



Dislocated quasi-metric stability of a multiplicative inverse functional equation



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Abstract

In this study, we employ the dislocated metric space stability result of an equation with one variable function to prove different stabilities of a two variable equation involving rational function in the codomain of complete dislocated quasi-metric spaces. We also extend the stabilities by taking different upper bounds.

Keywords: Functional equation, multiplicative inverse functional equation, Ulam stability, dislocated quasi-metric space.

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1. Introduction and preliminaries

The role of dislocated metric spaces is very significant in cybercrime and cryptography for protecting data and information. In particular, the applications of these spaces are of great importance in elliptic curve cryptography [35]. Hence, this raises many more applications in information and communication technologies and other domains of computing. Let us revocate few essential ideas of dislocated metric spaces.

Let B be a non-empty set with a metric $d : B \times B \rightarrow \mathbb{R}^+ \cup \{0\}$, which satisfies the subsequent conditions:

- (i) $d(\nu, \xi) = d(\xi, \nu) = 0 \implies \nu = \xi$;
- (ii) $d(\nu, \xi) \leq d(\nu, \tau) + d(\tau, \xi)$

for all $\nu, \xi, \tau \in B$. The mapping d defined with the set B is called a dislocated quasi-metric (in short dq-metric) and the pair (B, d) is said to be a dislocated quasi-metric space (in short dq-metric space) (Refer [37, 49]). The pair (B, d) is called a metric-like space if the dq-metric d defined in B satisfies the symmetric condition, that is, $d(\nu, \xi) = d(\xi, \nu)$ for $\nu, \xi \in B$ (see [6]).

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Example 1.1. A mapping $d : \mathbb{R}^2 \rightarrow \mathbb{R}^+ \cup \{0\}$ defined via $d(v, \xi) = |v|$ for $v, \xi \in \mathbb{R}$ is a dislocated quasi-metric.

Example 1.2. A mapping $d : \mathbb{R}^2 \rightarrow \mathbb{R}^+ \cup \{0\} = \max\{|v|, |\xi|\}$ for $v, \xi \in \mathbb{R}$, then the pair (\mathbb{R}, d) is a metric-like space.

Suppose (B, d) is a dq-metric space and suppose $\{v_m\}$ is a sequence of elements in B . Then the limit of the sequence $\{v_m\}$ converges to $v \in B$ if and only if $d(v_m, v) \rightarrow 0$ and $d(v, v_m) \rightarrow 0$ and it can be verified that this limit if it exists then it is unique. If the sequence $\{v_m\}$ converges to v , then we can write it as $v_m \rightarrow v$. Precisely, suppose there are two different limits $\alpha, \beta \in B$ of the sequence $\{v_m\}$, then we have $0 \leq d(v, \xi) \leq d(v, \xi_m) + d(v_m, \xi)$. This implies that $d(v, \xi) = 0$ and analogously, we have $d(\xi, v) = 0$, and hence we get $v = \xi$.

Let $\{v_m\}$ be a sequence. Then it is said to be Cauchy if for all $\epsilon > 0$, there exists $M \in \mathbb{N}$, for all $r, s \geq M$, $d(v_s, v_r) < \epsilon$. If every Cauchy sequence is convergent in a dq-metric space (B, d) , then it is said to be complete.

The classical stability result of a one variable functional equation is obtained in the following theorem. The proof of this theorem is available in [17]. This theorem plays a crucial key to prove our major results. The following notations are used in the theorem. Let (B, d) be a dq-metric space with completeness, L be a non-empty set, $\Phi : B \rightarrow B$, $g : L \rightarrow L$, and $f_k : L \rightarrow \mathbb{R}^+ \cup \{0\}$ be mappings. Suppose the mapping Φ satisfies the Lipschitz condition $d(\Phi(r), \Phi(s)) \leq \mu d(r, s)$ for all $r, s \in B$ and for $k = 1, 2$, where $\mu > 0$ is a Lipschitz constant, $F_k(r) = \sum_{k=0}^{\infty} \mu^k f_k(g^k(r)) < \infty$ for every $r \in L$.

Theorem 1.3. Suppose a mapping $h : L \rightarrow B$ satisfies the following inequalities

$$d(\Phi \circ h \circ g(r), h(r)) \leq f_1(r), \quad r \in L, \quad d(h(r), \Phi \circ h \circ g(r)) \leq f_2(r), \quad r \in L.$$

Then a mapping $H : L \rightarrow B$ exists defined by the existence of the limit $H(r) = \lim_{k \rightarrow \infty} \Phi^k \circ h \circ g^k(r)$ for all $r \in L$ and satisfies the equation $\Phi \circ H \circ g = H$ such that

$$d(H(r), h(r)) \leq F_1(r), \quad r \in L, \quad d(h(r), H(r)) \leq F_2(r), \quad r \in L.$$

The most active research area in the domain of analysis is stability theory. The investigation of validity of stabilities of functional equations has become the crucial question for various research problems and a number of motivating and novel results are published by many mathematicians. The vital question raised in [56] became the backbone for the evolution of stability theory of equations. This fundamental question is concerned with determining approximate group homomorphisms. The question can be stated as “When an approximate homomorphism exists near to the given homomorphism?” This question triggered to find the solution and it was first responded in [20] by considering fundamental Cauchy additive equation $\chi(v + \xi) = \chi(v) + \chi(\xi)$ in the framework of Banach spaces. Later on, the results obtained in [20] were dealt in different versions by many mathematicians [7, 10, 15, 16, 38, 40, 57]. A lot of fundamental ideas about functional equations and inequalities are published as books [5, 9, 13, 28, 39, 41, 48]. The stability results of various other forms of functional equations are published in the form of monographs in [21, 24]. There are lot of interesting information regarding stability results available in [11, 12, 14, 22, 23, 29, 31, 45–47]. Quite recently for the past one decade, there are new and interesting stability results of several functional equations are obtained in [3, 4, 25–27, 32–34, 36].

The pioneering multiplicative inverse functional equation in the dynamic research domain of stability theory of equations is

$$h(r + s) = \frac{h(r)h(s)}{h(r) + h(s)}. \quad (1.1)$$

The solution and stability results of equation (1.1) were obtained in [43]. Later on, there are numerous intriguing and noteworthy stability results, applications and interpretations of various forms of multiplicative inverse functional equations are published in [8, 18, 30, 42, 50, 51]. The solutions of these

multiplicative inverse functional equations are of rational form provide exceptional contributions in various disciplines such as to compute the combined resistance of a parallel electric circuit with resistors [44], to study physical properties of objects [52], to interpret them with a significant hypothesis arising in electromagnetic theory and the relation of stiffness and length of diving board [53], to compute the total focal length of combined lenses in optics [54], to interpret through an application in electromagnetism [55].

The considerable contribution of these type functional equations in other spheres and the remarkable results obtained in Theorem 1.3 influenced us to deal with the same approach to the stablesness of equation (1.1). Hence, in this article, by the approach of Theorem 1.3, we prove the stabilities of equation (1.1) where $h : L \rightarrow B$ is a mapping with L as a non-empty set and B as a dq-metric space. We also extend further stability results by taking different upper bounds.

2. Stability of equation (1.1) via the approach of Theorem 1.3

In this section, we apply the approach applied in Theorem 1.3 to prove stability results of equation (1.1). Let us consider $(A, +)$ is a square summetric groupoid which is divisible by 2 distinctively and (B, d) is a dq-metric space satisfying the completeness property with the continuous binary operator $+$. Let $L \subset A$ be a non-empty set such that $r \neq 0, r + s \neq 0$, for all $r, s \in L$. Further assume that $\zeta_1, \zeta_2 : L \times L \rightarrow \mathbb{R}^+ \cup \{0\}$ are mappings. We determine the stability result of equation (1.1) with these assumptions in the ensuing result.

Theorem 2.1. *Suppose $\rho_1, \rho_2, \theta \in (0, \infty)$ exist with the conditions $\rho_1\theta < 1$ and $\rho_2\theta < 1$ and*

$$\zeta_k \left(\frac{r}{2}, \frac{s}{2} \right) \leq \rho_k \zeta_k(r, s), \quad r, s \in L, \quad k = 1, 2, \quad d \left(\frac{1}{2}h(r), \frac{1}{2}h(s) \right) \leq \theta d(h(r), h(s)), \quad r, s \in L.$$

If a mapping $h : L \rightarrow B$ satisfies the following conditions:

$$d \left(\frac{h(r)h(s)}{h(r) + h(s)}, h(r + s) \right) \leq \zeta_1(r, s), \quad d \left(h(r + s), \frac{h(r)h(s)}{h(r) + h(s)} \right) \leq \zeta_2(r, s)$$

for all $r, s \in L, r + s \in L$, then a mapping $H : L \rightarrow B$ exists which is unique, close to h and is a solution of (1.1) satisfying the ensuing approximations

$$d(H(r), h(r)) \leq \frac{\rho_1 \zeta_1(r, r)}{1 - \rho_1 \theta} \quad \text{and} \quad d(h(r), H(r)) \leq \frac{\rho_2 \zeta_2(r, r)}{1 - \rho_2 \theta}$$

for all $r \in L$.

Proof. Replacing (r, s) by $(\frac{r}{2}, \frac{r}{2})$ in the assumption, one can find

$$d \left(\frac{1}{2}h \left(\frac{r}{2} \right), h(r) \right) \leq \zeta_1 \left(\frac{r}{2}, \frac{r}{2} \right)$$

for all $r \in L$, and analogously, one can also obtain

$$d \left(h(r), \frac{1}{2}h \left(\frac{r}{2} \right) \right) \leq \zeta_2 \left(\frac{r}{2}, \frac{r}{2} \right)$$

for all $r \in L$. Now, assuming $\Phi(r) = \frac{r}{2}, \mu = \theta, f_k(r) = \zeta_k \left(\frac{r}{2}, \frac{r}{2} \right), k=1,2$, and $g(r) = \frac{r}{2}$ in Theorem 1.3, then we have a mapping $H : L \rightarrow B$ defined by the existence of the limit $H(r) = \lim_{m \rightarrow \infty} 2^{-m}h(2^{-m}r)$ for all $r \in L$. The mapping H is unique and it satisfies the equation $2^{-1}H(2^{-1}r) = H(r), r \in L$, such that for all $r \in L$, we have

$$d((H(r), h(r)) \leq \sum_{k=0}^{\infty} \theta^k \zeta_1 \left(\frac{r}{2^{k+1}}, \frac{r}{2^{k+1}} \right) \leq \sum_{k=0}^{\infty} \theta^k \rho^k \zeta_1 \left(\frac{r}{2}, \frac{r}{2} \right) \leq \rho_1 \zeta_1(r, r) \sum_{k=0}^{\infty} (\rho_1 \theta)^k = \frac{\rho_1 \zeta_1(r, r)}{1 - \rho_1 \theta}.$$

Similarly, one can come up with for all $r \in L$,

$$d(h(r), H(r)) \leq \frac{\rho_2 \zeta_2(r, r)}{1 - \rho_2 \theta}.$$

Now, for all positive integers m , we find that

$$\begin{aligned} d\left(\frac{2^{-m}h(2^{-m}r)2^{-m}h(2^{-m}s)}{2^{-m}h(2^{-m}r) + 2^{-m}h(2^{-m}s)}, 2^{-m}h(2^{-m}(r+s))\right) &\leq \theta^m d\left(\frac{h(2^{-m}r)h(2^{-m}s)}{h(2^{-m}r) + h(2^{-m}s)}, h(2^{-m}(r+s))\right) \\ &\leq \theta^m \zeta_1(2^{-m}r, 2^{-m}s) \\ &\leq (\rho_1 \theta)^m \zeta_1(r, s) \end{aligned}$$

for all $r, s \in L$. Allowing m to ∞ in the above inequality and since $\rho \theta < 1$, we can observe that H satisfies equation (1.1). By similar approach of the arguments as in Theorem 1.3, it is easy to show that this mapping H is unique, which completes the proof. \square

In the following corollaries, we apply Theorem 2.1 by considering L to be the set of real numbers containing elements $r, s \in \mathbb{R}$ with the conditions $r \neq 0$ and $r + s \neq 0$.

Corollary 2.2. *Suppose that $k \in (0, \infty)$ be a fixed constant and there exists a constant $\theta \in (0, 1)$ such that*

$$d\left(\frac{1}{2}h(r), \frac{1}{2}h(s)\right) \leq \theta d(h(r), h(s)), \quad r, s \in L.$$

Let $h : L \rightarrow B$ be a mapping satisfying the following inequalities

$$d\left(\frac{h(r)h(s)}{h(r) + h(s)}, h(r+s)\right) \leq k \quad \text{and} \quad d\left(h(r+s), \frac{h(r)h(s)}{h(r) + h(s)}\right) \leq k$$

for all $r, s \in L$. Then the equation (1.1) has a unique solution $H : L \rightarrow B$ and this solution H satisfies the following approximations

$$d(H(r), h(r)) \leq \frac{k}{1 - \theta} \quad \text{and} \quad d(h(r), H(r)) \leq \frac{k}{1 - \theta}$$

for all $r \in L$.

Corollary 2.3. *Let $k_1 \in (0, \infty)$ be a fixed constant. Let there exist $\theta, \alpha \in (0, \infty)$ with the condition that $\frac{\theta}{2^\alpha} < 1$ and*

$$d\left(\frac{1}{2}h(r), \frac{1}{2}h(s)\right) \leq \theta d(h(r), h(s)), \quad r, s \in L.$$

Let $h : L \rightarrow B$ be a mapping satisfies the following inequalities

$$d\left(\frac{h(r)h(s)}{h(r) + h(s)}, h(r+s)\right) \leq k_1 (|r|^\alpha + |s|^\alpha) \quad \text{and} \quad d\left(h(r+s), \frac{h(r)h(s)}{h(r) + h(s)}\right) \leq k_1 (|r|^\alpha + |s|^\alpha)$$

for all $r, s \in L$. Then the equation (1.1) has a unique solution $H : L \rightarrow B$ and this solution H satisfies the following approximations

$$d(H(r), h(r)) \leq \frac{2k_1}{2^\alpha - \theta} |r|^\alpha \quad \text{and} \quad d(h(r), H(r)) \leq \frac{2k_1}{2^\alpha - \theta} |r|^\alpha$$

for all $r \in L$.

Corollary 2.4. *Let $k \in (0, \infty)$ be a fixed real constant. Let there exist $\theta, \alpha, b \in (0, \infty)$ with the condition that $c = \alpha + b$ and $\frac{\theta}{2^c} < 1$ and*

$$d\left(\frac{1}{2}h(r), \frac{1}{2}h(s)\right) \leq \theta d(h(r), h(s)), \quad r, s \in L.$$

Let $h : L \rightarrow B$ be a mapping satisfies the following inequalities

$$d\left(\frac{h(r)h(s)}{h(r)+h(s)}, h(r+s)\right) \leq k_1 (|r|^a|s|^b) \quad \text{and} \quad d\left(h(r+s), \frac{h(r)h(s)}{h(r)+h(s)}\right) \leq k_1 (|r|^a|s|^b)$$

for all $r, s \in L$. Then the equation (1.1) has a unique solution $H : L \rightarrow B$ and this solution H satisfies the following approximations

$$d(H(r), h(r)) \leq \frac{k_2}{2^c - \theta} |r|^c \quad \text{and} \quad d(h(r), H(r)) \leq \frac{k_2}{2^c - \theta} |r|^c$$

for all $r \in L$.

3. Discussion of the stability results of equation (1.1) obtained

In Theorem 1.3, the stability results of fundamental Cauchy additive equation are obtained by considering a common control mapping only as an upper bound. But in this study, we have extended the generalized stability by considering different upper bounds in the corollaries by taking L to be \mathbb{R} such that $r \neq 0$ and $r + s \neq 0$ to avoid singularities to achieve the main results.

4. Conclusion

So for in the literature, several functional equations in rational form are dealt to investigate their stabilities in the framework of different spaces. This is our first attempt to consider dq-metric space to analyze the validity of stableness of equation (1.1). The dq-metric spaces are useful in logic programming, theoretical computer science and electronics and hence the stability results obtained in this study can be utilized to approach problems arising in the above fields [1, 2, 19].

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