J. Math. Computer Sci., 24 (2022), 1–9

Online: ISSN 2008-949X

Journal of Mathematics and Computer Science

Journal Homepage: www.isr-publications.com/jmcs

Weak prime L–fuzzy filters of semilattices

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Abstract

The concept of weak prime L–fuzzy filter of a semilattice S is introduced and example are given. A characterization of weak prime L–fuzzy filters is established and prime filters of S are identified with weak prime L–fuzzy filters. Also, minimal weak prime L–fuzzy filters are characterized.

Keywords: Bounded semilattice, L–fuzzy filter, prime L–fuzzy filter, weak prime L–fuzzy filter, frame.

2020 MSC: 06D72, 06F15, 08A72.

1. Introduction

Zadeh, in his pioneering work [11] introduced the notion of a fuzzy subset A of a non-empty set X as a function from X into $[0, 1]$. Rosenfield [6] applied this notion to develop the theory of groups. Goguen [1] generalized and continued the work of Zadeh and realized that the unit interval $[0, 1]$ is not sufficient to take the truth values of general fuzzy statements. Therewith, several researchers took interest to the fuzzyfication of algebraic structures. In which, Kuroki [2], Liu [3], Malik and Mordersan [4], and Mukherjee and Sen [5] are engaged in fuzzifying various concepts and obtained significant results of algebras.

Further, Swamy and Swamy [10] have introduced the concept of a fuzzy prime ideal of a ring and developed the theory of fuzzy ideals by assuming truth values in a complete lattice L satisfying the infinite meet distributive law, such lattices are called frames. The concept of prime ideal is vital in the study of structure theory of distributive lattices. In [8], the authors have introduced and studied the notion of L–fuzzy filters of a semilattice S with truth values in a frame L. It is proved that S is distributive iff the lattice $\mathcal{F}(S)$ of all filters of S is distributive iff the lattice $\mathcal{F}_L(F(S))$ of all L–fuzzy filters of S is distributive. In [9], the authors have introduced the concept of prime L–fuzzy filters of a bounded semilattice S, which are meet-prime elements in the lattice $\mathcal{F}_L(\mathcal{F}(S))$. Further, in [7] the authors have introduced the notion of L–fuzzy ideals of a semilattice S and obtained significant results on this.

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doi: 10.22436/jmcs.024.01.01

Received: 2020-10-10 *Revised:* 2020-11-06 *Accepted:* 2020-11-11

The aim of this paper is to study the L–fuzzy filters A of a bounded semilattice S for which each α-cut A_α i.e., $A_\alpha = \{x \in S : A(x) \geqslant \alpha\}$ is either a prime filter of S or whole semilattice S. This paper consists of four sections. In the second section we recall some definitions and certain results. In third section we introduce the concept of a weak prime L–fuzzy filter (WPLF) of a bounded semilattice S and characrerize these. Fourth section deals with minimal weak prime L–fuzzy filters (Minimal WPLFs).

Throughout this paper, S stands for a bounded semilattice $(S, \wedge, 0, 1)$ unless otherwise stated. And, L stands for a non-trivial frame $(L, \wedge, \vee, 0, 1)$; i.e., a complete lattice satisfying the infinite meet distributive law

$$\alpha \wedge \Big(\bigvee_{\beta \in T} \beta\Big) = \bigvee_{\beta \in T} (\alpha \wedge \beta),$$

for all $\alpha \in L$ and any $T \subseteq L$. Here the operations $\wedge$ and $\vee$ are supremum and infimum in the lattice L. An element $1 \neq c \in L$ is said to be meet-prime if, for any $a, b \in L$ and $a \wedge b \leqslant c$ imply $a \leqslant c$ or $b \leqslant c$.

2. Preliminaries

In this section we collect basic definitions and certain results from [8, 9], that we need in sequel.

A semilattice (meet-semilattice) is an algebra $S = (S, \wedge)$ satisfying the axioms

(1) $x \wedge x = x$;

(2) $x \wedge y = y \wedge x$; and

(3) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$, for all $x, y, z \in S$.

If we define $x \leqslant y$ iff $x \wedge y = x$, then $\leqslant$ is a partial order on S in which $x \wedge y$ is the $\inf\{x, y\}$ in S. A non-empty subset F of S is said to be final segment of S if, for any $x \in F, y \in S$ and $x \leqslant y$ implies $y \in F$. A filter of a semilattice S is a final segment F of S such that $x \wedge y \in F$ for all $x, y \in F$. The principal filter generated by an element a of S, i.e., the set $\{x \in S : x \geqslant a\}$ will be denoted by $[a)$. A proper filter P of a semilattice S is said to be prime if whenever two filters G and H are such that $\phi \neq G \cap H \subseteq P$ imply either $G \subseteq P$ or $H \subseteq P$ (or equivalently, if, for any a, b are such that $a \notin P$ and $b \notin P$ imply the existence of $x \in S$ such that $a \leqslant x, b \leqslant x$ and $x \notin P$).

Definition 2.1. Let X be any non-empty set and L a frame. Any function $A : X \to L$ is called an L–fuzzy subset of X. For any L–fuzzy subset A of X and $\alpha \in L$, A_α denotes α-cut of A, i.e.,

$$A_\alpha = \{x \in X : \alpha \leqslant A(x)\}.$$

Definition 2.2. For any L–fuzzy subsets A and B of X, define

$$A \leqslant B \Leftrightarrow A(x) \leqslant B(x), \quad \text{for all } x \in X.$$

Then $\leqslant$ is a partial order on the set of L–fuzzy subsets of X and is called the point wise ordering.

Result 1. Let A and B be L–fuzzy subsets of X. Then

$$A \leqslant B \Leftrightarrow A_\alpha \subseteq B_\alpha, \quad \text{for all } \alpha \in L.$$

Definition 2.3. A proper L–fuzzy subset A of X is a non-constant L–fuzzy subset of X, i.e., $A(x) \neq 1$ for some $x \in X$.

Definition 2.4. An L–fuzzy subset A of S is said to be an L–fuzzy filter of S if,

$$A(x_0) = 1, \quad \text{for some } x_0 \in S,$$

and

$$A(x \wedge y) = A(x) \wedge A(y), \quad \text{for all } x, y \in S.$$

Result 2. The following are equivalent to each other, for any L–fuzzy subset A of S,

(1) A is an L–fuzzy filter of S.

(2) $A(x_0) = 1$ for some $x_0 \in S$, $A(x \wedge y) \geqslant A(x) \wedge A(y)$ and $x \leqslant y \Rightarrow A(y) \geqslant A(x)$.

(3) A_α is a filter of S, for all $\alpha \in L$.

Result 3. Let A be a fuzzy filter of S and X a non-empty subset of S, and $x, y \in S$. We have

(1) $x \in [X) \Rightarrow A(x) \geqslant \bigwedge_{i=1}^{m} A(a_i)$ for some $a_1, a_2, \cdots a_m \in X$, where

$$[X) = \{a \in S : \bigwedge_{i=1}^{n} x_i \leqslant a \text{ for some } x_i \in X\}.$$

(2) $x \in [y) \Rightarrow A(x) \geqslant A(y)$.

(3) If S is bounded then $A(0) < 1$ and $A(1) = 1$.

Result 4. Let $(S, \wedge)$ be a bounded semilattice and $\mathcal{F}_L(F(S))$ denote the lattice all L–fuzzy filters of S. Then the following are equivalent to each other:

(1) $\mathcal{F}_L(\mathcal{F}(S))$ is a distributive.

(2) $F(S)$ is a distributive, where $F(S)$ denotes the lattice of filters of S.

(3) S is distributive.

Definition 2.5. A proper L–fuzzy filter A of a bounded semilattice S is said to be prime L–fuzzy filter of S if, for any L–fuzzy filters B and C of S,

$$B \wedge C \leqslant A \Rightarrow B \leqslant A \text{ or } C \leqslant A,$$

where $(B \wedge C)(x) = B(x) \wedge C(x)$.

Result 5. Let A be an L–fuzzy fileter of S. Then A is prime L–fuzzy filter of S if and only if, the following are satisfied.

(1) $| Im(A) |= 2$, i.e., A is two-valued.

(2) For any $x \in S$, either $A(x) = 1$ or $A(x)$ is meet-prime element in L.

(3) A_1 is a prime filter of S.

Result 6. Let A be an L–fuzzy filter of S. Then A is a prime L–fuzzy filter of S iff there exists a prime filter P of S and a meet-prime element α in L such that $A = A^P_\alpha$, where

$$A^P_\alpha(x) = \begin{cases} 1 & \text{if } x \in P, \\ \alpha & \text{if } x \notin P. \end{cases}$$

3. Weak prime L–Fuzzy filters (WPLF)

Let us recall that an L–fuzzy subset A of S is an L–fuzzy filter of S iff A_α is a filter of S for each $\alpha \in L$.

Definition 3.1. A proper L–fuzzy filter A of S is called a weak prime L–fuzzy filter (WPLF), if for each $\alpha \in L$, A_α is a prime filter of S or $A_\alpha = S$.

Example 3.2. Consider the semilattice S whose Hasse-diagram is as depicted in Figure 1 and $L = [0, 1]$, the closed interval of real numbers which is a frame in which, for any $x, y \in L$,

$$x \vee y = \max\{x, y\},\ x \wedge y = \min\{x, y\}.$$

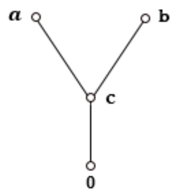

Figure 1: Hasse-diagram of Semilattice S.

Clearly $\{a\}, \{b\}$ and $\{a, b, c\}$ are all prime filters of S. Now, define $A : S \to L$ as follows:

$$A = \{(0, 0), (c, 0.5), (b, 0.5), (a, 1)\}.$$

Then A is a WPLF; since, the α-cuts of A are

$$\begin{aligned} A_0 &= S, \\ A_1 &= \{a\}, \\ A_{0.5} &= \{a, b, c\}, \\ A_\alpha &= \{a\}, \quad \text{for any } \alpha \in (0.5, 1), \end{aligned}$$

and

$$A_\alpha = \{a, b, c\}, \quad \text{for any } \alpha \in (0, 0.5).$$

Theorem 3.3. *Let A be a proper L–fuzzy filter of S. If A is a WPLF of S, then $\mathrm{Im}(A)$ is a chain.*

Proof. Let a and $b \in S$ and put $\alpha = A(a) \vee A(b)$. Then,

$$\begin{aligned} x \in [a) \cap [b) &\Rightarrow a \leqslant x \text{ and } b \leqslant x \\ &\Rightarrow A(a) \leqslant A(x) \text{ and } A(b) \leqslant A(x) \\ &\Rightarrow \alpha = A(a) \vee A(b) \leqslant A(x) \\ &\Rightarrow x \in A_\alpha. \end{aligned}$$

Therefore $[a) \cap [b) \subseteq A_\alpha$. Since A_α is prime, $[a) \subseteq A_\alpha$ or $[b) \subseteq A_\alpha$.

$$\begin{aligned} [a) \subseteq A_\alpha \Rightarrow a \in A_\alpha &\Rightarrow \alpha = A(a) \vee A(b) \leqslant A(a) \\ &\Rightarrow A(b) \leqslant A(a). \end{aligned}$$

Similarly, $[b) \subseteq A_\alpha \Rightarrow A(a) \leqslant A(b)$. Thus $\mathrm{Im}(A)$ is a chain in L. □

The converse of above theorem is not true. For, consider the following example.

Example 3.4. Consider two lattices S and L whose Hasse-diagrams are given in Figure 2 and Figure 3 respectively, where $S = \{0, c, a, b, 1\}$ and $L = \{0, s, 1\}$.

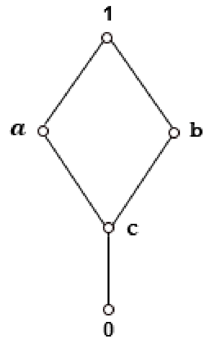

Figure 2: Hasse-diagram of lattice S.

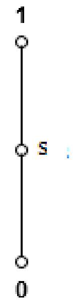

Figure 3: Hasse-diagram of lattice L.

Clearly for any L-fuzzy filter A of S, $\mathrm{Im}(A)$ is a chain. Define $A : S \to L$ as

$$A = \{(0,0),(c,s),(a,s),(b,s),(1,1)\}.$$

Then the α-cuts of A are $A_0 = S$, $A_s = \{c,a,b,1\}$ and $A_1 = \{1\}$, which are filters of S. Therefore A is an L–fuzzy filter of S. However A is not WPLF because A_1 is not prime since $[a) \cap [b) = \{1\}$.

The following gives a characterization of WPLFs.

Theorem 3.5. *For any* L*–fuzzy filter* A *of* S*, the following are equivalent:*

(1) A *is a WPLF of* S*.*

(2) *For any* a *and* $b \in S$*,*

$$\bigwedge \{A(x) : x \in [a) \cap [b)\} = A(a) \text{ or } A(b).$$

(3) *For any* a *and* $b \in S$*,*

$$\bigwedge \{A(x) : x \in [a) \cap [b)\} = A(a) \vee A(b),$$

and

$$\mathrm{Im}(A) \text{ is a chain in } L.$$

Proof. First note that for any a and $b \in S$,

$$A(a) \text{ and } A(b) \leqslant \bigwedge \{A(x) : x \in [a) \cap [b)\}.$$

$(1) \Rightarrow (2)$: Let a and $b \in S$ and put $\alpha = \bigwedge\{A(x) : x \in [a) \cap [b)\}$. Then $\alpha \leqslant A(x)$ for all $x \in [a) \cap [b)$, so that $[a) \cap [b) \subseteq A_\alpha$. By (1) A_α is a prime filter of S and hence $[a) \subseteq A_\alpha$ or $[b) \subseteq A_\alpha$. So that $a \in A_\alpha$ or $b \in A_\alpha$, i.e., $\alpha \leqslant A(a)$ or $\alpha \leqslant A(b)$. This implise $\alpha = A(a)$ or $A(b)$.

$(2) \Rightarrow (3)$: Let a and $b \in S$. Then, by (2),

$$\bigwedge \{A(x) : x \in [a) \cap [b)\} = A(a) \text{ or } A(b),$$

and hence $A(b) \leqslant A(a)$ or $A(a) \leqslant A(b)$. Therefore $\mathrm{Im}(A)$ is a chain in L. Also, by (2) and since $A(a),\ A(b)$ are lower bounds of $\{A(x) : x \in [a) \cap [b)\}$, it follows that

$$\bigwedge \{A(x) : x \in [a) \cap [b)\} = \max\{A(a), A(b)\} = A(a) \vee A(b).$$

(3) $\Rightarrow$ (1): Let $\alpha \in L$. Such that $A_\alpha \neq S$. Let G and H be two filters of S such that $G \nsubseteq A_\alpha$ and $H \nsubseteq A_\alpha$. Then, $\alpha \nleq A(a)$ and $\alpha \nleq A(b)$ for some $a, b \in S$. By (3), $A(a) \leqslant A(b)$ or $A(b) \leqslant A(a)$. Hence,

$$\alpha \nleq \max\{A(a), A(b)\} = A(a) \vee A(b).$$

Also, by (3),

$$\alpha \nleq \bigwedge \left\{A(x) : x \in [a) \cap [b)\right\}.$$

Hence $\alpha \nleq A(x)$ for some $x \in [a) \cap [b)$. This implise $G \cap H \nsubseteq A_\alpha$. Hence A_α is a prime filter of S. Thus A is a WPLF of S. □

Now, we slightly generalize an α-level L–fuzzy filter A^F_α corresponding to a filter F (see Result 6).

Definition 3.6. For any filter F of S and $\alpha, \beta \in L$, define an L–fuzzy subset $A^F_{\alpha,\beta}$ of S as follows:

$$A^F_{\alpha,\beta}(x) = \begin{cases} 1 & \text{if } x = 1, \\ \alpha & \text{if } 1 \neq x \in F, \\ \beta & \text{if } x \notin F. \end{cases}$$

Note that $A^F_{1,\beta} = A^F_\beta$ and $A^F_{1,0} = \chi_F$, the characteristic function corresponding to F.

The following is straight forward verification.

Lemma 3.7. *Let F be a proper filter of S and $\alpha, \beta \in L$. Then*

$$A^F_{\alpha,\beta} \textit{ is an } L\textit{–fuzzy filter of } S \quad \textit{iff} \quad \beta \leqslant \alpha,$$

and, in the case,

$$A^F_{\alpha,\beta} \textit{ is proper} \quad \textit{iff} \quad \beta < 1.$$

Theorem 3.8. *For any proper filter P of S, the following are equivalent:*

(1) P is a prime filter of S.

(2) $A^P_{1,\beta}$ is a WPLF of S for each $\beta < 1$.

(3) χ_P is a WPLF of S.

Proof. (1) $\Rightarrow$ (2): Suppose P is prime and let $\beta < 1$ in L. Put $A = A^P_{1,\beta}$. Then,

$$A(x) = \begin{cases} 1 & \text{if } x \in P, \\ \beta & \text{if } x \notin P. \end{cases}$$

Let a and $b \in S$. Then,

$$\begin{aligned} a \in P \text{ or } b \in P &\Rightarrow A(a) = 1 \text{ or } A(b) = 1 \text{ and } [a) \cap [b) \subseteq P \\ &\Rightarrow A(a) = 1 \text{ or } A(b) = 1 \text{ and } A(x) = 1 \text{ for all } x \in [a) \cap [b) \\ &\Rightarrow \bigwedge \left\{A(x) : x \in [a) \cap [b)\right\} = 1 = A(a) \text{ or } A(b). \\ a \notin P \text{ and } b \notin P &\Rightarrow A(a) = \beta = A(b) \text{ and } [a) \cap [b) \nsubseteq P \\ &\Rightarrow A(a) = \beta = A(b) \text{ and } x \notin P \text{ for some } x \in [a) \cap [b) \\ &\Rightarrow \bigwedge \left\{A(x) : x \in [a) \cap [b)\right\} = \beta = A(a) = A(b). \end{aligned}$$

Therefore

$$\bigwedge\{A(x) : x \in [a) \cap [b)\} = A(a) \text{ or } A(b).$$

Thus A is WPLF.

(2) ⇒ (3): It is clear by the fact that $\chi_P = A^P_{1,0}$.

(3) ⇒ (1): Suppose χ_P is WPLF. Let a and $b \in S$ such that $a \notin P$ and $b \notin P$. Then $\chi_P(a) = 0 = \chi_P(b)$. By supposition and hence by Theorem 3.5,

$$\bigwedge\{\chi_P(x) : x \in [a) \cap [b)\} = \chi_P(a) \text{ or } \chi_P(b).$$

So that

$$\bigwedge\{\chi_P(x) : x \in [a) \cap [b)\} = 0.$$

Hence $\chi_P(x) = 0$ for some $x \in [a) \cap [b)$. (for, $\chi_P(x) = 1$ for all $x \in [a) \cap [b) \Rightarrow \chi_P(a) = 1$ or $\chi_P(b) = 1$; a contradiction). Therefore $x \notin P$. So that $[a) \cap [b) \nsubseteq P$. Thus P is prime. □

Lemma 3.9. *For any bounded semilattice S, the following are equivalent:*

(1) [1) is a meet-prime element in the lattice $\mathcal{F}(S)$ of all filters of S.

(2) For any $1 \neq a$ and $1 \neq b \in S$, there exists $1 \neq c \in S$ such that $c \geqslant a$ and b, i.e., $c \in [a) \cap [b)$.

Theorem 3.10. *Le P be a proper filter of S and suppose that [1) is a meet-prime element in the lattice $\mathcal{F}(S)$ of filters of S. Then P is prime iff $A^P_{\alpha,\beta}$ is WPLF for all $1 \neq \beta \leqslant \alpha$ in L.*

Proof. Suppose P is prime and $1 \neq \beta \leqslant \alpha \in L$. Put $A = A^P_{\alpha,\beta}$. Then A is a proper L–fuzzy filter of S (by Lemma 3.7). Let a and $b \in S$. Then $A(a)$ and $A(b) \leqslant A(x)$ for all $x \in [a) \cap [b)$. Let $\gamma \in L$ such that $\gamma \leqslant A(x)$ for all $x \in [a) \cap [b)$. Now,

$$\begin{aligned}
a = 1 \text{ or } b = 1 &\Rightarrow A(a) = 1 \text{ or } A(b) \text{ and hence } \mathcal{V} \leqslant A(a) = 1 \text{ or } A(b)\\
&\Rightarrow \bigwedge\{A(x) : x \in [a) \cap [b)\} = A(a) \text{ or } A(b)\\
a \notin P \text{ and } b \notin P &\Rightarrow A(a) = \beta = A(b) \text{ and } [a) \cap [b) \nsubseteq P\\
&\Rightarrow A(a) = \beta = A(b) \text{ and } A(x) = \beta \text{ for some } x \in [a) \cap [b)\\
&\Rightarrow \gamma \leqslant A(x) = \beta = A(a) = A(b)\\
&\Rightarrow \bigwedge\{A(x) : x \in [a) \cap [b)\} = A(a) = A(b),
\end{aligned}$$

and

$$\begin{aligned}
1 \neq a \in P, 1 \neq b \in P &\Rightarrow A(a) = \alpha = A(b) \text{ and there exsits } 1 \neq c \in S\\
&\qquad \text{suchthat } c \in [a) \cap [b) \subseteq P\\
&\Rightarrow \gamma \leqslant A(c) = \alpha = A(a) = A(b)\\
&\Rightarrow \bigwedge\{A(x) : x \in [a) \cap [b)\} = A(a) = A(b).
\end{aligned}$$

Thus, by Theorem 3.5, A is WPLF. □

Finally in this section we discuss an inter-relationship between prime L–fuzzy filters (refer Result 6) and WPLFs.

Theorem 3.11. *Every prime L–fuzzy filter of S is WPLF.*

Proof. Let B be a Prime L–fuzzy filter of S. Then, $B = A^P_\alpha$ for some prime filter P of S and a meet-prime element α in L. Since P is prime and $\alpha < 1$, we have A^P_α is a WPLF of S (by Theorem 3.8). Thus B is WPLF. □

The converse of the above theorem is true. For, consider the example given in the following.

Example 3.12. Let S be the 5-element lattice $\{0, b, c, a, 1\}$ represented by the Hasse-diagram given below Figure 4 and L be the 3-element chain $\{0, s, 1\}$ with $0 < s < 1$.

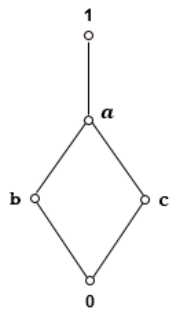

Figure 4: Hasse-diagram of 5-element lattice S.

Define $A : S \to L$ by $A = \{(0, 0), (b, s), (c, 0), (a, s), (1, 1)\}$. Then A is a proper L–fuzzy filter of S. Here the α-cuts of A are $A_0 = S$, $A_s = \{b, a, 1\}$ and $A_1 = \{1\}$, which are prime filters of S. Hence A is WPLF. But A is not Prime L–fuzzy filter since A is not two-valued.

4. Minimal WPLF

By a minimal prime filter M of S, we mean that there is no prime filter Q of S such that $Q \subset M$ and analogously, a minimal WPLF is a minimal element in the set of all WPLFs under the point-wise partial ordering.

Theorem 4.1. *Let A be a WPLF of S. If A is a minimal WPLF of S, then A_1, i.e., 1-cut of A is a minimal prime filter of S.*

Proof. Suppose that A is a minimal WPLF of S. Then $A_1 = \{x \in S : A(x) = 1\}$ is a prime filter of S. To prove A_1 is minimal, let Q be a prime ideal of S such that $Q \subset A_1$. Then, choose $x \in A_1$ such that $x \notin Q$. Since Q is prime and hence by Theorem 3.8, χ_Q is a WPLF of S and $\chi_Q(x) < A(x)$. Therefore $\chi_Q \lneqq A$. This shows that A is not minimal; a contradiction. Thus A_1 is a minimal prime filter of S.

Converse of above theorem is not true. For example, in Example 3.2, A is an WPLF and $A_1 = \{a\}$ which is a minimal prime filter of S. But A is not minimal. If we define $B : S \to L$ by $B = \{(0, 0), (c, 0.25), (b, 0.25), (a, 1)\}$, then B is a WPLF of S and $B \lneqq A$. □

Theorem 4.2. *Let A be a WPLF of S and $[1)$ is a meet-prime element in the lattice $\mathcal{F}(S)$ filters of S. Then, A is a minimal WPLF of S iff, A_α is a minimal prime filter of S, for each $\alpha \in L$.*

Proof. Assume that A is a minimal WPLF of S. If A_β is not a minimal prime filter of S for some $0 < \beta < 1$. Then, there exists a prime filter P of S such that $P \subset A_\beta$. Now, define $B : S \to L$ by

$$B(x) = \begin{cases} 1 & \text{if } x = 1, \\ \beta & \text{if } 1 \neq x \in P, \\ 0 & \text{if } x \notin P. \end{cases}$$

Clearly $B = A^P_{\beta,0}$. By Theorem 3.5, B is a WPLF of S. As $P \subset A_\beta$, choose $y \in A_\beta$ such that $y \notin P$. Then,

$$\beta \leqslant A(y), \quad \text{and} \quad B(y) < A(y).$$

Also $B(x) \leqslant A(x)$ for all $x \neq y \in S$. Therefore $B \lneqq A$; a contradiction to our assumption. Thus A_α is a minimal prime filter of S for all $\alpha \in L$.

Conversely, assume that A_α is a minimal prime filter of S for all $\alpha \in L$. If B is a WPLF of S such that $B \leqslant A$. Then, $B_\alpha \subseteq A_\alpha$ for all $\alpha \in L$. By assumption, $A_\alpha = B_\alpha$ for all $\alpha \in L$. Hence $B = A$. Thus A is a minimal WPLF of S. □

Acknowledgment

The authors thanks the editor and anonymous reviewers for their careful reading and useful comments that have resulted in a significant improvement of the manuscript.

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