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Computing the edge metric dimension of convex polytopes related graphs

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Abstract

Let $G = (V(G), E(G))$ be a connected graph and $d(f, y)$ denotes the distance between edge f and vertex y, which is defined as $d(f, y) = \min\{d(p, y), d(q, y)\}$, where $f = pq$. A subset $W_E \subseteq V(G)$ is called an edge metric generator for graph G if for every two distinct edges $f_1, f_2 \in E(G)$, there exists a vertex $y \in W_E$ such that $d(f_1, y) \neq d(f_2, y)$. An edge metric generator with minimum number of vertices is called an edge metric basis for graph G and the cardinality of an edge metric basis is called the edge metric dimension represented by $edim(G)$. In this paper, we study the edge metric dimension of flower graph $f_{n\times3}$ and also calculate the edge metric dimension of the prism related graphs D'_n and D^t_n. It has been concluded that the edge metric dimension of D'_n is bounded, while of $f_{n\times3}$ and D^t_n is unbounded.

Keywords: Edge metric dimension, edge metric generator, edge metric basis, resolving set, prism related graphs, flower graph.

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1. Introduction and preliminaries

Slater proposed the idea of metric dimension to find the location of intruder in a network (see [18, 23]). The proposed idea was further extended by Melter and Harary in [10]. Metric dimension is important in robot navigation, chemistry, problems of image processing and pattern recognition etc. (see [4, 5, 11, 12, 14, 15]). The use of metric dimension of graphs was also observed in games like mastermind and coin weighing (see [3]). Caceres et al. computed the metric dimension of cartesian product of graphs and Hallaway et al. had worked on the metric dimension of permutation of graphs (see [3, 9]). Further Zhang had worked on the theory and resolvability of graphs in [6].

Suppose G is connected graph having edge set $E(G)$ and vertex set $V(G)$, also $|E(G)|$ shows the size of graph G and $|V(G)|$ represents the order of graph G. Let $N(a) = \{b \in V(G) | ab \in E(G)\}$ denote the neighborhood of the vertex a, then $|N(a)|$ is called the degree of the vertex a. Moreover, $\Delta(G)$ and $\delta(G)$ represent the maximum and minimum degree of graph G, respectively.

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The distance between two distinct vertices s and t, is the number of edges in a shortest path between them and it is denoted by $d(s,t)$. A vertex $u \in V(G)$ is said to distinguish a pair of vertices $s,t \in V(G)$ if $d(s,u) \neq d(t,u)$. A set $W \subseteq V(G)$ is a metric generator for G if every pair of vertices of G can be distinguished by some vertex in W. A metric basis is the minimum metric generator for graph G and number of elements in metric basis is called the metric dimension of G, denoted by $\dim(G)$. It was shown that computing the metric dimension of a graph is NP-hard [14].

The edge metric dimension is introduced recently. The concept was brought by kelenc et al. and further studied by Zubrilina, peterin, Kratica, Yuezhong Zhang and Ahsan et al. [1, 7, 13, 16, 22, 23]. We can find the distance between an edge $f = pq$ and a vertex y as follows:

$$d(f,y) = \min\{d(p,y), d(q,y)\}.$$

A vertex $a \in V(G)$ is said to distinguish two distinct edges $e_1, e_2 \in E(G)$ if $d(e_1,a) \neq d(e_2,a)$. A set W_E is an edge metric generator of a graph G if every two distinct edges are distinguished by some vertex of W_E. An edge metric basis is the minimum edge metric generator of graph G and its cardinality is called edge metric dimension, denoted by $\mathrm{edim}(G)$.

For an ordered subset $W_E = \{a_1, a_2, \dots, a_k\}$ of the vertex set of $V(G)$, the k-tuple $r(e|W_E) = (d(e,a_1), d(e,a_2), \dots, d(e,a_k))$ is called the edge metric representation of an edge e with respect to W_E. In this sense, W_E is an edge metric generator for G if and only if for every pair of different edges e_1, e_2 of $E(G)$, we have $r(e_1|W_E) \neq r(e_2|W_E)$.

In this whole paper, all vertex indices are considered to be module n. The propositions given below are very helpful for calculating the edge metric dimension of graphs.

Proposition 1.1 ([13])**.** *If* G *is a connected graph, then* $\mathrm{edim}(G) \geqslant \lceil \log_2 \Delta(G) \rceil$.

Proposition 1.2 ([13])**.** *If* G *is a connected graph, then* $\mathrm{edim}(G) \geqslant 1 + \lceil \log_2 \delta(G) \rceil$.

In following proposition, we will demonstrate the metric dimension of flower graph.

Proposition 1.3 ([11])**.** *For the flower graph* $f_{n\times 3}$ *with* $n \geqslant 6$, *we have*

$$\dim(f_{n\times 3}) = \begin{cases} 2, & n \text{ is even,} \\ 3, & \text{otherwise.} \end{cases}$$

In the following propositions we calculate the metric dimensions of $D_n^{'}$ and D_n^t by showing its resolving sets and the results are obvious.

Proposition 1.4. *For the prism related graph* $D_n^{'}$ *with* $n \geqslant 4$, *we have* $\dim(D_n^{'}) = 3$ *and its metric basis is* $W = \{a_1, a_2, a_{k+2}\}$, *where either* $n = 2k$ *or* $n = 2k+1$.

Proposition 1.5. *For the prism related graph* D_n^t *with* $n \geqslant 4$, *we have* $\dim(D_n^t) = 3$ *and its metric basis is* $W = \{a_1, a_2, a_{k+1}\}$, *where either* $n = 2k$ *or* $n = 2k+1$.

The rest of paper is structured as follows. In Section 2, edge metric dimension of flower graph $f_{n\times 3}$ will be studied. In Section 3, edge metric dimension of prism related graph $D_n^{'}$ will be investigated. In Section 4, edge metric dimension of prism related graph D_n^t will be determined. In last Section, article will be concluded.

2. Edge metric dimension of flower graph

In this Section, we will investigate the edge metric dimension of flower graph $f_{n\times 3}$. We have the vertex set $V(f_{n\times 3}) = \{a_\alpha, b_\alpha | 1 \leqslant \alpha \leqslant n\}$ and the edge set $E(f_{n\times 3}) = \{a_\alpha a_{\alpha+1}, a_\alpha b_\alpha, a_\alpha b_{\alpha+1} | 1 \leqslant \alpha \leqslant n\}$ as shown in Figure 1.

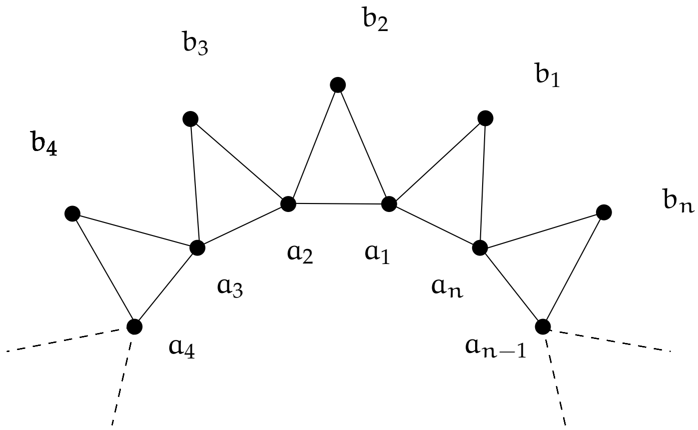

Figure 1: Graph of $f_{n\times 3}$.

Lemma 2.1. *Let* $Y = \{b_1, b_2, \ldots, b_n\}$ *be a subset of* $V(f_{n\times3})$. *Then any arbitrary edge metric generator* W_E *of* $f_{n\times3}$ *contains at least* $\lceil \frac{n}{2} \rceil$ *vertices of* Y.

Proof. Suppose that W_E contains at most $\lceil \frac{n}{2} \rceil - 1$ vertices of Y for a contradiction. Without loss of generality we assume that $b_\alpha, b_{\alpha+1} \notin W_E$, then we have $(a_\alpha b_\alpha|W) = (a_\alpha b_{\alpha+1}|W)$, a contradiction. □

Remark 2.2. Let W_E be any edge metric basis of $f_{n\times3}$. We note that W_E contains all odd vertices (vertex indices are odd) of Y for odd n, while W_E contains either all odd vertices or even vertices (vertex indices are even) of Y for even n.

Theorem 2.3. *For the flower graph* $f_{n\times3}$, *we have*

$$\text{edim}\,(f_{n\times3}) = \begin{cases} 3, & n = 4, \\ 4, & n = 3, 5, \\ \lceil \frac{n}{2} \rceil, & \textit{otherwise.} \end{cases}$$

Proof. For $n = 3, 4, 5$ we have calculated the edge metric dimension by total enumeration, $\text{edim}(f_{3\times3}) = 4$ and its edge metric basis is $\{a_1, a_2, b_1, b_2\}$, $\text{edim}(f_{4\times3}) = 3$ and its edge metric basis is $\{b_1, b_3, b_4\}$, and $\text{edim}(f_{5\times3}) = 4$ and its edge metric basis is $\{a_1, b_1, b_3, b_5\}$.

For $n \geqslant 6$, we discuss the following four cases.

Let $W_E = \{b_1, b_3, b_5, \ldots, b_{n-1}\}$. We will show that W_E is an edge metric basis of $f_{n\times3}$ in Case(I) and Case(II), respectively.

Case (I) When $n \equiv 0 \pmod 4$. We can write $n = 2k$, $k \geqslant 4$, and k is even. Let $W_1 = \{b_1, b_3, b_{k+1}, b_{k+3}\}$. Next, we give representation of edges of $f_{n\times3}$ with respect to W_1.

$$r\big(a_\alpha a_{\alpha+1}|W_1\big) = \begin{cases} \big(\alpha, 1, k-\alpha, \alpha+k-2\big), & 1 \leqslant \alpha \leqslant 2, \\ \big(\alpha, \alpha-2, k-\alpha, k-\alpha+2\big), & 3 \leqslant \alpha \leqslant k-1, \\ \big(k, k-2, 1, 2\big), & \alpha = k, \\ \big(n-\alpha, \alpha-2, \alpha-k, 1\big), & k+1 \leqslant \alpha \leqslant k+2, \\ \big(n-\alpha, n-\alpha+2, \alpha-k, \alpha-k-2\big), & k+3 \leqslant \alpha \leqslant n-1, \\ \big(1, 2, k, k-2\big), & \alpha = n, \end{cases}$$

$$r(a_\alpha b_\alpha|W_1) = \begin{cases} (0,2,k,k-1), & \alpha = 1, \\ (\alpha, 3-\alpha, k-\alpha+1, k), & 2 \leqslant \alpha \leqslant 3, \\ (\alpha, \alpha-2, k-\alpha+1, k-\alpha+3), & 4 \leqslant \alpha \leqslant k, \\ (n-k, k-1, 0, 2), & \alpha = k+1, \\ (n-k-1, k, 2, 1), & \alpha = k+2, \\ (n-k-2, n-k, 3, 0), & \alpha = k+3, \\ (n-\alpha+1, n-\alpha+3, \alpha-k, \alpha-k-2), & k+4 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(a_\alpha b_{\alpha+1}|W_1) = \begin{cases} (1,2,k,k-1), & \alpha = 1, \\ (2,0,k-1,k), & \alpha = 2, \\ (\alpha, \alpha-2, k-\alpha+1, k-\alpha+3), & 3 \leqslant \alpha \leqslant k-1, \\ (k, k-2, 0, 3), & \alpha = k, \\ (n-k, k-1, 1, 2), & \alpha = k+1, \\ (n-k-1, k, 2, 0), & \alpha = k+2, \\ (n-\alpha+1, n-\alpha+3, \alpha-k, \alpha-k-2), & k+3 \leqslant \alpha \leqslant n-1, \\ (0,3,k,k-2), & \alpha = n. \end{cases}$$

From above representation we see that $r(a_\alpha b_\alpha|W_1) = r(a_\alpha b_{\alpha+1}|W_1)$ when $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1, 2, 3, k, k+1, k+2, k+3, n$ and no other edges have same representation. If we take odd α, where $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1, 3, k+1, n-1$ such that $W_E = W_1 \cup \{b_\alpha\}$ then $r(a_\alpha b_\alpha|W_E) \neq r(a_\alpha b_{\alpha+1}|W_E)$ which implies that $\text{edim}\,(f_{n\times 3}) \leqslant \lceil \frac{n}{2} \rceil$. So from Lemma 2.1, W_E is an edge metric generator for $f_{n\times 3}$ and $\text{edim}(f_{n\times 3}) = \lceil \frac{n}{2} \rceil$.

Case (II) When $n \equiv 2 \pmod 4$. We can write $n = 2k$, $k \geqslant 3$, and k is odd. Let $W_1 = \{b_1, b_3, b_{k+2}\}$. Next, we give representation of edges of $f_{n\times 3}$ with respect to W_1.

$$r(a_\alpha a_{\alpha+1}|W_1) = \begin{cases} (\alpha, 1, k-\alpha+1), & 1 \leqslant \alpha \leqslant 2, \\ (\alpha, \alpha-2, k-\alpha+1), & 3 \leqslant \alpha \leqslant k, \\ (n-k-1, k-1, 1), & \alpha = k+1, \\ (n-\alpha, n-\alpha+2, \alpha-k-1), & k+2 \leqslant \alpha \leqslant n-1, \\ (1, 2, n-k-1), & \alpha = n, \end{cases}$$

$$r(a_\alpha b_\alpha|W_1) = \begin{cases} (0,2,k), & \alpha = 1, \\ (\alpha, 3-\alpha, k-\alpha+2), & 2 \leqslant \alpha \leqslant 3, \\ (\alpha, \alpha-2, k-\alpha+2), & 4 \leqslant \alpha \leqslant k, \\ (n-\alpha+1, \alpha-2, k-\alpha+2), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha+1, n-\alpha+3, \alpha-k-1), & k+3 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(a_\alpha b_{\alpha+1}|W_1) = \begin{cases} (1,2,k), & \alpha = 1, \\ (\alpha, \alpha-2, k-\alpha+2), & 2 \leqslant \alpha \leqslant k, \\ (n-\alpha+1, \alpha-2, \alpha-k-1), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha+1, n-\alpha+3, \alpha-k-1), & k+3 \leqslant \alpha \leqslant n-1, \\ (0,3,k-1), & \alpha = n. \end{cases}$$

From above representation we see that $r(a_\alpha b_\alpha|W_1) = r(a_\alpha b_{\alpha+1}|W_1)$ when $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1, 2, 3, k+1, k+2, n$ and no other edges have same representation. If we take odd α, where $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1, 3, k+2$ such that $W_E = W_1 \cup \{b_\alpha\}$ then $r(a_\alpha b_\alpha|W_E) \neq r(a_\alpha b_{\alpha+1}|W_E)$ which implies that $\text{edim}\,(f_{n\times 3}) \leqslant \lceil \frac{n}{2} \rceil$. So from Lemma 2.1, W_E is an edge metric generator for $f_{n\times 3}$ and $\text{edim}(f_{n\times 3}) = \lceil \frac{n}{2} \rceil$.

Let $W_E = \{b_1, b_3, b_5, \dots, b_{n-2}, b_n\}$. We will show that W_E is an edge metric basis of $f_{n\times 3}$ in Cases (III) and (IV), respectively.

Case (III) When $n \equiv 1 \pmod 4$. We can write $n = 2k+1$, $k \geqslant 4$, and k is even. Let $W_1 = \{b_1, b_3, b_{k+1}, b_{k+3}, b_{k+5}\}$. Next, we give representation of edges of $f_{n\times 3}$ with respect to W_1.

$$r(a_\alpha a_{\alpha+1}|W_1) = \begin{cases} (1,1,k-1,k,k-2), & \alpha = 1, \\ (\alpha,1,k-\alpha,k-\alpha+2,\alpha+k-3), & 2 \leqslant \alpha \leqslant 3, \\ (\alpha,\alpha-2,k-\alpha,k-\alpha+2,k-\alpha+4), & 4 \leqslant \alpha \leqslant k-1, \\ (k,k-2,1,2,4), & \alpha = k, \\ (n-\alpha,\alpha-2,\alpha-k,1,k-\alpha+4), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha,n-\alpha+2,\alpha-k,\alpha-k-2,1), & k+3 \leqslant \alpha \leqslant k+4, \\ (n-\alpha,n-\alpha+2,\alpha-k,\alpha-k-2,\alpha-k-4), & k+5 \leqslant \alpha \leqslant n-1, \\ (1,2,k,k-1,k-3), & \alpha = n, \end{cases}$$

$$r(a_\alpha b_\alpha|W_1) = \begin{cases} (0,2,k,k,k-2), & \alpha = 1, \\ (\alpha,3-\alpha,k-\alpha+1,k-\alpha+3,\alpha+k-3), & 2 \leqslant \alpha \leqslant 3, \\ (\alpha,\alpha-2,k-\alpha+1,k-\alpha+3,k-\alpha+5), & 4 \leqslant \alpha \leqslant k+1, \\ (n-\alpha+1,\alpha-2,\alpha-k,k-\alpha+3,k-\alpha+5), & k+2 \leqslant \alpha \leqslant k+3, \\ (n-\alpha+1,n-\alpha+3,\alpha-k,\alpha-k-2,k-\alpha+5), & k+4 \leqslant \alpha \leqslant k+5, \\ (n-\alpha+1,n-\alpha+3,\alpha-k,\alpha-k-2,\alpha-k-4), & k+6 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(a_\alpha b_{\alpha+1}|W_1) = \begin{cases} (1,2,k,k,k-2), & \alpha = 1, \\ (\alpha,\alpha-2,k-\alpha+1,k-\alpha+3,\alpha+k-3), & 2 \leqslant \alpha \leqslant 3, \\ (\alpha,\alpha-2,k-\alpha+1,k-\alpha+3,k-\alpha+5), & 4 \leqslant \alpha \leqslant k-1, \\ (\alpha,\alpha-2,\alpha-k,k-\alpha+3,k-\alpha+5), & k \leqslant \alpha \leqslant k+1, \\ (n-\alpha+1,\alpha-2,\alpha-k,\alpha-k-2,k-\alpha+5), & k+2 \leqslant \alpha \leqslant k+3, \\ (n-\alpha+1,n-\alpha+3,\alpha-k,\alpha-k-2,\alpha-k-4), & k+4 \leqslant \alpha \leqslant n-1, \\ (0,3,k+1,k-1,k-3), & \alpha = n. \end{cases}$$

From above representation we see that $r(a_\alpha b_\alpha|W_1) = r(a_\alpha b_{\alpha+1}|W_1)$ when $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1,2,3,k,k+1,k+2,k+3,k+4,k+5,n$ and no other edges have same representation. If we take odd α, where $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1,3,k+1,k+3,n-1$ such that $W_E = W_1 \cup \{b_\alpha\}$ then $r(a_\alpha b_\alpha|W_E) \neq r(a_\alpha b_{\alpha+1}|W_E)$ which implies that $\mathrm{edim}(f_{n\times 3}) \leqslant \lceil \frac{n}{2} \rceil$. So from Lemma 2.1, W_E is an edge metric generator for $f_{n\times 3}$ and $\mathrm{edim}(f_{n\times 3}) = \lceil \frac{n}{2} \rceil$.

Case (IV) When $n \equiv 3 \pmod 4$. We can write $n = 2k+1$, $k \geqslant 3$, k is odd. Let $W_1 = \{b_1, b_3, b_{k+2}, b_{k+4}\}$. Next, we give representation of edges of $f_{n\times 3}$ with respect to W_1.

$$r(a_\alpha a_{\alpha+1}|W_1) = \begin{cases} (\alpha,1,k-\alpha+1,\alpha+k-2), & 1 \leqslant \alpha \leqslant 2, \\ (\alpha,\alpha-2,k-\alpha+1,k-\alpha+3), & 3 \leqslant \alpha \leqslant k-1, \\ (k,\alpha-2,1,k-\alpha+3), & k \leqslant \alpha \leqslant k+1, \\ (n-\alpha,k,\alpha-k-1,1), & k+2 \leqslant \alpha \leqslant k+3, \\ (n-\alpha,n-\alpha+2,\alpha-k-1,\alpha-k-3), & k+4 \leqslant \alpha \leqslant n-1, \\ (1,2,n-k-1,n-k-3), & \alpha = n, \end{cases}$$

$$r(a_\alpha b_\alpha|W_1)=\begin{cases}(0,2,k+1,n-k-2), & \alpha=1,\\ (\alpha,3-\alpha,k-\alpha+2,n-k+\alpha-3), & 2\leqslant\alpha\leqslant 3,\\ (\alpha,\alpha-2,k-\alpha+2,k-\alpha+4), & 4\leqslant\alpha\leqslant k+1,\\ (n-k-1,k,0,2), & \alpha=k+2,\\ (n-\alpha+1,n-\alpha+3,\alpha-k-1,k-\alpha+4), & k+3\leqslant\alpha\leqslant k+4,\\ (n-\alpha+1,n-\alpha+3,\alpha-k-1,\alpha-k-3), & k+5\leqslant\alpha\leqslant n,\end{cases}$$

$$r(a_\alpha b_{\alpha+1}|W_1)=\begin{cases}(1,2,k+1,n-k-2), & \alpha=1,\\ (2,0,k,n-k-1), & \alpha=2,\\ (\alpha,\alpha-2,k-\alpha+2,k-\alpha+4), & 3\leqslant\alpha\leqslant k,\\ (n-\alpha+1,\alpha-2,\alpha-k-1,k-\alpha+4), & k+1\leqslant\alpha\leqslant k+2,\\ (n-\alpha+1,n-\alpha+3,\alpha-k-1,\alpha-k-3), & k+3\leqslant\alpha\leqslant n-1,\\ (0,3,n-k-1,n-k-3), & \alpha=n.\end{cases}$$

From above representation we see that $r(a_\alpha b_\alpha|W_1)=r(a_\alpha b_{\alpha+1}|W_1)$ when $1\leqslant\alpha\leqslant n$ and $\alpha\neq 1,2,3,k+1,k+2,k+3,k+4,n$ and no other edges have same representation. If we take odd α, where $1\leqslant\alpha\leqslant n$ and $\alpha\neq 1,3,k+2,k+4$ such that $W_E=W_1\cup\{b_\alpha\}$ then $r(a_\alpha b_\alpha|W_E)\neq r(a_\alpha b_{\alpha+1}|W_E)$ which implies that $\mathrm{edim}(f_{n\times 3})\leqslant\lceil\frac{n}{2}\rceil$. So from Lemma 2.1, W_E is an edge metric generator for $f_{n\times 3}$ and $\mathrm{edim}(f_{n\times 3})=\lceil\frac{n}{2}\rceil$. □

3. Edge metric dimension of $D_n^{'}$

The prism related graph $D_n^{'}$ has vertex set $V(D_n^{'})=\{a_\alpha,b_\alpha,c_\alpha,d_\alpha,e_\alpha|1\leqslant\alpha\leqslant n\}$ and the edge set $E(D_n^{'})=\{a_\alpha a_{\alpha+1},a_\alpha b_\alpha,c_\alpha b_\alpha,c_\alpha b_{\alpha+1},c_\alpha c_{\alpha+1},c_\alpha d_\alpha,d_\alpha d_{\alpha+1},d_\alpha e_\alpha|1\leqslant\alpha\leqslant n\}$ as shown in Figure 2. In this Section, we determine the edge metric of the graph $D_n^{'}$.

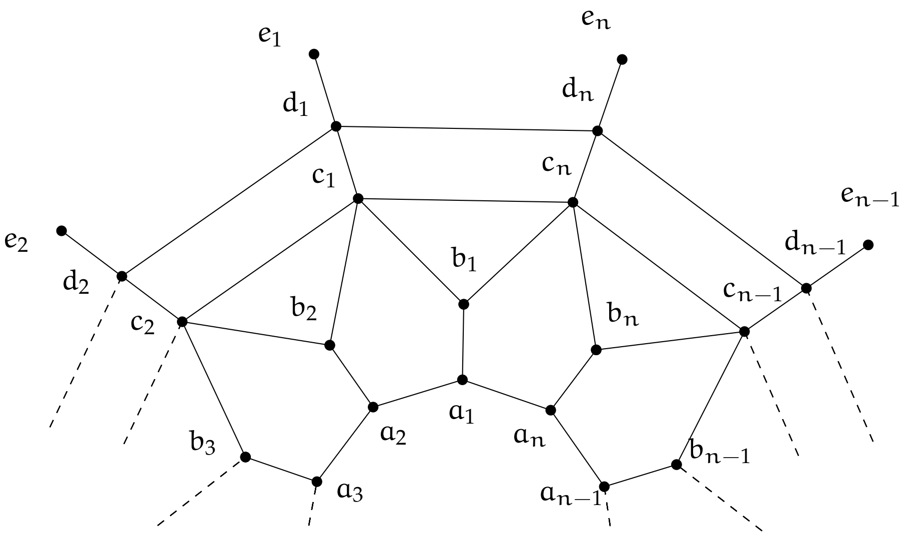

Figure 2: Graph of $D_n^{'}$.

Theorem 3.1. *For the graph $D_n^{'}$ with $n\geqslant 4$, we have*

$$\mathrm{edim}\left(D_n^{'}\right)=\begin{cases}3, & n \text{ is even and } n\neq 4,\\ 4, & n \text{ is odd and } n=4.\end{cases}$$

Proof. For $n = 4$, we have calculated the edge metric dimension by total enumeration, $\mathrm{edim}(D_4') = 4$ and its edge metric basis is $\{a_1, a_2, b_3, b_4\}$.

For $n > 4$, we discuss the two cases.

Case (i) When n is even. We can write $n=2k$, for $k \geqslant 3$. Let $W_E=\{a_1, e_{k-1}, e_{n-1}\}$ is an edge metric basis for D_n'. We give representations of any edge of $E(D_n')$ with respect to W_E.

$$r(a_\alpha a_{\alpha+1}|W_E) = \begin{cases} (\alpha-1, k-\alpha+2, \alpha+4), & 1 \leqslant \alpha \leqslant k-2, \\ (\alpha-1, 4, n-\alpha+2), & k-1 \leqslant \alpha \leqslant k, \\ (n-\alpha, \alpha-k+4, n-\alpha+2), & k+1 \leqslant \alpha \leqslant n-2, \\ (n-\alpha, n-\alpha+k+2, 4), & n-1 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(a_\alpha b_\alpha|W_E) = \begin{cases} (\alpha-1, k-\alpha+2, \alpha+3), & 1 \leqslant \alpha \leqslant k-1, \\ (k-1, 3, n-k+2), & \alpha = k, \\ (n-\alpha+1, \alpha-k+3, n-\alpha+2), & k+1 \leqslant \alpha \leqslant n-1, \\ (1, k+2, 3), & \alpha = n, \end{cases}$$

$$r(b_\alpha c_\alpha|W_E) = \begin{cases} (\alpha, k-\alpha+1, \alpha+3), & 1 \leqslant \alpha \leqslant k-1, \\ (k, 3, n-k+1), & \alpha = k, \\ (n-\alpha+2, \alpha-k+3, n-\alpha+1), & k+1 \leqslant \alpha \leqslant n-1, \\ (2, k+1, 3), & \alpha = n, \end{cases}$$

$$r(c_\alpha b_{\alpha+1}|W_E) = \begin{cases} (\alpha+1, k-\alpha+1, \alpha+3), & 1 \leqslant \alpha \leqslant k-1, \\ (k+1, 3, n-k+1), & \alpha = k, \\ (n-\alpha+1, \alpha-k+3, n-\alpha+1), & k+1 \leqslant \alpha \leqslant n-1, \\ (1, k+1, 3), & \alpha = n, \end{cases}$$

$$r(c_\alpha c_{\alpha+1}|W_E) = \begin{cases} (\alpha+1, k-\alpha, \alpha+3), & 1 \leqslant \alpha \leqslant k-2, \\ (\alpha+1, \alpha-k+3, n-\alpha), & k-1 \leqslant \alpha \leqslant k, \\ (n-\alpha+1, \alpha-k+3, n-\alpha), & k+1 \leqslant \alpha \leqslant n-2, \\ (2, n-\alpha+k, \alpha-n+3), & n-1 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(c_\alpha d_\alpha|W_E) = \begin{cases} (\alpha+1, k-\alpha, \alpha+2), & 1 \leqslant \alpha \leqslant k-1, \\ (k+1, 2, n-k), & \alpha = k, \\ (n-\alpha+2, \alpha-k+2, n-\alpha), & k+1 \leqslant \alpha \leqslant n-1, \\ (2, k, 2), & \alpha = n, \end{cases}$$

$$r(d_\alpha e_\alpha|W_E) = \begin{cases} (\alpha+2, k-\alpha, \alpha+2), & 1 \leqslant \alpha \leqslant k-2, \\ (k+1, 0, k+1), & \alpha = k-1, \\ (k+2, 2, n-k), & \alpha = k, \\ (n-\alpha+3, \alpha-k+2, n-\alpha), & k+1 \leqslant \alpha \leqslant n-2, \\ (4, n-k+1, 0), & \alpha = n-1, \\ (3, k, 2), & \alpha = n, \end{cases}$$

$$r(d_\alpha d_{\alpha+1}|W_E) = \begin{cases} (\alpha+2, k-\alpha-1, \alpha+2), & 1 \leqslant \alpha \leqslant k-2, \\ (\alpha+2, \alpha-k+2, n-\alpha-1), & k-1 \leqslant \alpha \leqslant k, \\ (n-\alpha+2, \alpha-k+2, n-\alpha-1), & k+1 \leqslant \alpha \leqslant n-2, \\ (3, n-\alpha+k-1, \alpha-n+2), & n-1 \leqslant \alpha \leqslant n. \end{cases}$$

We note that there are no vertices having the same edge metric representation implying that $edim({D_n}') \leqslant 3$. Using Proposition 1.1, $edim({D_n}') \geqslant 3$, which implies $edim({D_n}') = 3$.

Case (ii) When n is odd. We can write $n = 2k+1$, for $k \geqslant 2$. Let $W_E = \{a_1, a_{k+2}, b_2, b_{k+1}\}$ is an edge metric basis for D'_n. We give representations of any edge with respect to W_E.

$$r(a_\alpha a_{\alpha+1}|W_E) = \begin{cases} (0,k,1,k), & \alpha = 1, \\ (\alpha-1, k-\alpha+1, \alpha-1, k-\alpha+1), & 2 \leqslant \alpha \leqslant k, \\ (n-k-1, 0, k, 1), & \alpha = k+1, \\ (n-\alpha, \alpha-k-2, n-\alpha+2, \alpha-k), & k+2 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(a_\alpha b_\alpha|W_E) = \begin{cases} (0,k,2,k+1), & \alpha = 1, \\ (1,k,0,k), & \alpha = 2, \\ (\alpha-1, k-\alpha+2, \alpha-1, k-\alpha+2), & 3 \leqslant \alpha \leqslant k, \\ (k,1,k,0), & \alpha = k+1, \\ (n-k-1, 0, k+1, 2), & \alpha = k+2, \\ (n-\alpha+1, \alpha-k-2, n-\alpha+3, \alpha-k), & k+3 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(b_\alpha c_\alpha|W_E) = \begin{cases} (1,k+1,1,k), & \alpha = 1, \\ (2,k+1,0,k-1), & \alpha = 2, \\ (\alpha, k-\alpha+3, \alpha-1, k-\alpha+1), & 3 \leqslant \alpha \leqslant k+1, \\ (n-\alpha+2, \alpha-k-1, n-\alpha+2, \alpha-k), & k+2 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(c_\alpha b_{\alpha+1}|W_E) = \begin{cases} (\alpha+1, k-\alpha+2, \alpha-1, k-\alpha+1), & 1 \leqslant \alpha \leqslant k-1, \\ (k+1, 2, k-1, 0), & \alpha = k, \\ (k+2, 1, k, 1), & \alpha = k+1, \\ (n-\alpha+1, \alpha-k, n-\alpha+2, \alpha-k), & k+2 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(c_\alpha c_{\alpha+1}|W_E) = \begin{cases} (2,k+1,1,k-1), & \alpha = 1, \\ (\alpha+1, k-\alpha+2, \alpha-1, k-\alpha), & 2 \leqslant \alpha \leqslant k-1, \\ (k+1, 2, \alpha-1, 1), & k \leqslant \alpha \leqslant k+1, \\ (n-\alpha+1, \alpha-k, n-\alpha+1, \alpha-k), & k+2 \leqslant \alpha \leqslant n-1, \\ (2,k+1,1,k), & \alpha = n, \end{cases}$$

$$r(c_\alpha d_\alpha|W_E) = \begin{cases} (2,k+2,1,k), & \alpha = 1, \\ (\alpha+1, k-\alpha+3, \alpha-1, k-\alpha+1), & 2 \leqslant \alpha \leqslant k, \\ (k+2, 2, k, 1), & \alpha = k+1, \\ (n-\alpha+2, \alpha-k, n-\alpha+2, \alpha-k), & k+2 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(d_\alpha d_{\alpha+1}|W_E) = \begin{cases} (3,k+2,2,k), & \alpha = 1, \\ (\alpha+2, k-\alpha+3, \alpha, k-\alpha+1), & 2 \leqslant \alpha \leqslant k-1, \\ (k+2, 3, \alpha, 2), & k \leqslant \alpha \leqslant k+1, \\ (n-\alpha+2, \alpha-k+1, n-\alpha+2, \alpha-k+1), & k+2 \leqslant \alpha \leqslant n-1, \\ (3,k+2,2,k+1), & \alpha = n, \end{cases}$$

$$r(d_\alpha e_\alpha|W_E) = \begin{cases} (3,k+3,2,k+1), & \alpha = 1, \\ (\alpha+2, k-\alpha+4, \alpha, k-\alpha+2), & 2 \leqslant \alpha \leqslant k, \\ (k+3, 3, k+1, 2), & \alpha = k+1, \\ (n-\alpha+3, \alpha-k+1, n-\alpha+3, \alpha-k+1), & k+2 \leqslant \alpha \leqslant n. \end{cases}$$

We note that there are no vertices having the same edge metric representation implying that $edim({D_n}') \leqslant 4$.

Suppose on contrary that $edim({D_n}') = 3$, then the Table 1 shows all order pairs of edges (e, f) for which $r(e|W_E) = r(f|W_E)$.

Table 1: (e, f) for which $r(e|W_E) = r(f|W_E)$.

W_E	(e, f)
$\{a_1, a_j, a_l\}$ where $2 \leqslant j \leqslant k+1$ and $3 \leqslant l \leqslant n$	For $3 \leqslant l \leqslant k+1$, $r(a_1a_n|W_E) = r(a_1b_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(b_1c_1|W_E) = r(b_1c_n|W_E)$.
$\{b_1, b_j, b_l\}$ where $2 \leqslant j \leqslant k+1$ and $3 \leqslant l \leqslant n$	For $3 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_n|W_E) = r(d_ne_n|W_E)$.
$\{c_1, c_j, c_l\}$ where $2 \leqslant j \leqslant k+1$ and $3 \leqslant l \leqslant n$	For $3 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_2|W_E) = r(d_1e_1|W_E)$.
$\{d_1, d_j, d_l\}$ where $2 \leqslant j \leqslant k+1$ and $3 \leqslant l \leqslant n$	This case is same to case (3) due to symmetry of the graph.
$\{e_1, e_j, e_l\}$ where $2 \leqslant j \leqslant k+1$ and $3 \leqslant l \leqslant n$	For $3 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(c_1d_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_2|W_E) = r(c_1d_1|W_E)$.
$\{a_1, a_j, b_l\}$ where $2 \leqslant j \leqslant k+1$ and $1 \leqslant l \leqslant n$	For $1 \leqslant l \leqslant k+1$, $r(e_kd_k|W_E) = r(d_kd_{k+1}|W_E)$. For $k+2 \leqslant l \leqslant n-1$, $r(b_2c_2|W_E) = r(c_1b_2|W_E)$. And for $l = n$, $r(c_{k+1}d_{k+1}|W_E) = r(d_kd_{k+1}|W_E)$.
$\{a_1, a_j, c_l\}$ where $2 \leqslant j \leqslant k+1$ and $1 \leqslant l \leqslant n$	For $1 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_2|W_E) = r(d_1e_1|W_E)$.
$\{a_1, a_j, e_l\}$ where $2 \leqslant j \leqslant k+1$ and $1 \leqslant l \leqslant n$	For $1 \leqslant l \leqslant k$, $r(d_{n-1}d_n|W_E) = r(d_ne_n|W_E)$. For $l = k+1$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_2|W_E) = r(d_1e_1|W_E)$.
$\{a_1, b_j, b_l\}$ where $1 \leqslant j \leqslant k+1$ and $2 \leqslant l \leqslant n$	For $2 \leqslant l \leqslant k$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. For $k+2 \leqslant l \leqslant n-1$, $r(c_1b_2|W_E) = r(b_2c_2|W_E)$. And for $l = n$, $r(d_1d_2|W_E) = r(d_1e_1|W_E)$.
$\{a_1, b_j, c_l\}$ where1 $\leqslant j \leqslant k+1$ and $1 \leqslant l \leqslant n$	For $1 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_2|W_E) = r(d_1e_1|W_E)$.
$\{a_1, b_j, e_l\}$ where $1 \leqslant j \leqslant k+1$ and $1 \leqslant l \leqslant n$	For $1 \leqslant l \leqslant k$, $r(d_{n-1}d_n|W_E) = r(d_ne_n|W_E)$. For $l = k+1$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_2|W_E) = r(d_1e_1|W_E)$.
$\{a_1, c_j, c_l\}$ where $1 \leqslant j \leqslant k+1$ and $2 \leqslant l \leqslant n$	For $2 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_2|W_E) = r(d_1e_1|W_E)$.
$\{a_1, c_j, d_l\}$ where $1 \leqslant j \leqslant k+1$ and $1 \leqslant l \leqslant n$	For $1 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(d_1e_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_1d_2|W_E) = r(d_1e_1|W_E)$.
$\{a_1, e_j, e_l\}$ where $1 \leqslant j \leqslant k+1$ and $2 \leqslant l \leqslant n$	For $2 \leqslant l \leqslant k$, $r(d_nd_{n-1}|W_E) = r(d_ne_n|W_E)$. For $l = k+1$, $r(d_{n-1}d_n|W_E) = r(c_2d_2|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_{k+1}c_{k+1}|W_E) = r(c_{k+1}b_{k+2}|W_E)$.
$\{b_1, e_j, e_l\}$ where $1 \leqslant j \leqslant k+1$ and $2 \leqslant l \leqslant n$	For $2 \leqslant l \leqslant k$, $r(d_nd_{n-1}|W_E) = r(d_ne_n|W_E)$. And for $k+1 \leqslant l \leqslant n$, $r(d_2e_2|W_E) = r(c_2d_2|W_E)$.
$\{d_1, d_j, e_l\}$ where $2 \leqslant j \leqslant k+1$ and $1 \leqslant l \leqslant n$	For $1 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(c_1d_1|W_E)$. And for $k+2 \leqslant l \leqslant n$, $r(d_2e_2|W_E) = r(c_2d_2|W_E)$.
$\{d_1, e_j, e_l\}$ where $1 \leqslant j \leqslant k+1$ and $2 \leqslant l \leqslant n$	For $2 \leqslant l \leqslant k+1$, $r(d_1d_n|W_E) = r(c_1d_1|W_E)$. For $k+2 \leqslant l \leqslant n$, $r(d_2e_2|W_E) = r(c_2d_2|W_E)$.

Hence, Table 1 follows that there is no resolving set with three vertices for $V(D_n{}')$ implying that $edim(D_n{}') = 4$. Which completes the proof. □

4. Edge metric dimension of D_n^t

The prism related graph D_n^t has vertex set $V(D_n^t) = \{a_\alpha, b_\alpha, c_\alpha, d_\alpha : 1 \leqslant \alpha \leqslant n\}$ and the edge set $E(D_n^t) = \{a_\alpha a_{\alpha+1}, a_\alpha b_\alpha, b_\alpha b_{\alpha+1}, b_\alpha c_\alpha, b_\alpha c_{\alpha+1}, c_\alpha d_\alpha : 1 \leqslant \alpha \leqslant n\}$ as shown in Figure 3. In this Section, we determine the edge metric dimension of the graph D_n^t.

Lemma 4.1. *Let* $Y = \{d_1, d_2, \ldots, d_n\}$ *be a subset of* $V(D_n^t)$. *Then any arbitrary edge metric generator* W_E *of* D_n^t *contains at least* $\lceil \frac{n}{2} \rceil$ *vertices.*

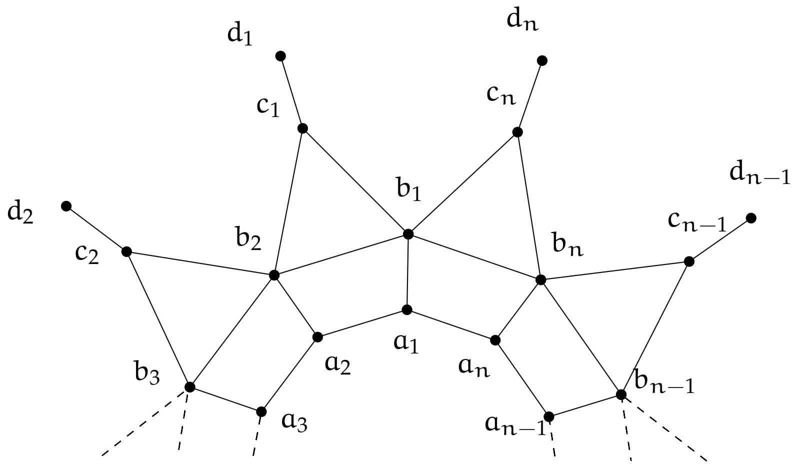

Figure 3: Graph of D_n^t.

Proof. Suppose that W_E contains at most $\lceil \frac{n}{2} \rceil - 1$ vertices of Y for a contradiction. Without loss of generality we assume that $d_\alpha, d_{\alpha+1} \notin W_E$, then we have $(b_\alpha c_\alpha | W_E) = (b_\alpha c_{\alpha+1} | W_E)$, a contradiction. □

Remark 4.2. Let W_E be any edge metric basis of D_n^t. We note that W_E contains all odd vertices (vertex indices are odd) of Y for odd n, while W_E contains either all odd vertices or even vertices (vertex indices are even) of Y for even n.

Lemma 4.3. *For* $n \geqslant 5$, *we have* $\operatorname{edim}\left(D_n^t\right) \geqslant \lceil \frac{n}{2} \rceil + 1$

Proof. We assume for a contradiction that the cardinality of subset W_E is equal to $\lceil \frac{n}{2} \rceil$ by Lemma 4.1. Using Remark 4.2, we take $W_E = \{d_\alpha \in W_E | \text{Vertices indices } \alpha \text{ is odd}\}$ such that $W_E = \lceil \frac{n}{2} \rceil$. We have $(a_\alpha b_\alpha | W_E) = (b_\alpha c_\alpha | W_E)$ for even α $(1 \leqslant \alpha \leqslant n)$, a contradiction. So, $\operatorname{edim}\left(D_n^t\right) \geqslant \lceil \frac{n}{2} \rceil + 1$. □

Theorem 4.4. *For the graph* D_n^t *with* $n \geqslant 3$, *we have*

$$\operatorname{edim}\left(D_n^t\right) = \begin{cases} 4, & n = 3, 4, \\ \lceil \frac{n}{2} \rceil + 1, & \textit{otherwise}. \end{cases}$$

Proof. For $n = 3, 4$ we have calculated the edge metric dimension by total enumeration that is 4 and its edge metric basis is $\{c_1, c_2, c_3, c_4\}$ and $\{a_1, b_3, c_1, c_2\}$, respectively.

For $n \geqslant 5$, we discuss the following four cases.

Let $W_E = \{a_1, d_1, d_3, d_5, \ldots, d_{n-1}\}$. We will show that W_E is an edge metric basis of D_n^t in Case(I) and Case(II), respectively.

Case (I) When $n \equiv 0 \pmod 4$. We can write $n = 2k$, $k \geqslant 4$, and k is even. Let $W_1 = \{a_1, d_1, d_3, d_{k+1}, d_{k+3}\}$. Next, we give representation of edges of D_n^t with respect to W_1.

$$r(a_\alpha a_{\alpha+1} | W_1) = \begin{cases} (\alpha-1, \alpha+2, 3, k-\alpha+2, \alpha+k), & 1 \leqslant \alpha \leqslant 2, \\ (\alpha-1, \alpha+2, \alpha, k-\alpha+2, k-\alpha+4), & 3 \leqslant \alpha \leqslant k-1, \\ (k-1, k+2, k, 3, 4), & \alpha = k, \\ (n-\alpha, n-\alpha+2, \alpha, \alpha-k+2, 3), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha, n-\alpha+2, n-\alpha+4, \alpha-k+2, \alpha-k), & k+3 \leqslant \alpha \leqslant n-1, \\ (0, 3, 4, k+2, k), & \alpha = n, \end{cases}$$

$$r(a_\alpha b_\alpha | W_1) = \begin{cases} (\alpha-1, \alpha+1, 4-\alpha, k-\alpha+2, \alpha+k-1), & 1 \leqslant \alpha \leqslant 2, \\ (\alpha-1, \alpha+1, \alpha-1, k-\alpha+2, k-\alpha+4), & 3 \leqslant \alpha \leqslant k, \\ (n-\alpha+1, n-\alpha+2, \alpha-1, \alpha-k+1, k-\alpha+4), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha+1, n-\alpha+2, n-\alpha+4, \alpha-k+1, \alpha-k-1), & k+3 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(b_\alpha b_{\alpha+1}|W_1)=\begin{cases}(\alpha,\alpha+1,2,k-\alpha+1,\alpha+k-1), & 1\leqslant\alpha\leqslant 2,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+1,k-\alpha+3), & 3\leqslant\alpha\leqslant k-1,\\ (k,k+1,k-1,2,3), & \alpha=k,\\ (n-\alpha+1,n-\alpha+1,\alpha-1,\alpha-k+1,2), & k+1\leqslant\alpha\leqslant k+2,\\ (n-\alpha+1,n-\alpha+1,n-\alpha+3,\alpha-k+1,\alpha-k-1), & k+3\leqslant\alpha\leqslant n-1,\\ (1,2,3,k+1,k-1), & \alpha=n,\end{cases}$$

$$r(b_\alpha c_\alpha|W_1)=\begin{cases}(1,1,3,k+1,k), & \alpha=1,\\ (\alpha,\alpha+1,4-\alpha,k-\alpha+2,k+1), & 2\leqslant\alpha\leqslant 3,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+2,k-\alpha+4), & 4\leqslant\alpha\leqslant k,\\ (n-k+1,n-k+1,k,1,3), & \alpha=k+1,\\ (n-k,n-k,k+1,3,2), & \alpha=k+2,\\ (n-k-1,n-k-1,n-k+1,4,1), & \alpha=k+3,\\ (n-\alpha+2,n-\alpha+2,n-\alpha+4,\alpha-k+1,\alpha-k-1), & k+4\leqslant\alpha\leqslant n,\end{cases}$$

$$r(b_\alpha c_{\alpha+1}|W_1)=\begin{cases}(1,2,3,k+1,k), & \alpha=1,\\ (2,3,1,k,k+1), & \alpha=2,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+2,k-\alpha+4), & 3\leqslant\alpha\leqslant k-1,\\ (k,k+1,k-1,1,4), & \alpha=k,\\ (n-k+1,n-k+1,k,2,3), & \alpha=k+1,\\ (n-k,n-k,k+1,3,1), & \alpha=k+2,\\ (n-\alpha+2,n-\alpha+2,n-\alpha+4,\alpha-k+1,\alpha-k-1), & k+3\leqslant\alpha\leqslant n-1,\\ (2,1,4,k+1,k-1), & \alpha=n,\end{cases}$$

$$r(c_\alpha d_\alpha|W_1)=\begin{cases}(2,0,4,k+2,k), & \alpha=1,\\ (2,3,3,k+1,k+1), & \alpha=2,\\ (3,4,0,k,k+2), & \alpha=3,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+3,k-\alpha+5), & 4\leqslant\alpha\leqslant k,\\ (k+1,k+2,k,0,4), & \alpha=k+1,\\ (n-k+1,n-k+1,k+1,3,3), & \alpha=k+2,\\ (n-k,n-k,n-k+2,4,0), & \alpha=k+3,\\ (n-\alpha+3,n-\alpha+3,n-\alpha+5,\alpha-k+1,\alpha-k-1), & k+4\leqslant\alpha\leqslant n.\end{cases}$$

From above representation we see that $r(b_\alpha c_\alpha|W_1)=r(b_\alpha c_{\alpha+1}|W_1)$ when $1\leqslant\alpha\leqslant n$ and $\alpha\neq 1,2,3,k,k+1,k+2,k+3,n$ and no other edges have same representation. If we take odd α, where $1\leqslant\alpha\leqslant n$ and $\alpha\neq 1,3,k+1,n-1$ such that $W_E=W_1\cup\{d_\alpha\}$ then $r(b_\alpha c_\alpha|W_E)\neq r(b_\alpha c_{\alpha+1}|W_E)$ which implies that $\text{edim}\left(D_n^t\right)\leqslant\lceil\frac{n}{2}\rceil+1$. So from Lemma 4.3 W_E is an edge metric basis for D_n^t and $\text{edim}(D_n^t)=\lceil\frac{n}{2}\rceil+1$.

Case (II) When $n\equiv 2 \pmod 4$. We can write $n=2k$, $k\geqslant 3$, and k is odd. Let $W_1=\{a_1,d_1,d_3,d_{k+2}\}$. Next, we give representation of edges of D_n^t with respect to W_1.

$$r(a_\alpha a_{\alpha+1}|W_1)=\begin{cases}(\alpha-1,\alpha+2,3,k-\alpha+3), & 1\leqslant\alpha\leqslant 2,\\ (\alpha-1,\alpha+2,\alpha,k-\alpha+3), & 3\leqslant\alpha\leqslant k,\\ (k-1,k+1,k+1,3), & \alpha=k+1,\\ (n-\alpha,n-\alpha+2,n-\alpha+4,\alpha-k+1), & k+2\leqslant\alpha\leqslant n-1,\\ (0,3,4,k+1), & \alpha=n,\end{cases}$$

$$r(a_\alpha b_\alpha|W_1)=\begin{cases}(\alpha-1,\alpha+1,4-\alpha,k+1), & 1\leqslant\alpha\leqslant 2,\\ (\alpha-1,\alpha+1,\alpha-1,k-\alpha+3), & 3\leqslant\alpha\leqslant k,\\ (n-\alpha+1,n-\alpha+2,\alpha-1,2), & k+1\leqslant\alpha\leqslant k+2,\\ (n-\alpha+1,n-\alpha+2,n-\alpha+4,\alpha-k), & k+3\leqslant\alpha\leqslant n,\end{cases}$$

$$r(b_\alpha b_{\alpha+1}|W_1) = \begin{cases} (\alpha, \alpha+1, 2, k-\alpha+2), & 1 \leqslant \alpha \leqslant 2, \\ (\alpha, \alpha+1, \alpha-1, k-\alpha+2), & 3 \leqslant \alpha \leqslant k, \\ (k, k, k, 2), & \alpha = k+1, \\ (n-\alpha+1, n-\alpha+1, n-\alpha+3, \alpha-k), & k+2 \leqslant \alpha \leqslant n-1, \\ (1, 2, 3, k), & \alpha = n, \end{cases}$$

$$r(b_\alpha c_\alpha|W_1) = \begin{cases} (1, 1, 3, k+1), & \alpha = 1, \\ (\alpha, \alpha+1, 4-\alpha, k-\alpha+3), & 2 \leqslant \alpha \leqslant 3, \\ (\alpha, \alpha+1, \alpha-1, k-\alpha+3), & 4 \leqslant \alpha \leqslant k, \\ (n-\alpha+2, n-\alpha+2, \alpha-1, k-\alpha+3), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha+2, n-\alpha+2, n-\alpha+4, \alpha-k), & k+3 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(b_\alpha c_{\alpha+1}|W_1) = \begin{cases} (1, 2, 3, k+1), & \alpha = 1, \\ (\alpha, \alpha+1, \alpha-1, k-\alpha+3), & 2 \leqslant \alpha \leqslant k, \\ (n-\alpha+2, n-\alpha+2, \alpha-1, \alpha-k), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha+2, n-\alpha+2, n-\alpha+4, \alpha-k), & k+3 \leqslant \alpha \leqslant n-1, \\ (2, 1, 4, k), & \alpha = n, \end{cases}$$

$$r(c_\alpha d_\alpha|W_1) = \begin{cases} (2, 0, 4, k+1), & \alpha = 1, \\ (2, 3, 3, k+2), & \alpha = 2, \\ (3, 4, 0, k+1), & \alpha = 3, \\ (\alpha, \alpha+1, \alpha-1, k-\alpha+4), & 4 \leqslant \alpha \leqslant k+1, \\ (n-k+1, n-k+1, k+1, 0), & \alpha = k+2, \\ (n-\alpha+3, n-\alpha+3, n-\alpha+5, \alpha-k), & k+3 \leqslant \alpha \leqslant n. \end{cases}$$

From above representation we see that $r(b_\alpha c_\alpha|W_1) = r(b_\alpha c_{\alpha+1}|W_1)$ when $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1, 2, 3, k+1, k+2, n$ and no other edges have same representation. If we take odd α, where $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1, 3, k+2$ such that $W_E = W_1 \cup \{d_\alpha\}$ then $r(b_\alpha c_\alpha|W_E) \neq r(b_\alpha c_{\alpha+1}|W_E)$ which implies that edim $(D_n^t) \leqslant \lceil \frac{n}{2} \rceil + 1$. So from Lemma 4.3 W_E is an edge metric basis for D_n^t and edim$(D_n^t) = \lceil \frac{n}{2} \rceil + 1$.

Let $W_E = \{a_1, d_1, d_3, d_5, \ldots, d_{n-2}, d_n\}$. We will show that W_E is an edge metric basis of D_n^t in Cases (III) and (IV), respectively.

Case (III) When $n \equiv 1$ (mod 4). We can write $n = 2k+1$, $k \geqslant 2$, and k is even. Let $W_1 = \{a_1, d_1, d_3, d_{k+3}\}$. Next, we give representation of edges of D_n^t with respect to W_1.

$$r(a_\alpha a_{\alpha+1}|W_1) = \begin{cases} (\alpha-1, \alpha+2, 3, k+2), & 1 \leqslant \alpha \leqslant 2, \\ (\alpha-1, \alpha+2, \alpha, k-\alpha+4), & 3 \leqslant \alpha \leqslant k, \\ (n-\alpha, n-\alpha+2, \alpha, 3), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha, n-\alpha+2, n-\alpha+4, \alpha-k), & k+3 \leqslant \alpha \leqslant n-1, \\ (0, 3, 4, k+1), & \alpha = n, \end{cases}$$

$$r(a_\alpha b_\alpha|W_1) = \begin{cases} (\alpha-1, \alpha+1, 4-\alpha, k+\alpha), & 1 \leqslant \alpha \leqslant 2, \\ (\alpha-1, \alpha+1, \alpha-1, k-\alpha+4), & 3 \leqslant \alpha \leqslant k+1, \\ (k, k+1, k+1, 2), & \alpha = k+2, \\ (n-\alpha+1, n-\alpha+2, n-\alpha+4, \alpha-k-1), & k+3 \leqslant \alpha \leqslant n, \end{cases}$$

$$r(b_\alpha b_{\alpha+1}|W_1) = \begin{cases} (\alpha, \alpha+1, 2, k+1), & 1 \leqslant \alpha \leqslant 2, \\ (\alpha, \alpha+1, \alpha-1, k-\alpha+3), & 3 \leqslant \alpha \leqslant k, \\ (n-\alpha+1, n-\alpha+1, \alpha-1, 2), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha+1, n-\alpha+1, n-\alpha+3, \alpha-k-1), & k+3 \leqslant \alpha \leqslant n-1, \\ (1, 2, 3, k), & \alpha = n, \end{cases}$$

$$r(b_\alpha c_\alpha|W_1)=\begin{cases}(1,1,3,k+1), & \alpha=1,\\ (\alpha,\alpha+1,4-\alpha,k-\alpha+4), & 2\leqslant\alpha\leqslant 3,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+4), & 4\leqslant\alpha\leqslant k+1,\\ (n-\alpha+2,n-\alpha+2,\alpha-1,k-\alpha+4), & k+2\leqslant\alpha\leqslant k+3,\\ (n-\alpha+2,n-\alpha+2,n-\alpha+4,\alpha-k-1), & k+4\leqslant\alpha\leqslant n,\end{cases}$$

$$r(b_\alpha c_{\alpha+1}|W_1)=\begin{cases}(1,2,3,k+1), & \alpha=1,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+4), & 2\leqslant\alpha\leqslant k+1,\\ (n-\alpha+2,n-\alpha+2,\alpha-1,\alpha-k-1), & k+2\leqslant\alpha\leqslant k+3,\\ (n-\alpha+2,n-\alpha+2,n-\alpha+4,\alpha-k-1), & k+4\leqslant\alpha\leqslant n-1,\\ (2,1,4,k), & \alpha=n,\end{cases}$$

$$r(c_\alpha d_\alpha|W_1)=\begin{cases}(2,0,4,k+1), & \alpha=1,\\ (2,3,3,k+2), & \alpha=2,\\ (3,4,0,k+2), & \alpha=3,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+5), & 4\leqslant\alpha\leqslant k+1,\\ (k+2,n-k+1,k+1,3), & \alpha=k+2,\\ (n-k,n-k,k+2,0), & \alpha=k+3,\\ (n-\alpha+3,n-\alpha+3,n-\alpha+5,\alpha-k-1), & k+4\leqslant\alpha\leqslant n.\end{cases}$$

From above representation we see that $r(b_\alpha c_\alpha|W_1)=r(b_\alpha c_{\alpha+1}|W_1)$ when $1\leqslant\alpha\leqslant n$ and $\alpha\neq 1,2,3,k+2,k+3,n$ and no other edges have same representation. If we take odd α, where $1\leqslant\alpha\leqslant n$ and $\alpha\neq 1,3,k+3$ such that $W_E=W_1\cup\{d_\alpha\}$ then $r(b_\alpha c_\alpha|W_E)\neq r(b_\alpha c_{\alpha+1}|W_E)$ which implies that $\operatorname{edim}(D_n^t)\leqslant\lceil\frac{n}{2}\rceil+1$. So from Lemma 4.3 W_E is an edge metric basis for D_n^t and $\operatorname{edim}(D_n^t)=\lceil\frac{n}{2}\rceil+1$.

Case (IV) When $n\equiv 3 \pmod 4$. We can write $n=2k+1$, $k\geqslant 3$, and k is odd. Let $W_1=\{a_1,d_1,d_3,d_{k+2},d_{k+4}\}$. Next, we give representation of edges of D_n^t with respect to W_1.

$$r(a_\alpha a_{\alpha+1}|W_1)=\begin{cases}(\alpha-1,\alpha+2,3,k-\alpha+3,\alpha+k), & 1\leqslant\alpha\leqslant 2,\\ (\alpha-1,\alpha+2,\alpha,k-\alpha+3,k-\alpha+5), & 3\leqslant\alpha\leqslant k,\\ (n-\alpha,n-\alpha+2,\alpha,3,k-\alpha+5), & k+1\leqslant\alpha\leqslant k+2,\\ (n-k-3,n-k-1,n-k+1,4,3), & \alpha=k+3,\\ (n-\alpha,n-\alpha+2,n-\alpha+4,\alpha-k+1,\alpha-k-1), & k+4\leqslant\alpha\leqslant n-1,\\ (0,3,4,n-k+1,n-k-1), & \alpha=n,\end{cases}$$

$$r(a_\alpha b_\alpha|W_1)=\begin{cases}(\alpha-1,\alpha+1,4-\alpha,k-\alpha+3,k+\alpha-1), & 1\leqslant\alpha\leqslant 2,\\ (\alpha-1,\alpha+1,\alpha-1,k-\alpha+3,k-\alpha+5), & 3\leqslant\alpha\leqslant k+1,\\ (n-\alpha+1,n-\alpha+2,\alpha-1,\alpha-k,k-\alpha+5), & k+2\leqslant\alpha\leqslant k+3,\\ (n-\alpha+1,n-\alpha+2,n-\alpha+4,\alpha-k,\alpha-k-2), & k+4\leqslant\alpha\leqslant n,\end{cases}$$

$$r(b_\alpha b_{\alpha+1}|W_1)=\begin{cases}(\alpha,\alpha+1,2,k-\alpha+2,\alpha+k-1), & 1\leqslant\alpha\leqslant 2,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+2,k-\alpha+4), & 3\leqslant\alpha\leqslant k,\\ (k+1,n-k,k,2,3), & \alpha=k+1,\\ (n-\alpha+1,n-\alpha+1,k+1,\alpha-k,2), & k+2\leqslant\alpha\leqslant k+3,\\ (n-\alpha+1,n-\alpha+1,n-\alpha+3,\alpha-k,\alpha-k-2), & k+4\leqslant\alpha\leqslant n-1,\\ (1,2,3,n-k,n-k-2), & \alpha=n,\end{cases}$$

$$r(b_\alpha c_\alpha|W_1)=\begin{cases}(1,1,3,k+2,k), & \alpha=1,\\ (\alpha,\alpha+1,4-\alpha,k-\alpha+3,\alpha+k-1), & 2\leqslant\alpha\leqslant 3,\\ (\alpha,\alpha+1,\alpha-1,k-\alpha+3,k-\alpha+5), & 4\leqslant\alpha\leqslant k+1,\\ (n-k,n-k,k+1,1,3), & \alpha=k+2,\\ (n-\alpha+2,n-\alpha+2,n-\alpha+4,\alpha-k,k-\alpha+5), & k+3\leqslant\alpha\leqslant k+4,\\ (n-\alpha+2,n-\alpha+2,n-\alpha+4,\alpha-k,\alpha-k-2), & k+5\leqslant\alpha\leqslant n,\end{cases}$$

$$r(b_\alpha c_{\alpha+1}|W_1) = \begin{cases} (1,2,3,k+2,k), & \alpha = 1, \\ (\alpha,\alpha+1,\alpha-1,k-\alpha+3,k+\alpha-1), & 2 \leqslant \alpha \leqslant 3, \\ (\alpha,\alpha+1,\alpha-1,k-\alpha+3,k-\alpha+5), & 4 \leqslant \alpha \leqslant k, \\ (k+1,n-\alpha+2,\alpha-1,\alpha-k,k-\alpha+5), & k+1 \leqslant \alpha \leqslant k+2, \\ (n-\alpha+2,n-\alpha+2,n-\alpha+4,\alpha-k,\alpha-k-2), & k+3 \leqslant \alpha \leqslant n-1, \\ (2,1,4,k+1,k-1), & \alpha = n, \end{cases}$$

$$r(c_\alpha d_\alpha|W_1) = \begin{cases} (2,0,4,k+2,k), & \alpha = 1, \\ (2,3,3,k+2,k+1), & \alpha = 2, \\ (3,4,0,k+1,k+2), & \alpha = 3, \\ (\alpha,\alpha+1,\alpha-1,k-\alpha+4,k-\alpha+6), & 4 \leqslant \alpha \leqslant k+1, \\ (n-k+1,n-k+1,k+1,0,4), & \alpha = k+2, \\ (n-k,n-k,k+2,3,3), & \alpha = k+3, \\ (n-k-1,n-k-1,n-k+1,4,0), & \alpha = k+4, \\ (n-\alpha+3,n-\alpha+3,n-\alpha+5,\alpha-k,\alpha-k-2), & k+5 \leqslant \alpha \leqslant n. \end{cases}$$

From above representation we see that $r(b_\alpha c_\alpha|W_1) = r(b_\alpha c_{\alpha+1}|W_1)$ when $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1,2,3,k+1,k+2,k+3,k+4,n$ and no other edges have same representation. If we take odd α, where $1 \leqslant \alpha \leqslant n$ and $\alpha \neq 1,3,k+2,k+4$ such that $W_E = W_1 \cup \{d_\alpha\}$ then $r(b_\alpha c_\alpha|W_E) \neq r(b_\alpha c_{\alpha+1}|W_E)$ which implies that $\mathrm{edim}\left(D_n^t\right) \leqslant \lceil \frac{n}{2} \rceil + 1$. So from Lemma 4.3 W_E is an edge metric basis for D_n^t and $\mathrm{edim}(D_n^t) = \lceil \frac{n}{2} \rceil + 1$. □

5. Conclusion

In this paper, we have computed the exact value of the edge metric dimension of flower graph $f_{n\times3}$, the prism related graphs $D_n^{'}$ and D_n^t. It has been observed that the edge metric dimension of these graphs is greater than the metric dimension and we concluded that the edge metric dimension of $D_n^{'}$ is constant while $f_{n\times3}$ and D_n^t have unbounded edge metric dimensions.

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