_a(t) = \frac{1}{s^2.(\log\, a)^2}, \quad (s > 0).$$

(3) *If* $f(t) = t^n$, *then*

$$\mathcal{L}_a(t^n) = \frac{\Gamma(n+1)}{s^{n+1}.(\log\, a)^{n+1}}, \quad (s > 0, n = 0, 1, 2, \ldots).$$

(4) *If* $f(t) = e^{bt}$, *then*

$$\mathcal{L}_a(e^{bt}) = \int_0^{\infty} a^{-st} e^{bt} dt = \frac{1}{s \log a - b}, \quad (s \log a > |b|).$$

(5) *If* $f(t) = \sinh\, bt$, *then*

$$\begin{aligned}\mathcal{L}_a(\sinh bt) &= \int_0^\infty a^{-st}\left(\frac{e^{bt}-e^{bt}}{2}\right)dt\\ &=\frac{1}{2}\int_0^\infty e^{-st\log a}(e^{bt}-e^{-bt})dt\\ &=\frac{1}{2}\left[\int_0^\infty e^{-t(s\log a-b)}-e^{-t(s\log a+b)}\right]dt\\ &=\frac{b}{s^2(\log a)^2-b^2},\quad (s\log a>|b|).\end{aligned}$$

(6) *If* $f(t) = \cosh\, bt$, *then*

$$\begin{aligned}\mathcal{L}_a(\cosh bt) &= \int_0^\infty a^{-st}\left(\frac{e^{bt}+e^{bt}}{2}\right)dt\\ &=\frac{1}{2}\int_0^\infty e^{-st\log a}(e^{bt}+e^{-bt})dt\\ &=\frac{1}{2}\left[\int_0^\infty e^{-t(s\log a-b)}+e^{-t(s\log a+b)}\right]dt\\ &=\frac{b}{s^2(\log a)^2-b^2},\quad (s\log a>|b|).\end{aligned}$$

(7) *If* $f(t) = \sin bt$, *then*

$$\begin{aligned}\mathcal{L}_a(\sin bt) &= \int_0^\infty a^{-st}\left(\frac{e^{bt}-e^{bt}}{2i}\right)dt\\ &=\frac{1}{2i}\int_0^\infty e^{-st\log a}(e^{bt}-e^{-bt})dt\\ &=\frac{1}{2i}\left[\int_0^\infty e^{-t(s\log a-b)}-e^{-t(s\log a+b)}\right]dt\\ &=\frac{b}{s^2(\log a)^2+b^2},\quad (s\log a>0).\end{aligned}$$

(8) *If* $f(t) = \cos bt$, *then*

$$\begin{aligned}\mathcal{L}_a(\cos bt) &= \int_0^\infty a^{-st}\left(\frac{e^{bt}-e^{bt}}{2}\right)dt\\ &=\frac{1}{2}\int_0^\infty e^{-st\log a}(e^{bt}-e^{-bt})dt\\ &=\frac{1}{2}\left[\int_0^\infty e^{-t(s\log a-b)}-e^{-t(s\log a+b)}\right]dt\\ &=\frac{s}{s^2(\log a)^2+b^2},\quad (s\log a>0).\end{aligned}$$

2.2. *Unit step function*

We now present a method for solving certain initial value problem with discontinuous forcing function. For example, an external force acting on a mechanical system or a voltage applied to an electrical circuit can be turned off after a certain period of time. Therefore, we shall discuss the case of function having jump type discontinuity at $t = c$.

$$u(t-c) = \begin{cases} 0, & x < c, \\ 1, & x \geqslant c, \end{cases} \tag{2.2}$$

which, on applying the modified Laplace transform gives

$$\mathcal{L}_a\left(u(t-c)\right) = \int_0^\infty a^{-st} u(t-c)\,dt.$$

Now in view of (2.2), we get

$$\mathcal{L}_a\left(u(t-c)\right) = \int_0^c a^{-st} 0\,dt + \int_c^\infty a^{-st}\,dt = \frac{a^{-sc}}{s\log a}.$$

2.3. First shifting property

If $\mathcal{L}_a(f(t)) = \mathcal{F}(s;a)$, then for $a > 0\,(a \neq 1)$, we have

$$\mathcal{L}_a(e^{bt} f(t))dt = \mathcal{F}(s\log a - b).$$

Proof. By using (2.1), we write

$$\mathcal{L}_a(f(t)) = \int_0^\infty a^{-st} e^{bt} f(t)dt = \int_0^\infty e^{-t(s\log a - a)} f(t)dt.$$

Now by integrating and using limit, we obtain

$$\mathcal{L}_a(e^{bt} f(t))dt = \mathcal{F}(s\log a - b).$$

□

2.4. Second shifting property

If $f(t)$ be a piece-wise continuous function and of exponential order such that $\mathcal{L}_a(f(t)) = \mathcal{F}(s;a)$, then we have the expression

$$\mathcal{L}_a[f(t-b)u_b(t)] = e^{-sb}\mathcal{L}_a(f(t)).$$

Proof. With the help of the definition of modified Laplace transform in (2.1), we can write

$$\mathcal{L}_a[f(t-b)u_b(t)] = \int_0^\infty a^{-st\,\log a} f(t-b)dt.$$

Further, by putting $\tau = t - b$ implies $d\tau = dt$, therefore

$$\mathcal{L}_a[f(t-b)u_b(t)] = \int_b^\infty a^{-st\,\log a} f(t-b)dt = \int_0^\infty e^{-s(\tau+b)\log a} f(\tau)d\tau = e^{-sb}\int_0^\infty a^{-st} f(t)dt,$$

which gives

$$\mathcal{L}_a[f(t-b)u_b(t)] = e^{-sb}\mathcal{L}_a(f(t)).$$

□

Definition 2.2.

Convolution: The convolution of $f(t)$ and $g(t)$ ($f(t)$ and $g(t)$ are piece-wise continuous and of exponential order) is denoted by $(f * g)$ and is defined as

$$(f * g)(t) = \int_0^t f(t-u)g(u)du. \tag{2.3}$$

It can be easily seen that the set of all modified Laplace transformable functions form commutative semi group with respect to the convolution operation $*$.

Commutativity: The operation $*$ defined above for modified Laplace transform is commutative, that is,

$$(f * g)(t) = (g * f)(t).$$

Associativity: The operation $*$ defined above for modified Laplace transform is associative, that is,

$$\{f * (g * h)\} = \{(f * g) * h\}.$$

Distributivity: The operation $*$ defined above for modified Laplace transform is distributive, that is,

$$\{f * (g + h)\} = (f * g) + (f * h).$$

Identity: The operation $*$ defined above for modified Laplace transform has identity, that is,

$$(f * \delta)(t) = \delta = (\delta * f)(t),$$

where δ is Kronecker delta.

Theorem 2.3 (Convolution theorem)**.** *Let $\mathcal{L}_a(f(t)) = \mathcal{F}(s; a)$ and $\mathcal{L}_a(g(t)) = \mathcal{G}(s; a)$ be such that $f(t)$ and $g(t)$ are piece-wise continuous functions on $[0, \infty)$. Then their convolution $(f * g)$ is defined by:*

$$\mathcal{L}_a(f * g)(t) = \mathcal{F}(s; a)\, \mathcal{G}(s; a).$$

Proof. By simply using (2.1) and (2.3), we obtain

$$\{\mathcal{L}_a(f * g)(t)\} = \mathcal{L}_a \left[\int_0^\infty f(u)g(t-u)du\right] = \int_0^\infty a^{-st} \left(\int_0^t f(u)g(t-u)du\right) dt = \int_0^\infty a^{-st} \int_0^t f(u)g(t-u)dudt.$$

Putting $t - u = z$ implies $dt = dz$, and we write

$$\{\mathcal{L}_a(f * g)(t)\} = \int_{u=0}^\infty \int_{t=u}^\infty a^{-st} f(u)g(t-u)du\, dt = \int_{u=0}^\infty \int_{z=0}^\infty a^{-p(u+z)} f(u)g(z)dz\, du.$$

After separating the variables, we get

$$\{\mathcal{L}_a(f * g)(t)\} = \left(\int_0^\infty a^{-pu} f(u)du\right) + \left(\int_0^\infty a^{-pz} g(z)dz\right),$$

from which the result easily follows. □

2.5. Connection with Bessel function of order zero

Modified Laplace transform in (2.1) is connected with Bessel function of order zero by the relation

$$\mathcal{L}_a [J_0(t)] = \frac{1}{\sqrt{s^2(\log a)^2 + 1}},$$

where, $J_0(t)$ is defined by (1.5).

2.6. Connection with Laguerre polynomial

Modified Laplace transform in (2.1) is connected with simple Laguerre polynomial

$$\int_0^\infty a^{-st} L_n(t)\, dt = \frac{1}{s}\left(1 - \frac{\log a}{s}\right)^n,$$

where, $L_n(t)$ is given by (1.4).

2.7. Modified Laplace transform of hypergeometric function

The modified Laplace transform of hypergeometric function given in (1.1), is given as:

$$\mathcal{L}_a\, {}_2F_1[a, b; c; z] = \mathcal{L}_a \left\{\sum_{n=0}^\infty \frac{(a)_n (b)_n}{(c)_n} \frac{t^n}{n!}\right\} = \sum_{n=0}^\infty \frac{(a)_n (b)_n}{(c)_n} \int_0^\infty a^{-st} t^n dt = \left(\frac{1}{s \log a}\right) {}_2F_0\left[a, b; \frac{1}{s \log a}\right].$$

2.8. Modified Laplace transform of confluent hypergeometric function

The modified Laplace transform of confluent hypergeometric function given in (1.2) is connected as:

$$\mathcal{L}_a\, {}_2F_1[a,-,1,t] = \mathcal{L}_a\left\{\sum_{n=0}^{\infty}\frac{(a)_n}{(1)_n}\frac{t^n}{n!}\right\} = \sum_{n=0}^{\infty}\frac{(a)_n}{(1)_n}\int_0^{\infty} a^{-st}t^n dt = \left(\frac{1}{s\log a}\right)\, {}_1F_0\left[a,b;\frac{1}{s\log a}\right].$$

2.9. Modified Laplace transform of generalized hypergeometric function

The modified Laplace transform of generalized hypergeometric function given in (1.3) is given as:

$$\begin{aligned}\mathcal{L}_a\, {}_pF_q[a_1,a_2\dots,a_p;b_1,b_2\dots,b_{q-1};t] &=\mathcal{L}_a\left\{\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_p)_n}{(b_1)_n,(b_2)_n,\dots,(b_{q-1})_n(1)_n}\frac{t^n}{n!}\right\}\\ &=\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_p)_n}{(b_1)_n,(b_2)_n,\dots,(b_{q-1})_n(1)_n(1)_n}\int_0^{\infty}a^{-st}t^n dt\\ &=\left(\frac{1}{s\log\, a}\right)\, {}_pF_{q-1}\left[a_1,a_2,\dots a_p,b_1,b_2,\dots,b_{q-1};\frac{1}{s\log a}\right].\end{aligned}$$

2.10. Modified Laplace transform of Legendre polynomials

The modified Laplace transform of Legendre polynomial given in (1.6) is given as:

$$\begin{aligned}\mathcal{L}_a\left\{t^{\beta}P_n(t)dt\right\} &=2^n\sum_{k=0}^{[\frac{n}{2}]}\frac{(-n)_{2k}(\frac{1}{2})_n}{k!n!(1-\frac{1}{2}-n)_k 2^{2k}}\int_0^{\infty}a^{-st}t^{\beta+n-2k}dt\\ &=\frac{2^n(\frac{1}{2})_n}{n!}\sum_{k=0}^{[\frac{n}{2}]}\frac{(-n)_{2k}}{k!(\frac{1}{2}-n)_k 2^{2k}}\frac{\Gamma(1+n-2k+\beta)}{s^{1+n-2k+\beta}(\log a)^{1+n-2k+\beta}}\\ &=\frac{2^n(\frac{1}{2})_n\Gamma(1+\beta+n)}{n!s^{1+\beta+n}(\log a)^{1+\beta+n}}\, {}_2F_3\left[\begin{matrix}\frac{-n}{2},\frac{-n}{2}+\frac{1}{2};\\ \frac{1}{2}-n,\frac{1}{2}(-\beta-n),\frac{1}{2}(1-\beta-n);\end{matrix}\quad -\frac{(s\log a)^2}{4}\right].\end{aligned}$$

3. Modified inverse Laplace transform

In this section, we define the modified inverse Laplace transform of a function $f(t)$ given by:

$$\mathcal{L}_a^{-1}(f(t)) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} a^{st}\mathcal{F}(s;a)\, dt, \quad (c>0).$$

Proposition 3.1. *Let $f(t)$ be a piece wise continuous function of exponential order. Then, the following properties are valid for modified inverse Laplace transform.*

(1) *If $f(s)=\frac{1}{s}$, then*

$$\mathcal{L}_a^{-1}\left(\frac{1}{s}\right) = \log a, \quad (s>0).$$

(2) *If $f(s)=\frac{1}{s^n}$, then*

$$\mathcal{L}_a^{-1}\left(\frac{1}{s^n}\right) = \frac{(\log a)^{n+1}t^n}{\Gamma(n+1)}, \quad (s>0).$$

(3) *If $f(s)=\frac{1}{s\log a-b}$, then*

$$\mathcal{L}_a^{-1}\left(\frac{1}{s\log a-b}\right) = e^{bt}, \quad (s\log a>|b|).$$

(4) *If* $f(s) = \frac{b}{s^2(\log a)^2 - b^2}$, *then*

$$\mathcal{L}_a^{-1}\left(\frac{b}{s^2(\log a)^2 - b^2}\right) = \sinh bt, \quad (s \log a > |b|).$$

(5) *If* $f(s) = \frac{s}{s^2(\log a)^2 - b^2}$, *then*

$$\mathcal{L}_a^{-1}\left(\frac{s}{s^2(\log a)^2 - b^2}\right) = \cosh bt, \quad (s \log a > |b|).$$

(6) *If* $f(s) = \frac{b}{s^2(\log a)^2 + b^2}$, *then*

$$\mathcal{L}_a^{-1}\left(\frac{b}{s^2(\log a)^2 + b^2}\right) = \sin bt, \quad (s \log a > 0).$$

(7) *If* $f(s) = \frac{s}{s^2(\log a)^2 + b^2}$, *then*

$$\mathcal{L}_a^{-1}\left(\frac{s}{s^2(\log a)^2 + b^2}\right) = \cos bt, \quad (s \log a > 0).$$

Theorem 3.2.

Linearity property: Let α, β *be constants and* $f_1(s; a), f_2(s; a)$ *be the modified Laplace transform of* $\mathcal{F}_1(s, a)$ *and* $\mathcal{F}_2(s, a)$*, respectively, then*

$$\mathcal{L}_a^{-1}\left(\alpha f_1(s; a) + \beta f_2(s; a)\right) = \alpha\left(\mathcal{L}_a^{-1} f_1(s; a)\right) + \beta\left(\mathcal{L}_a^{-1} f_2(s; a)\right) = \alpha\,\mathcal{F}_1(s, a) + \beta\,\mathcal{F}_2(s, a).$$

First shifting property: If $\mathcal{L}_a^{-1}\mathcal{F}(s; a) = f(t)$*, then*

$$\mathcal{L}_a^{-1}\{\mathcal{F}(s \log a - b)\} = e^{at} f(t).$$

Second shifting property: If $\mathcal{L}_a^{-1}\mathcal{F}(s; a) = f(t)$*, then*

$$\mathcal{L}_a^{-1}\left\{e^{-sb}\mathcal{L}_a(f(t))\right\} = f(t - b)\, u_b(t).$$

Convolution theorem: If $\mathcal{L}_a^{-1}(F(s; a)) = f(t)$, $\mathcal{L}_a^{-1}(G(s; a)) = g(t)$*, then*

$$\mathcal{L}_a^{-1}\{\mathcal{F}(s; a)\, \mathcal{G}(s; a)\} = (f * g).$$

4. Conclusion

Laplace transform is an important tool applied in vast areas of science and engineering. The applications of Laplace transform are not limited to areas of applied mathematics, statistics and engineering but to a much wider extent for, e.g., a very simple application of modified Laplace transform in the area of physics could be to find out the harmonic vibration of a beam. Being useful, a very extensive work on Laplace transform is available in literature (see, for details, [3–5, 12]) and several authors have dug into its properties and applications. Following up, in this paper, our motive is to present a modified form of Laplace transform that is more general and works on a larger domain ($a \in (0, \infty) \setminus \{1\}$). The results obtained by this transform in potential problems of physics and applied mathematics will be more precised and accurate. We will discuss some advanced problems involving modified Laplace transform in the subsequent paper.

For future work, we introduce a modified double Laplace transform and show its linear property. Interested authors can further study advanced properties of the double Laplace transform.

The modified double Laplace transform of a function $f(x,y)$ of two variable x and y is defined as:

$$\mathcal{L}_a(f(x,y)) = \int_0^\infty \int_0^\infty a^{-(px+qy)} f(x,y)\,dx\,dy \tag{4.1}$$

provided the integral in (4.1) exists.

Eq. (4.1) satisfies the following linear property:

$$\mathcal{L}_a\{\alpha(f(x,y)) + \beta(g(x,y))\} = \alpha\left(\mathcal{L}_a f(x,y) + \beta\mathcal{L}_a g(x,y)\right),$$

where α and β are constant.

Acknowledgment

The author K. S. Nisar thanks to Prince Sattam bin Abdulaziz University, Saudi Arabia for providing facilities and support.

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