J. Math. Computer Sci., 24 (2022), 299–307

Online: ISSN 2008-949X

Journal of Mathematics and Computer Science

Journal Homepage: www.isr-publications.com/jmcs

Nörlund statistical convergence and Tauberian conditions for statistical convergence from statistical summability using Nörlund means in non-Archimedean fields

D. Eunice Jemima[a,*], V. Srinivasan[b]

[a] Department of Mathematics, Faculty of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chennai-603203, India.

[b] (Retd. Professor) Department of Mathematics, Faculty of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chennai-603203, India.

Abstract

In this paper, we define the concept of statistical convergence of sequences by Nörlund summability method and obtain a few results on the relationship between Nörlund summability and Nörlund statistical convergence in a complete, non-trivially valued, non-archimedean field K. Also, the necessary and sufficient Tauberian conditions under which statistical convergence follows from statistical summability by Nörlund means over K are discussed.

Keywords: Non-archimedean fields, Nörlund mean, statistical convergence, statistical summability (N, p_n), Tauberian conditions.

2020 MSC: 40A35, 40E05, 40G05, 40G15, 46S10.

1. Introduction

In 1951, Fast [4] introduced the notion of statistical convergence. The relation between summability theory and statistical convergence was brought in by Schoenberg, which was later studied in detail by Fridy [5], Kolk, Freedman, Savas, Fridy and Miller [6], Mursaleen [10], Salat [12], Fridy and Orhan, Cakalli [3] etc. Monna [7] started a systematic study of Functional Analysis over a field other than the Real or Complex fields. A detailed study on the p-Adic numbers and Valuation theory was done by Bachman [1]. Suja and Srinivasan [14] introduced statistical convergence in non-archimedean fields.

Nörlund method of summability in non-archimedean fields was introduced by Srinivasan [13]. Natarajan [11] studied the relation between regular Nörlund methods and Nörlund summability. Braha [2], Fekete, Totur, Canak, Loku, etc. worked on Tauberian theorems using different methods of summability. Moricz [8] established the Tauberian conditions under which statistical convergence follows from statistical summability $(C, 1)$ and also by weighted means along with Orhan [9], in classical analysis. In this

*Corresponding author
Email addresses: eunicejem@gmail.com (D. Eunice Jemima), drvsrinivas.5@gmail.com (V. Srinivasan)

doi: 10.22436/jmcs.024.04.02

Received: 2021-01-07 *Revised:* 2021-02-19 *Accepted:* 2021-02-21

paper, the concept of statistical convergence of sequences by Nörlund summability method (N, p_n) is defined, and a few results on the relation between (N, p_n) summability and (N, p_n) statistical convergence are found. Also, Tauberian conditions for sequences that are statistically summable by Nörlund means over non-archimedean fields are studied.

1.1. Preliminaries

Let K be a complete, non-trivially valued, non-archimedean field. (Recall that a valued field $(K, |.|)$ is non-archimedean if $|a + b| \leqslant \max\{|a|, |b|\}$, for all $a, b \in K$). A sequence $x = (x_k)$, $x_k \in K$, $k = 0, 1, 2, \cdots$ is said to be statistically convergent [14] to a limit 'l' if, for every $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leqslant n : |x_k - l| \geqslant \epsilon\}| = 0,$$

(where the outer vertical bars indicate the cardinality of the set), which we write as

$$st - \lim_{k\to\infty} x_k = l.$$

Let $p = (p_k)$, $k = 0, 1, 2, \cdots$ be a sequence in K such that $p_0 \neq 0$, $|p_0| > |p_j|$, $j = 1, 2, \cdots$ and

$$P_n = \sum_{k=0}^{n} p_k\,, \qquad n = 0, 1, 2, \cdots .$$

It is clear that $|P_n| = |p_0| \neq 0$, so $P_n \neq 0$, $n = 0, 1, 2, \cdots$. Srinivasan [13] introduced the Nörlund method of summability, that is, the (N, p_n) method in K by the infinite matrix $(a_{n,k})$ where

$$a_{n,k} = \begin{cases} \frac{p_{n-k}}{P_n}, & k \leqslant n, \\ 0, & k > n. \end{cases}$$

Definition 1.1. The Nörlund mean (N, p_n) of the sequence $x = (x_n)$ is defined by

$$t_n = \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}x_k, \qquad n = 0, 1, 2, \cdots .$$

Definition 1.2. The sequence (x_k) is said to be statistically (N, p_n) summable to a limit 'l' if

$$st - \lim_{n\to\infty} t_n = l. \tag{1.1}$$

That is,

$$\lim_{M\to\infty} \frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}x_k - l\right| \geqslant \epsilon\right\}\right| = 0.$$

Definition 1.3. A sequence $x = (x_k)$ is said to be Nörlund statistically convergent to l if, for every $\epsilon > 0$

$$\lim_{n\to\infty} \frac{1}{P_n}|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}| = 0.$$

Definition 1.4. A sequence $x = (x_k)$ is said to be (N, p_n) summable to l if,

$$\lim_{n\to\infty} \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}|x_k - l| = 0.$$

Natarajan [11] proved that, if sequence (x_k) is (N, p_n) summable, then (x_k) is bounded, and also proved the necessary and sufficient conditions for a regular (N, p_n) method, stated in the definition below.

Definition 1.5. The (N, p_n) method is regular if and only if $p_n \to 0$ as $n \to \infty$.

In this section, we consider the (N, p_n) method to be regular.

2. New results

Theorem 2.1. *Let* $\frac{P_n}{n} > 1$*, for every* $n \in \mathbb{N}$*. If* (x_k) *is statistically convergent to* l*, then* (x_k) *is statistically* (N, p_n) *convergent to* l*.*

Proof. Given, (x_k) is statistically convergent to l. That is,

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leqslant n : |x_k - l| \geqslant \epsilon\}| = 0. \tag{2.1}$$

To prove (x_k) is statistically (N, p_n) convergent to l, that is to prove

$$\lim_{n\to\infty} \frac{1}{P_n}\left|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}\right| = 0,$$

consider

$$\begin{aligned}\frac{1}{P_n}\left|\{k \leqslant n :\ p_{n-k}|x_k - l| \geqslant \epsilon\}\right| &= \frac{n}{P_n} \times \frac{1}{n}\left|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}\right| \\ &\leqslant \frac{1}{n}\left|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}\right| \qquad \left(\text{since } \frac{n}{P_n} < 1\right) \\ &\to 0, \ \text{ as } \ n \to \infty. \qquad (\text{since } p_n \to 0,\ n \to \infty \text{ and by (2.1)})\end{aligned}$$

Therefore,

$$\lim_{n\to\infty} \frac{1}{P_n}\left|\left\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\right\}\right| = 0,$$

or, (x_k) is statistically (N, p_n) convergent to l. □

The following example is an illustration of this theorem.

Example 2.2. Consider the sequence $x = (x_k)$ defined by

$$x_k = \begin{cases} \frac{k-1}{k^2+1}, & \text{if } k \text{ is a perfect square,} \\ 0, & \text{otherwise.} \end{cases}$$

Choosing the non-archimedean valuation to be 2-adic, the terms of the sequence are

$$(0, 0, 0, 1, 0, 0, 0, 0, \frac{1}{4}, 0, 0, \cdots).$$

This sequence is clearly statistically convergent to 0, since,

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leqslant n : |x_k - 0| \geqslant \epsilon\}| = 0.$$

Let $(p_n) = (3^n)$, $n = 0, 1, 2, \cdots$ be a (N, p_n) method in the 2-adic field Q_2. Then, $(p_n) = (1, 1, 1, \cdots)$. Therefore,

$$\begin{aligned}P_n &= p_0 + p_1 + \cdots + p_n \\ &= 1 + 1 + \cdots + 1 \\ &= |n + 1|_2.\end{aligned}$$

Now,

$$\lim_{n\to\infty} \frac{1}{P_n}|\{k \leqslant n : p_{n-k}|x_k - 0| \geqslant \epsilon\}| = \lim_{n\to\infty} \frac{1}{|n+1|_2}|\{k \leqslant n : 3^{n-k}|x_k| \geqslant \epsilon\}| = 0,$$

which shows that (x_k) is statistically (N, p_n) convergent to 0.

Theorem 2.3. *If the sequence* (P_n) *is bounded such that* $\lim_{n\to\infty}\sup \frac{P_n}{n} < \infty$, *and if* (x_k) *is statistically* (N, p_n) *convergent to* l, *then* (x_k) *is statistically convergent to* l.

Proof. Given, (x_k) is statistically (N, p_n) convergent to l; that is,

$$\lim_{n\to\infty} \frac{1}{P_n}\left|\left\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\right\}\right| = 0. \tag{2.2}$$

To prove (x_k) is statistically convergent to l; that is, to prove

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leqslant n : |x_k - l| \geqslant \epsilon\}| = 0,$$

consider

$$\begin{aligned}
\frac{1}{n}|\{k \leqslant n : |x_k - l| \geqslant \epsilon\}| &\leqslant \frac{1}{n}|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}| \\
&\leqslant \frac{P_n}{n} \times \frac{1}{P_n}|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}| \\
&\leqslant \frac{1}{P_n}|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}| \qquad \left(\text{since } \lim_{n\to\infty}\sup \frac{P_n}{n} < \infty\right) \\
&\to 0 \quad \text{as} \quad n \to \infty \quad (\text{by } (2.2)).
\end{aligned}$$

Thus, (x_k) is statistically convergent to l. □

Theorem 2.4. *If the sequence* (x_k) *is* (N, p_n) *summable to* l, *then* (x_k) *is statistically* (N, p_n) *convergent to* l.

Proof. Given, $\lim_{n\to\infty} t_n = l$. That is,

$$\lim_{n\to\infty} \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}x_k = l,$$

i.e.,

$$\lim_{n\to\infty}(p_n x_0 + p_{n-1}x_1 + \cdots + p_0 x_n) = \lim_{n\to\infty} P_n l = \lim_{n\to\infty}(p_0 + p_1 + \cdots + p_n)l,$$

i.e.,

$$\lim_{n\to\infty}[p_n(x_0 - l) + p_{n-1}(x_1 - l) + \cdots + p_0(x_n - l)] = 0,$$

i.e.,

$$\lim_{n\to\infty}|p_n(x_0 - l) + p_{n-1}(x_1 - l) + \cdots + p_0(x_n - l)| = 0,$$

which implies that

$$\lim_{n\to\infty} \max\{|p_n(x_0 - l)|, |p_{n-1}(x_1 - l)|, \cdots, |p_0(x_n - l)|\} = 0.$$

That is,

$$\lim_{n\to\infty}|p_{n-k}||x_k - l| = 0, \quad k = 0, 1, \cdots, n,$$

implies,

$$\lim_{n\to\infty}|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}| = 0,$$

or,

$$\lim_{n\to\infty} \frac{1}{P_n}\left|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}\right| = 0.$$

This proves that (x_k) is statistically (N, p_n) convergent to l. □

Theorem 2.5. *If (x_k) is statistically (N, p_n) convergent to l, then (x_k) is (N, p_n) summable to l.*

Proof. Given,

$$\lim_{n\to\infty} \frac{1}{P_n}\left|\left\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\right\}\right| = 0. \tag{2.3}$$

Let us assume the contrary that (x_k) is not (N, p_n) summable to l. That is,

$$\lim_{n\to\infty} \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}x_k > l.$$

i.e.,

$$\begin{aligned}\lim_{n\to\infty}(p_n x_0 + p_{n-1}x_1 + \cdots + p_0 x_n) &> \lim_{n\to\infty} P_n l\\ &> \lim_{n\to\infty}(p_0 + p_1 + \cdots + p_n)l,\end{aligned}$$

i.e.,

$$\lim_{n\to\infty}[p_n(x_0 - l) + p_{n-1}(x_1 - l) + \cdots + p_0(x_n - l)] > 0,$$

implies,

$$\lim_{n\to\infty}|p_n(x_0 - l) + p_{n-1}(x_1 - l) + \cdots + p_0(x_n - l)| > 0,$$

which further implies that

$$\lim_{n\to\infty}\max\{|p_n(x_0 - l)|, |p_{n-1}(x_1 - l)|, \cdots, |p_0(x_n - l)|\} > 0,$$

or,

$$\lim_{n\to\infty}|p_{n-k}||x_k - l| > 0, \quad k = 0, 1, \cdots, n.$$

implies,

$$\lim_{n\to\infty}|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}| > 0.$$

Also,

$$\lim_{n\to\infty}\frac{1}{P_n}\left|\{k \leqslant n : p_{n-k}|x_k - l| \geqslant \epsilon\}\right| > 0.$$

But this cannot happen by (2.3). Thus, (x_k) is (N, p_n) summable to l. □

This theorem is illustrated by the following example.

Example 2.6. For the sequence $x = (x_k)$ together with the sequence (p_k) and the 2-adic valuation discussed in the previous example, which is statistically (N, p_n) convergent to 0, we have

$$\lim_{n\to\infty}\frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}|x_k - 0| = \lim_{n\to\infty}\frac{1}{|n+1|_2}\sum_{k=0}^{n} 3^{n-k}|x_k| = 0.$$

Thus it is clear that (x_k) is (N, p_n) summable to 0.

Theorem 2.7. *Let $p = (p_k)$ be a sequence in K such that $p_0 \neq 0$, $|p_0| > |p_j|$, $j = 1, 2, \cdots$. Let (λ_k) be a sequence in K such that $\lim_{k\to\infty}\lambda_k = 0$ and*

$$st - \lim_{n\to\infty}\frac{P_n}{P_{\lambda_n}} < 1, \quad \textit{for every} \quad 0 < \lambda_n < 1. \tag{2.4}$$

Let $x = (x_k)$, $x_k \in K$, $k = 0, 1, 2, \cdots$, be a sequence which is statistically (N, p_n) summable to a limit l. Then (x_k) is statistically convergent to l if and only if for every $\epsilon > 0$,

$$\lim_{M\to\infty}\frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k)\right| \geqslant \epsilon\right\}\right| = 0.$$

The following Lemmas are required in proving the theorem.

Lemma 2.8. *Let* $p = (p_k)$ *be a sequence in* K *such that* $p_0 \neq 0$, $|p_0| > |p_j|$, $j = 1, 2, \cdots$, *and*

$$st - \lim_{n\to\infty} \frac{P_n}{P_{\lambda_n}} < 1, \quad \textit{for every} \quad 0 < \lambda_n < 1,$$

where $\{\lambda_k\}$ *is a sequence in* K *such that* $\lim_{k\to\infty} \lambda_k = 0$. *Let* $x = (x_k)$, $x_k \in K$, $k = 0, 1, 2, \cdots$ *be a sequence which is statistically* (N, p_n) *summable to a limit* l. *Then for every* $0 < \lambda_n < 1$,

$$st - \lim_{n\to\infty} t_{\lambda_n} = l, \tag{2.5}$$

where (P_n) *and* (t_{λ_n}) *are non-decreasing sequences.*

Proof. Given that the sequence (x_n) is statistically (N, p_n) summable to a limit l. This means that

$$st - \lim_{n\to\infty} t_n = l.$$

That is

$$\lim_{M\to\infty} \frac{1}{M} |\{n \leqslant M : |t_n - l| \geqslant \epsilon\}| = 0,$$

or,

$$\lim_{M\to\infty} \frac{1}{M} \left| \left\{ n \leqslant M : \left| \frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} x_k - l \right| \geqslant \epsilon \right\} \right| = 0. \tag{2.6}$$

To prove, $st - \lim_{n\to\infty} t_{\lambda_n} = l$, that is to prove

$$\lim_{M\to\infty} \frac{1}{M} |\{\lambda_n \leqslant M : |t_{\lambda_n} - l| \geqslant \epsilon\}| = 0,$$

(or) to prove

$$\lim_{M\to\infty} \frac{1}{M} \left| \left\{ \lambda_n \leqslant M : \left| \frac{1}{P_{\lambda_n}} \sum_{k=0}^{\lambda_n} p_{\lambda_n - k} x_k - l \right| \geqslant \epsilon \right\} \right| = 0,$$

let us consider

$$\begin{aligned} \frac{1}{M} \left| \left\{ \lambda_n \leqslant M : \left| \frac{1}{P_{\lambda_n}} \sum_{k=0}^{\lambda_n} p_{\lambda_n - k} x_k - l \right| \geqslant \epsilon \right\} \right| &= \frac{1}{M} \left| \left\{ \lambda_n \leqslant M : \left| \left(\frac{P_n}{P_{\lambda_n}} \right) \frac{1}{P_n} \sum_{k=0}^{\lambda_n} p_{\lambda_n - k} x_k - l \right| \geqslant \epsilon \right\} \right| \\ &\leqslant \frac{1}{M} \left| \left\{ n \leqslant M : \left| \frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} x_k - l \right| \geqslant \epsilon \right\} \right| \quad \text{(using (2.4))} \\ &\to 0 \ \text{ as } M \to \infty. \quad \text{(using (2.6))} \end{aligned}$$

Therefore,

$$\lim_{M\to\infty} \frac{1}{M} \left| \left\{ \lambda_n \leqslant M : \left| \frac{1}{P_{\lambda_n}} \sum_{k=0}^{\lambda_n} p_{\lambda_n - k} x_k - l \right| \geqslant \epsilon \right\} \right| = 0,$$

which shows that $st - \lim_{n\to\infty} t_{\lambda_n} = l$. This proves the lemma. □

We shall now prove,

Lemma 2.9. *For* $0 < \lambda_n < 1$,

$$\frac{1}{(P_n - P_{\lambda_n})} \sum_{k=\lambda_n+1}^{n} p_{n-k} x_k = t_n + \frac{P_{\lambda_n}}{(P_n - P_{\lambda_n})} (t_n - t_{\lambda_n}),$$

provided $P_n > P_{\lambda_n}$.

Proof. Consider the right-hand side:

$$\begin{aligned}
t_n &+ \frac{P_{\lambda_n}}{P_n - P_{\lambda_n}}(t_n - t_{\lambda_n}) \\
&= \frac{P_n t_n - P_{\lambda_n} t_n + P_{\lambda_n} t_n - P_{\lambda_n} t_{\lambda_n}}{P_n - P_{\lambda_n}} \\
&= \frac{1}{P_n - P_{\lambda_n}}\left[P_n\left(\frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}x_k\right) - P_{\lambda_n}\left(\frac{1}{P_{\lambda_n}}\sum_{k=0}^{\lambda_n} p_{\lambda_n-k}x_k\right)\right] \\
&= \frac{1}{P_n - P_{\lambda_n}}\left[\sum_{k=0}^{\lambda_n} p_{\lambda_n-k}x_k + \sum_{k=\lambda_n+1}^{n} p_{n-k}x_k - \sum_{k=0}^{\lambda_n} p_{\lambda_n-k}x_k\right] \\
&= \frac{1}{P_n - P_{\lambda_n}}\sum_{k=\lambda_n+1}^{n} p_{n-k}x_k.
\end{aligned}$$

Thus,

$$\frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}x_k = t_n + \frac{P_{\lambda_n}}{P_n - P_{\lambda_n}}(t_n - t_{\lambda_n}).$$

□

Now, adding x_n to the above equation we get,

$$x_n - t_n = \frac{P_{\lambda_n}}{P_n - P_{\lambda_n}}(t_n - t_{\lambda_n}) + \frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k). \tag{2.7}$$

Proof of Theorem 2.7. Necessity: Here, we assume that

$$st - \lim_{n\to\infty} x_n = l,$$

and prove that, for every $0 < \lambda_n < 1$,

$$\lim_{M\to\infty}\frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k)\right| \geqslant \epsilon\right\}\right| = 0.$$

Now, since $st - \lim_{n\to\infty} x_n = l$ and $st - \lim_{n\to\infty} t_n = l$, we have

$$st - \lim_{n\to\infty}(x_n - t_n) = 0.$$

That is,

$$\lim_{M\to\infty}\frac{1}{M}|\{n \leqslant M : |x_n - t_n| \geqslant \epsilon\}| = 0.$$

This shows that

$$\begin{aligned}
\lim_{M\to\infty}\frac{1}{M}\Bigg|\Bigg\{n \leqslant M : \Bigg|&\frac{P_{\lambda_n}}{(P_n - P_{\lambda_n})}(t_n - t_{\lambda_n}) \\
&+ \frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k)\Bigg| \geqslant \epsilon\Bigg\}\Bigg| = 0. \quad \text{(using (2.7))}
\end{aligned}$$

Since the valuation is non-archimedean wherein $|a+b| = |a|$ if $|a| > |b|$, and since

$$\frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{P_{\lambda_n}}{(P_n - P_{\lambda_n})}(t_n - t_{\lambda_n})\right| \geqslant \epsilon\right\}\right| \to 0 \text{ as } M \to \infty,$$

by (1.1) and (2.5), we have that

$$\lim_{M\to\infty}\frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k)\right| \geqslant \epsilon\right\}\right| = 0.$$

Sufficiency: We now assume that

$$\lim_{M\to\infty}\frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k)\right| \geqslant \epsilon\right\}\right| = 0,$$

and prove that

$$st - \lim_{n\to\infty} x_n = l.$$

To this end, it is enough if we prove that

$$st - \lim_{n\to\infty}(x_n - t_n) = 0.$$

That is to prove,

$$\lim_{M\to\infty}\frac{1}{M}|\{n \leqslant M : |x_n - t_n| \geqslant \epsilon\}| = 0.$$

Using (2.7) we have,

$$\begin{aligned}\frac{1}{M}|\{n \leqslant M : |x_n - t_n| \geqslant \epsilon\}| &= \frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{P_{\lambda_n}}{(P_n - P_{\lambda_n})}(t_n - t_{\lambda_n})\right.\right.\right. \\ &\qquad \left.\left.\left. + \frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k)\right| \geqslant \epsilon\right\}\right| \\ &\leqslant \max\left\{\begin{array}{l}\frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{P_{\lambda_n}}{(P_n - P_{\lambda_n})}(t_n - t_{\lambda_n})\right| \geqslant \epsilon\right\}\right|, \\ \frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k)\right| \geqslant \epsilon\right\}\right|\end{array}\right\}.\end{aligned}$$

By our assumption,

$$\frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{1}{(P_n - P_{\lambda_n})}\sum_{k=\lambda_n+1}^{n} p_{n-k}(x_n - x_k)\right| \geqslant \epsilon\right\}\right| \to 0 \text{ as } M \to \infty.$$

Therefore,

$$\begin{aligned}\frac{1}{M}|\{n \leqslant M : |x_n - t_n| \geqslant \epsilon\}| &\leqslant \max\left\{\frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{P_{\lambda_n}}{(P_n - P_{\lambda_n})}(t_n - t_{\lambda_n})\right| \geqslant \epsilon\right\}\right|, 0\right\} \\ &\leqslant \frac{1}{M}\left|\left\{n \leqslant M : \left|\frac{P_{\lambda_n}}{(P_n - P_{\lambda_n})}(t_n - t_{\lambda_n})\right| \geqslant \epsilon\right\}\right| \\ &\to 0 \text{ as } M \to \infty, \quad \text{(by (1.1) and (2.5))}\end{aligned}$$

which implies that

$$\lim_{M\to\infty} \frac{1}{M}|\{n \leqslant M : |x_n - t_n| \geqslant \epsilon\}| = 0,$$

which means that

$$st - \lim_{n\to\infty} (x_n - t_n) = 0.$$

Thus, sequence (x_n) is statistically convergent to 'l'. This completes the proof of the theorem. □

Acknowledgment

I am truly grateful to my Research supervisor Prof. Srinivasan Vaithinathasamy for his insightful comments. His knowledge, expertise and exacting attention to detail brought this article to its final form.

References

[1] G. Bachman, *Introduction to p-Adic Numbers and Valuation Theory*, Academic Press, New York-London, (1964). 1
[2] N. L. Braha, *A Tauberian theorem for the generalized Nörlund-Euler summability method*, J. Inequal. Spec. Funct., **7** (2016), 137–142. 1
[3] H. Çakallı, *A Study on statistical convergence*, Funct. Anal. Approx. Comput., **1** (2009), 19–24. 1
[4] H. Fast, *Sur la convergence statistique*, Colloq. Math., **2** (1951), 241–244. 1
[5] J. A. Fridy, *On statistical Convergence*, Analysis, **5** (1985), 301–313. 1
[6] J. A. Fridy, H. I. Miller, *A matrix characterization of statistical convergence*, Analysis, **11** (1991), 59–66. 1
[7] A. F. Monna, *Sur le theoreme de Banach-Steinhaus*, Nederl. Akad. Wetensch. Proc. Ser. A 66=Indag. Math., **25** (1963), 121–131. 1
[8] F. Moricz, *Tauberian conditions under which statistical convergence follows from statistical summability* $(C, 1)$, J. Math. Anal. Appl., **275** (2002), 277–287. 1
[9] F. Moricz, C. Orhan, *Tauberian conditions under which statistical convergence follows from statistical summability by weighted means*, Studia Sci. Math. Hungar., **41** (2004), 391–403. 1
[10] M. Mursaleen, O. H. H. Edely, *Generalized statistical convergence*, Inform. Sci., **162** (2004), 287–294. 1
[11] P. N. Natarajan, *On Nörlund method of summability in non-archimedean fields*, J. Anal., **2** (1994), 97–102. 1, 1.1
[12] T. Šalát, *On statistically convergent sequences of real numbers*, Math. Slovaca, **30** (1980), 139–150. 1
[13] V. K. Srinivasan, *On certain summation processes in the p-adic field*, Nederl. Akad. Wetensch. Proc. Ser. A 68=Indag. Math., **27** (1965), 319–325. 1, 1.1
[14] K. Suja, V. Srinivasan, *On statistically convergent and statistically cauchy sequences in non-archimedean fields*, J. Adv. Math., **6** (2014), 1038–1043. 1, 1.1