J. Math. Computer Sci., 24 (2022), 358–369

Online: ISSN 2008-949X

Journal of Mathematics and Computer Science

Journal Homepage: www.isr-publications.com/jmcs

On characterization of χ-single valued neutrosophic subgroups

M. Shazib Hameed[a], Shahbaz Ali[a,*], Salman Mukhtar[b], Muhammad Shoaib[c], M. Kashif Ishaq[a], Umer Mukhtiar[a]

[a]Department of Mathematics, Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan, Punjab 64200, Pakistan.

[b]Department of Mathematics, The Islamia University of Bahawalpur, Bahawalpur, Punjab 63100, Pakistan.

[c]Department of Mathematics, Quaid-e-Azam University, Islamabad, 15300, Pakistan.

Abstract

In this paper, we investigate the notion of χ-single valued neutrosophic set and subgroups. Also, several properties related to algebraic structure are discussed. Moreover, many characterizations are proposed on χ-single valued neutrosophic sets and subgroups.

Keywords: Single valued neutrosophic set, χ-single valued neutrosophic set, χ-single valued neutrosophic subgroups.

2020 MSC: 20N25, 03E72, 08A72.

1. Introduction

Generally, the inconvenience of previously established strategies and designs is overcome by recently established fuzzy algebraic structure. Routine mathematics cannot always be used because of unclear and missing knowledge in certain regular structures. Various methodologies were seen as alternative groups to deal with these issues and to avoid vulnerabilities, like probability, rough set as well as a fuzzy set hypothesis. Unfortunately, each of these alternate mathematics has a side and inconveniences such as the majority of words like real, beautiful, famous that are not clearly observed or indeed vague. Henceforth, the rules for such terms are vary from person to person.

Zadeh [43], proposed the idea of fuzzy set which is focused on possibility on the support highlight doling out an enrollment grade in $[0, 1]$ to deal with such sort of vague and questionable data. Taking into account the possibility of enrollment and non-investment, Atanassov [9, 10] proposed intuitionistic fuzzy set which is a augmentation of fuzzy set. As an extension of intuitionistic fuzzy set, Smarandache's [33, 36, 38] introduced a neutrosophic logics, and sets. A neutrosophic set based on three degrees, that is, the level of participation, indeterminacy, and non-enrollment degree. The notion of a soft set is introduced in [28]

*Corresponding author

Email addresses: shazib.hameed@kfueit.edu.pk (M. Shazib Hameed), shahbaz.ali@kfueit.edu.pk (Shahbaz Ali), salman.mukhtar@iub.edu.pk (Salman Mukhtar), muhammadshoaibe14@gmail.com (Muhammad Shoaib), mkashif14700@gmail.com (M. Kashif Ishaq), umermukhtiar.math@gmail.com (Umer Mukhtiar)

doi: 10.22436/jmcs.024.04.08

Received: 2021-01-20 *Revised:* 2021-03-04 *Accepted:* 2021-03-27

by Molodtsov. Several operations added by Ali et al. in soft set in [2]. In [40–42], Yager was executed the idea of Pythagorean fuzzy set. Peng et al. presented several findings in [29, 30] on the measurements of the Pythagorean fuzzy and soft sets. Moreover, several new models have been investigated in [3, 32, 39, 46, 47].

In 1971, the concept of a fuzzy subgroup was proposed and investigated by Rosenfeld in [31]. Latter on, many algebraic structures; like as groups, rings, fields, and modules have been developed in [1, 6–8, 11, 12, 21–23, 25, 31, 33–38, 44]. In [4, 5, 13–20, 24, 26, 27, 45] many algebraic structures over groups, rings, graphs, and decision-making problems are discussed.

In this piece of work, we investigate the notion of χ-neutrosophic fuzzy sets and subgroups. The paper is arranged as follows. We provide some basic concepts related to single valued neutrosophic sets in Section 2. We give an overview of χ-single valued neutrosophic sets and subgroups in Sections 3 and 4, respectively, and then we suggested several characterizations on them. In Section 5, we discuss the concept of homomorphism on proposed algebraic structure χ-single valued neutrosophic subgroups.

2. Preliminaries

In this section, we discuss some basic concepts related to single valued neutrosophic set.

Definition 2.1 ([38])**.** A single valued neutrosophic set N on the universe set S is defined as: $N = \langle l,\ \alpha_N(l),\ \beta_N(l),\ \gamma_N(l)\rangle, l \in S$, where $\alpha, \beta, \gamma : S \to [0,1]$ and $0 \leqslant \alpha_N(l) + \beta_N(l) + \gamma_N(l) \leqslant 3$.

Definition 2.2 ([38])**.** Let S be a non empty set, and $L = \langle l, \alpha_L(l),\ \beta_L(l),\ \gamma_L(l)\rangle$, $M = \langle l, \alpha_M(l),\ \beta_M(l),\ \gamma_M(l)\rangle$,

(i) $L \subseteq M, \forall\ l$ if $\alpha_L(l) \leqslant \alpha_M(l),\ \beta_L(l) \leqslant \beta_M(l),\ \gamma_L(l) \geqslant \gamma_M(l)$;
(ii) $L \cup M = \langle l, \max(\alpha_L(l), \alpha_M(l)), \max(\beta_L(l), \beta_M(l)), \min(\gamma_L(l), \gamma_M(l))\rangle$;
(iii) $L \cap M = \langle l, \min(\alpha_L(l), \alpha_M(l)), \min(\beta_L(l), \beta_M(l)), \max(\gamma_L(l), \gamma_M(l))\rangle$;
(iv) $L \setminus M(l) = \langle l, \min(\alpha_L(l),\ \gamma_M(l)), \min(\beta_L(l), 1 - \beta_M(l)), \max(\gamma_L(l),\ \alpha_M(l))\rangle$.

Definition 2.3 ([38])**.** A single valued neutrosophic set L is called null or empty single valued neutrosophic set over the universe S if

$$\alpha_L(l) = 0,\ \ \beta_L(l) = 0,\ \ \gamma_L(l) = 1,\ \ \forall\ \ l \in S.$$

It is indicated with O_N.

Definition 2.4 ([38])**.** A single valued neutrosophic set of L is an absolute single valued neutrosophic set over the universe of S, if

$$\alpha_L(l) = 1,\ \ \beta_L(l) = 1,\ \ \gamma_L(l) = 0,\ \ \forall\ \ l \in S.$$

It is indicated with 1_N.

Definition 2.5 ([38])**.** L^c is the complement of single valued neutrosophic set L which is expressed as $L^c = \langle l, \alpha_{L^c}(l),\ \ \beta_{L^c}(l),\ \ \gamma_{L^c}(l)\rangle$, where $\alpha_{L^c}(l) = \ \ \gamma_L(l),\ \ \beta_{L^c}(l) = 1 - \beta_L(l),\ \ \gamma_{L^c}(l) = \alpha_L(l)$. It is also possible to describe the complement of the single valued neutrosophic set L as $L^c = 1_N - L$.

3. χ-single valued neutrosophic set

In this section, we define the notion of χ-single valued neutrosophic set and then discuss several operation on it.

Definition 3.1. If L be a neutrosophic fuzzy subset of G, then χ-single valued neutrosophic subset G is described as,

$$L^\chi = \{\alpha^\chi(l) = \min\{\alpha(l),\ \chi\},\quad \beta^\chi(l) = \min\{\beta(l), \chi\},\quad \gamma^\chi(l) = \max\{\gamma(l), \chi\}\},$$

where $\chi \in [0,1]$.

Proposition 3.2. *If* L *and* M *be two single valued neutrosophic subsets of group* G*, then*

1. $(L \cap M)^{\chi} = L^{\chi} \cap M^{\chi}$;
2. *defining a map* $\hbar : G \to H$, *where* L *and* M *two single valued neutrosophic subsets of* G *and* H, *respectively, then*
   (a) $\hbar^{-1}(M^{\chi}) = (\hbar^{-1}(M))^{\chi}$;
   (b) $\hbar(L^{\chi}) = (\hbar(L))^{\chi}$.

Proof. Assume that L and M are two single valued neutrosophic subset of group G.

$$\begin{aligned}
(1) \qquad (L \cap M)^{\chi}(l) &= \min\{(L \cap M)(l), \chi\} \\
&= \min\{\min\{L(l), M(l)\}, \chi\} \\
&= \min\{\min\{L(l), \chi\}, \min\{M(l), \chi\}\} = \min\{L^{\chi}(l) \cap M^{\chi}(l)\} = L^{\chi}(l) \cap M^{\chi}(l), \quad \forall \ l \in G. \\
(2) \qquad (a) \ \hbar^{-1}(M^{\chi}) &= M^{\chi}(\hbar(l)) = \min\{M(\hbar(l), \chi\} = \min\{\hbar^{-1}(M)(l), \chi\} = \hbar^{-1}(M)(l), \ \forall \ l \in G. \\
(b) \ \hbar(L^{\chi})(m) &= \sup\{(L^{\chi})(l) : \hbar(l) = m\} \\
&= \sup\{\min\{L(l), \chi\} : \hbar(l) = m\} \\
&= \min\{\sup\{L(l), \chi\} : \hbar(l) = m\} = \min\{\hbar(L)(m), \chi\} = (\hbar(L))^{\chi}(m), \ \forall \ m \in H.
\end{aligned}$$

□

4. χ-single valued neutrosophic subgroups

In this section, we define the notion of χ-single valued neutrosophic subgroups and then develop some results on it.

Definition 4.1. Let L is a single valued neutrosophic subset of a group G. The subset L^{χ} of G is called χ-single valued neutrosophic subgroup (χ-SVNSG) of G, if

- $\alpha^{\chi}(lm) \geqslant \min\{\alpha^{\chi}(l), \alpha^{\chi}(m)\}, \quad \alpha^{\chi}(l^{-1}) = \alpha^{\chi}(l), \ \forall \ l, m \in G,$
- $\beta^{\chi}(lm) \geqslant \min\{\beta^{\chi}(l), \beta^{\chi}(m)\}, \quad \beta^{\chi}(l^{-1}) = \beta^{\chi}(l), \ \forall \ l, m \in G,$
- $\gamma^{\chi}(lm) \leqslant \max\{\gamma^{\chi}(l), \gamma^{\chi}(m)\}, \quad \gamma^{\chi}(l^{-1}) = \gamma^{\chi}(l), \ \forall \ l, m \in G.$

Proposition 4.2. *If* $L : G \longrightarrow [0, 1]$ *is a* χ*-SVNSG of a group* G*, then*

(a) $\alpha^{\chi}(l) \geqslant \alpha^{\chi}(e), \ \beta^{\chi}(l) \geqslant \beta^{\chi}(e), \ \gamma^{\chi}(l) \leqslant \gamma^{\chi}(e), \ \forall \ l \in G$;
(b)

$$\begin{aligned}
\alpha^{\chi}(lm^{-1}) = \alpha^{\chi}(e) &\Longrightarrow \alpha^{\chi}(l) = \alpha^{\chi}(m), \\
\beta^{\chi}(lm^{-1}) = \beta^{\chi}(e) &\Longrightarrow \beta^{\chi}(l) = \beta^{\chi}(m), \\
\gamma^{\chi}(lm^{-1}) = \gamma^{\chi}(e) &\Longrightarrow \gamma^{\chi}(l) = \gamma^{\chi}(m), \ \ \forall \ l \in G.
\end{aligned}$$

Proof. Let L is a χ-SVNSG of a group G and $\chi \in [0, 1]$,

(a)

$$\begin{aligned}
\alpha^{\chi}(e) = \alpha^{\chi}(ll^{-1}) \geqslant \min\{\alpha^{\chi}(l), \alpha^{\chi}(l^{-1})\} = \min\{\alpha^{\chi}(l), \alpha^{\chi}(l)\} = \alpha^{\chi}(l), \\
\beta^{\chi}(e) = \beta^{\chi}(ll^{-1}) \geqslant \min\{\beta^{\chi}(l), \beta^{\chi}(l^{-1})\} = \min\{\beta^{\chi}(l), \beta^{\chi}(l)\} = \beta^{\chi}(l), \\
\gamma^{\chi}(e) = \gamma^{\chi}(ll^{-1}) \leqslant \max\{\gamma^{\chi}(l), \gamma^{\chi}(l^{-1})\} = \min\{\gamma^{\chi}(l), \gamma^{\chi}(l)\} = \gamma^{\chi}(l),
\end{aligned}$$

(b)

$$
\begin{aligned}
\alpha^{\chi}(l) = \alpha^{\chi}(lmm^{-1}) &\geqslant \min\{\alpha^{\chi}(lm^{-1}), \alpha^{\chi}(l)\} \\
&= \min\{\alpha^{\chi}(e), \alpha^{\chi}(m)\} \\
&= \alpha^{\chi}(m) = \alpha^{\chi}(mll^{-1}) \geqslant \min\{\alpha^{\chi}(ml^{-1}), \alpha^{\chi}(l)\} \geqslant \min\{\alpha^{\chi}(lm^{-1}), \alpha^{\chi}(l)\} = \alpha^{\chi}(l), \\
\beta^{\chi}(l) = \beta^{\chi}(lmm^{-1}) &\geqslant \min\{\beta^{\chi}(lm^{-1}), \beta^{\chi}(l)\} \\
&= \min\{\beta^{\chi}(e), \beta^{\chi}(m)\} \\
&= \beta^{\chi}(m) = \beta^{\chi}(mll^{-1}) \geqslant \min\{\beta^{\chi}(ml^{-1}), \beta^{\chi}(l)\} \geqslant \min\{\beta^{\chi}(lm^{-1}), \beta^{\chi}(l)\} = \beta^{\chi}(l), \\
\gamma^{\chi}(l) = \gamma^{\chi}(lmm^{-1}) &\leqslant \max\{\gamma^{\chi}(lm^{-1}), \gamma^{\chi}(l)\} \\
&= \max\{\gamma^{\chi}(e), \gamma^{\chi}(m)\} \\
&= \gamma^{\chi}(m) = \gamma^{\chi}(mll^{-1}) \leqslant \max\{\gamma^{\chi}(ml^{-1}), \gamma^{\chi}(l)\} \leqslant \max\{\gamma^{\chi}(lm^{-1}), \gamma^{\chi}(l)\} = \gamma^{\chi}(l).
\end{aligned}
$$

□

Proposition 4.3. *If* L *is a SVNSG of the group* G*, then* L *is also* χ*-SVNSG of* G*.*

Proof. Let $l, m \in G$ be any elements of group G.

$$
\begin{aligned}
\alpha^{\chi}(lm) = \min\{\alpha(lm), \chi\} = \min\{\alpha^{\chi}(e), \alpha^{\chi}(m)\} &= \alpha^{\chi}(m)\} \\
&\geqslant \min\{\min\{\alpha(l), \alpha(m)\}, \chi\} \\
&= \min\{\min\{\alpha(l), \chi\}, \min\{\alpha(m), \chi\}\} = \min\{\alpha^{\chi}(l) = \alpha^{\chi}(m)\}.
\end{aligned}
$$

Also

$$
\begin{aligned}
\alpha^{\chi}(l^{-1}) &= \min\{\alpha^{\chi}(l^{-1}), \chi\} = \min\{\alpha(l), \chi\} = \alpha^{\chi}(l), \\
\beta^{\chi}(lm) &= \min\{\beta(lm), \chi\} \\
&= \min\{\beta^{\chi}(e), \beta^{\chi}(m)\} \\
&= \beta^{\chi}(m)\} \geqslant \min\{\min\{\beta(l), \beta(m)\}, \chi\} = \min\{\min\{\beta(l), \chi\}, \min\{\beta(m), \chi\}\} = \min\{\beta^{\chi}(l) = \beta^{\chi}(m)\}.
\end{aligned}
$$

Also

$$
\begin{aligned}
\beta^{\chi}(l^{-1}) &= \min\{\beta^{\chi}(l^{-1}), \chi\} = \min\{\beta(l), \chi\} = \beta^{\chi}(l), \\
\gamma^{\chi}(lm) &= \max\{\gamma(lm), \chi\} \\
&= \max\{\gamma^{\chi}(e), \gamma^{\chi}(m)\} \\
&= \gamma^{\chi}(m)\} \\
&\leqslant \max\{\max\{\gamma(l), \gamma(m)\}, \chi\} = \max\{\max\{\gamma(l), \chi\}, \max\{\gamma(m), \chi\}\} = \max\{\gamma^{\chi}(l) = \gamma^{\chi}(m)\}.
\end{aligned}
$$

Also

$$\gamma^{\chi}(l^{-1}) = \max\{\gamma^{\chi}(l^{-1}), \chi\} = \min\{\gamma(l), \chi\} = \gamma^{\chi}(l).$$

□

Remark 4.4. The converse of above proposition is not true in general.

Proposition 4.5. *Let* L *be a fuzzy subset of a group* G *such that* $L(l^{-1}) = L(l)$ *hold for all* $l \in G$*. Then* $L^{\chi} = \{\alpha^{\chi}(l), \beta^{\chi}(l), \gamma^{\chi}(l)\}$*,* χ*-SVNSG of* G*. If* $\chi \leqslant p$*,* $\chi \leqslant q$*,* $\chi \geqslant r$*, with* $p = \inf\{\alpha(l) : l \in G\}$*,* $q = \inf\{\beta(l) : l \in G\}$*,* $r = \sup\{\gamma(l) : l \in G\}$ *holds for* $\chi \in [0, 1]$*.*

Proof. Since $\chi \leqslant p$, so, $\inf\{\alpha(l) : l \in G\} \geqslant \chi \Longrightarrow \alpha(l) \geqslant \chi$. As $\alpha^{\chi}(l) = \min\{\alpha(l), \chi\}$. Thus, $\alpha^{\chi}(l) = \chi\ \forall\ l \in G$. Therefore, $\alpha^{\chi}(lm) \geqslant \min\{\alpha^{\chi}(l), \alpha^{\chi}(m)\}$ hold, $\forall\ l, m \in G$. Moreover $\alpha(l^{-1}) = \alpha(l)$, $\forall\ l \in G$. Thus, $\alpha(l^{\chi}) = \alpha^{\chi}(l)$. Similarly for mapping β. When $\chi \geqslant r$, $\sup\{\beta(l) : l \in G\} \geqslant \chi \Longrightarrow \alpha(l) \geqslant \chi$. As $\gamma^{\chi}(l) = \max\{\gamma(l), \chi\}$. Therefore, $\gamma^{\chi}(l) = \chi$, $\forall\ l \in G$. Hence, $\gamma^{\chi}(lm) \leqslant \max\{\gamma^{\chi}(l), \gamma^{\chi}(m)\}$ hold, $\forall\ l, m \in G$. Moreover, $\gamma(l^{-1}) = \gamma(l)$ hold for all $l \in G \Longrightarrow \gamma(l^{\chi}) = \gamma^{\chi}(l)$. □

Proposition 4.6. *Intersection of two* χ-SVNSGs *of* G *is also* χ*-SVNSG of* G.

Proof. Let L and M be two χ-SVNSG of group G. Let $l, m \in G$ be any element, then

$$\begin{aligned}(L \cap M)(l, m) &= (L^{\chi} \cap M^{\chi})(l, m)\\ &= \min\{L^{\chi}(l, m), M^{\chi}(l, m)\}\\ &\geqslant \min\{\min\{L^{\chi}(l), L^{\chi}(m)\}, \min\{M^{\chi}(l), M^{\chi}(m)\}\}\\ &= \min\{\min\{L^{\chi}(l), M^{\chi}(l)\}, \min\{L^{\chi}(m), M^{\chi}(m)\}\}\\ &= \min\{(L \cap M)^{\chi}(l), (L \cap M)^{\chi}(m)\}.\end{aligned}$$

Thus

$$(L \cap M)(l, m) = (L^{\chi} \cap M^{\chi})(l, m).$$

Also

$$(L \cap M)(l^{-1}) = (L^{\chi} \cap M^{\chi})(l^{-1}) = \min\{L^{\chi}(l^{-1}), M^{\chi}(l^{-1})\} = \min\{L^{\chi}(l^{1}), M^{\chi}(l^{1})\} = (L^{\chi} \cap M^{\chi})(l).$$

Hence $L \cap M$ is χ-SVNSG of G. □

Corollary 4.7. *Intersection of a family of* χ*-SVNSG of a group* G *is again a* χ*-SVNSG of* G.

Remark 4.8. Union of two χ-SVNSGs of a group G need not to be χ-SVNSG of G.

Example 4.9. Let G=Z, the group of integer under ordinary addition of integers. Define two SVNSGs L and M by

$$L(l) = \begin{cases} \langle 0.3, 0, 0\rangle, & \text{if } l \in 3Z,\\ \langle 0, 0, 0\rangle, & \text{otherwise,}\end{cases} \qquad M(l) = \begin{cases} \langle 0.15, 0, 0\rangle, & \text{if } l \in 2Z,\\ \langle 0.05, 0, 0\rangle, & \text{otherwise.}\end{cases}$$

It is obvious to verify that L and M are 1-SVNSG of Z. Therefore,

$$(L \cup M)(l) = \begin{cases} \langle 0.3, 0, 0\rangle, & \text{if } l \in 3Z,\\ \langle 0.15, 0, 0\rangle, & \text{if } l \in 2Z - 3Z,\\ \langle 0.05, 0, 0\rangle, & \text{if } l \notin 2Z \text{ or } l \notin 3Z.\end{cases}$$

Take $l = 9$ and $m = 4$ then $(L \cup M)(l) = \langle 0.3, 0, 0\rangle$, $(L \cup M)(m) = \langle 0.15, 0, 0\rangle$. $(L \cup M)(l - m) = (L \cup M)(9 - 4) = (L \cup M)(5) = \langle 0.05, 0, 0\rangle$. Clearly, $\alpha^{1}_{L\cup M}(l) = 0.3$, $\alpha^{1}_{L\cup M}(m) = 0.15$, $\alpha^{1}_{L\cup M}(l - m) = 0.05$. It is clear that, $\alpha^{1}_{L\cup M}(l - m) \geqslant \min\{\alpha^{1}_{L\cup M}(l), \alpha^{1}_{L\cup M}(m)\}$ is not hold. Hence, $(L \cup M)$ is not 1-SVNSG.

Example 4.10. Let G=Z, the group of integer under ordinary addition of integers. Define two N fuzzy sets L and M by

$$L(l) = \begin{cases} \langle 0.7, 0.2, 0.3\rangle, & \text{if } l \in 2Z,\\ \langle 0.6, 0.1, 0.3\rangle, & \text{otherwise,}\end{cases}$$

$$M(l) = \begin{cases} \langle 0.35, 0.1, 0.15\rangle, & \text{if } l \in 2Z,\\ \langle 0, 0, 0\rangle, & \text{otherwise,}\end{cases}$$

$$(L \cup M)(l) = \begin{cases} \langle 0.7, 0.2, 0.3\rangle, & \text{if } l \in 2Z,\\ \langle 0.6, 0.1, 0.3\rangle, & \text{otherwise.}\end{cases}$$

It is easily to verified that L, M, and $(L \cap M)$ are 1-SVNSG of G.

Definition 4.11. Let L be χ-SVNSG of a group G, where $\chi \in [0,1]$ for any $l \in G$, define a fuzzy set L_l^{χ} of G, called χ-SVNSG right coset of L in G as follows, where,

$$L = \{\langle \alpha\{l\}, \beta\{l\}, \gamma\{l\}\rangle, \quad l \in G\},$$
$$L_l^{\chi}(g) = \langle \min\{\alpha(gl^{-l})\}, \min\{\beta(gl^{-l})\}, \max\{\gamma(gl^{-l})\}\rangle, \ \forall\ l, g \in G.$$

Similarly, we define the χ-SVNSG left coset ${}_lL^{\chi}$ as

$${}_lL^{\chi}(g) = \langle \min\{\alpha(l^{-1}g)\}, \min\{\beta(l^{-1}g)\}, \max\{\gamma(l^{-1}g)\}\rangle, \ \forall\ l, g \in G.$$

Definition 4.12. Let L be χ-SVNSG of a group G, where $\chi \in [0,1]$, then L is called χ-single valued neutrosophic normal subgroup of G $\Leftrightarrow {}_lL^{\chi}(g) = L_l^{\chi}(g), \forall\ l \in G$.

Note: (1) Clearly χ-SVNSG is ordinary NSVNSG of G.
(2) ${}_lL^{\chi}(g) = L^{\chi}(l^{-1}g)$ and $L_l^{\chi}(g) = L^{\chi}(gl^{-1}), \ \forall\ l, g \in G$.

Remark 4.13. If L is NSVNSG of group G, then L is also a χ-NSVNSG of G.

Proof. Let L be a NSVNSG of G. Then for any $l \in G$, we have ${}_lL = L_l$, where $L = \{\langle l, \alpha\{l\}, \beta\{l\}, \gamma\{l\}\rangle, \quad l \in G\}$, therefor for any $g \in G$ we have $({}_lL)g = (L_l)g$ that is $L(l^{-1}g) = L(gl^{-1})$, so

$$\langle \min\{\alpha(l^{-1}g)\}, \min\{\beta(l^{-1}g)\}, \max\{\gamma(l^{-1}g)\}\rangle = \langle \min\{\alpha(l^{-1}g)\}, \min\{\beta(l^{-1}g)\}, \max\{\gamma(l^{-1}g)\}\rangle, \ \forall\ l, g \in G$$
$$\Rightarrow \ \min\{L(l^{-1}g), \chi\} = \min\{L(gl^{-1}), \chi\},$$

where, $L = \{\langle l, \alpha\{l\}, \beta\{l\}, \gamma\{l\}\rangle, \quad l \in G\}$, hence ${}_lL^{\chi}(g) = L_l^{\chi}(g)$, so we have ${}_lL^{\chi}(g) = L_l^{\chi}(g), \forall\ l \in G$, hence L is a χ-NSVNSG of G. □

Remark 4.14. The converse of above result is not true in general.

Proposition 4.15. *Let* L *be* χ*-SVNSG of a group* G*. Then* $L^{\chi}(m^{-1}lm) = L^{\chi}(l)$ *or equivalently,* $L^{\chi}(lm) = L^{\chi}(ml)$ *holds for all* $l, m \in G$*, where* $L = \{\langle l, \alpha\{l\}, \beta\{l\}, \gamma\{l\}\rangle, \ l \in G\}$.

Proof. Since L is χ-NSVNSG of group G, therefore ${}_lL^{\chi}(g) = L_l^{\chi}(g)$ holds for all $l \in G$. Then

$${}_lL^{\chi}(m^{-1}) = L_l^{\chi}(m^{-1}) \text{ holsd for all } m^{-1} \in G,$$

so

$$\min\{L(l^{-1}m^{-1}), \chi\} = \min\{L(gl^{-1}m^{-1}), \chi\} \ \forall \ m^{-1}l^{-1} \in G.$$

Then

$$\langle \min\{\alpha(l^{-1}m^{-1})\}, \min\{\beta(l^{-1}m^{-1})\}, \max\{\gamma(l^{-1}m^{-1})\}\rangle = \langle \min\{\alpha(m^{-1}l^{-1})\}, \min\{\beta(m^{-1}l^{-1})\}, \max\{\gamma(m^{-1}l^{-1})\}\rangle,$$

for all $m^{-1}l^{-1} \in G$. So,

$$\min\{L(l^{-1}m^{-1}), \chi\} = \min\{L(gl^{-1}m^{-1}), \chi\}, \ \forall \ m^{-1}l^{-1} \in G \Longrightarrow L^{\chi}(l^{-1}m^{-1}) = L^{\chi}(m^{-1}l^{-1}),$$
$$\Longrightarrow L^{\chi}(lm)^{-1} = L^{\chi}(ml)^{-1},$$
$$\Longrightarrow L^{\chi}(lm) = L^{\chi}(ml) \ \ l, m \in G.$$

As L is χ-SVNSG of G. So $L^{\chi}(g^{-1}) = L^{\chi}(g), \ \ g \in G$. □

Note: Next we show that for some specific value of χ, every χ-SVNSG L of G will always be χ-NSVNSG of G. In this direction, we have the following.

Proposition 4.16. *Let* L *be* χ*-SVNSG of a group* G *such that* $\chi \leqslant P$*, where* $P = \inf\{L(l) \ \ \forall\ l \in G\}$ *and* $L = \{\langle l, \alpha\{l\}, \beta\{l\}, \gamma\{l\}\rangle, \ l \in G\}$*. Then* L *is also* χ*-NSVNSG of* G.

Proof. Since $\chi \leqslant P$ implies that $P \geqslant d$, $\inf\{L(l), \forall\, l \in G\} \leqslant \chi \implies L(l) \leqslant \chi$ and so, $\min\{L(l), d\} = \chi, \forall\, l \in G$, thus

$$L^{\chi}_{l}(g) = \min\{L^{\chi}(gl^{-1}), \chi\} = \chi.$$

Similarly,

$$_{l}L^{\chi}(g) = \min\{L^{\chi}(l^{-1}g), \chi\} = \chi.$$

That is

$$L^{\chi}_{l}(g) =_{l} L^{\chi}(g), \quad \forall\, g \in G.$$

Therefore

$$L^{\chi}_{l} =_{l} L^{\chi}, \quad \forall\, g \in G.$$

Hence L is also χ-NSVNSG of G. □

Proposition 4.17. *Let* L *be* χ*-NSVNSG of a group* G*, then the set* $G_{L^{\chi}} = \{l \in G,\ \ L^{\chi}(l) = L^{\chi}(e)\}$ *is a normal sub group of* G*, where* $L = \{\langle l, \alpha\{l\}, \beta\{l\}, \gamma\{l\}\rangle,\ l \in G\}$.

Proof. Clearly $G_{L^{\chi}} \neq \phi$ for $e \in G_{L^{\chi}}$. Let $l, m \in G_{L^{\chi}}$ be any element. Then we have

$$\begin{aligned} L^{\chi}(lm^{-1}) &\geqslant \min\{L^{\chi}(l), L^{\chi}(m)\} = \min\{L^{\chi}(e), L^{\chi}(e)\} = L^{\chi}(e),\\ L^{\chi}(lm^{-1}) &\geqslant \langle \min\{\alpha^{\chi}(l), \alpha^{\chi}(m)\}, \min\{\beta^{\chi}(l), \beta^{\chi}(m)\}, \max\{\gamma^{\chi}(l), \gamma^{\chi}(l)\}\rangle,\\ L^{\chi}(lm^{-1}) &\geqslant \langle \min\{\alpha^{\chi}(e), \alpha^{\chi}(e)\}, \min\{\beta^{\chi}(e), \beta^{\chi}(e)\}, \max\{\gamma^{\chi}(e), \gamma^{\chi}(e)\}\rangle\\ &= \langle \alpha^{\chi}(e), \beta^{\chi}(e), \gamma^{\chi}(e)\rangle = L^{\chi}(e), \end{aligned}$$

so, $L^{\chi}(lm^{-1}) \geqslant L^{\chi}(e)$, but $L^{\chi}(lm^{-1}) \leqslant L^{\chi}(e)$. Therefore $L^{\chi}(lm^{-1}) = L^{\chi}(e) \implies lm^{-1} \in G_{L^{\chi}}$. Thus $G_{L^{\chi}}$ is a subgroup of G. Further, let $l \in G_{L^{\chi}}$ and $m \in G$. We have $L^{\chi}(m^{-1}lm) = L^{\chi}(l) = L^{\chi}(e) \implies m^{-1}lm \in G_{L^{\chi}}$. So $G_{L^{\chi}}$ is a normal sub group of G. □

Proposition 4.18. *Let* L *be* χ*-NSVNSG of a group* G*,*

(1) $lL^{\chi} = mL^{\chi}$ *if and only if* $l^{-1}m \in G_{L^{\chi}}$*;*
(2) $Ll^{\chi} = L^{\chi}m$ *if and only if* $m, l^{-1} \in G_{L^{\chi}}$.

Proof.

(1) Firstly, let $lL^{\chi} = mL^{\chi}$,

$$\begin{aligned} L^{\chi}(lm^{-1}) &= \min\{L(l^{-1}m), \chi\}\\ &= \langle \min\{\alpha(l^{-1}m), \chi\}, \min\{\beta(l^{-1}m), \chi\}, \max\{(l^{-1}m), \chi\}\rangle\\ &= \langle (l\alpha^{\chi})(m), (l\beta^{\chi})(m), (l\gamma^{\chi})(m)\rangle = (lL^{\chi})(m) = (mL^{\chi})(m) = \min\{L(m^{-1}m), \chi\}\\ &= \langle \min\{\alpha(m^{-1}m), \chi\}, \min\{\beta(m^{-1}m), \chi\}, \max\{\gamma(m^{-1}m), \chi\}\rangle\\ &= \langle \min\{\alpha(e), \chi\}, \min\{\beta(e), \chi\}, \max\{\gamma(e), \chi\}\rangle\\ &= \min\{L(e), \chi\} = L^{\chi}(e). \end{aligned}$$

Thus $L(l^{-1}m) = L^{\chi}(e) \implies l^{-1}m \in G_{L^{\chi}}$. Conversely, let $l^{-1}m \in G_{L^{\chi}}$, then $L^{\chi}(l^{-1}m) = L^{\chi}(e)$, so $\langle \alpha^{\chi}(l^{-1}m), \beta^{\chi}(l^{-1}m), \gamma^{\chi}(l^{-1}m)\rangle = \langle \alpha^{\chi}(e), \beta^{\chi}(m), \gamma^{\chi}(m)\rangle$. Let $z \in G$ be any element, now,

$$\begin{aligned} {}_{l}L^{\chi}(z) &= \min\{L(l^{-1}z), \chi\}\\ &= \langle \min\{\alpha(l^{-1}z), \chi\}, \min\{\beta(l^{-1}z), \chi\}, \max\{\gamma(l^{-1}z), \chi\}\rangle\\ &= \langle \alpha^{\chi}(l^{-1}z), \beta^{\chi}(l^{-1}z), \gamma^{\chi}(l^{-1}z)\rangle\\ &= L^{\chi}(l^{-1}z) = L^{\chi}((l^{-1}m), (m^{-1}z)) \end{aligned}$$

$$
\begin{aligned}
&\geqslant \min\{L^{\chi}(l^{-1}m), L^{\chi}(m^{-1}z)\}\\
&= \langle \min\{\alpha^{\chi}(l^{-1}m), \alpha^{\chi}(m^{-1}z)\}, \min\{\beta^{\chi}(l^{-1}m), \beta^{\chi}(m^{-1}z)\}, \max\{\gamma^{\chi}(l^{-1}m), \gamma^{\chi}(m^{-1}z)\}\rangle\\
&= \langle \min\{\alpha^{\chi}(e), \alpha^{\chi}(m^{-1}z)\}, \min\{\beta^{\chi}(e), \beta^{\chi}(m^{-1}z)\}, \max\{\gamma^{\chi}(e), \gamma^{\chi}(m^{-1}z)\}\rangle\\
&= \langle \min\{L^{\chi}(e), L^{\chi}(m^{-1}z)\}\\
&= L^{\chi}(m^{-1}z),\\
{}_{l}L^{\chi}(z) &= mL^{\chi}(z).
\end{aligned}
$$

Interchanging the role of l and m, we get $lL^{\chi}(z) = mL^{\chi}(z)$, $\forall\ z \in G$. Hence $L^{\chi}l = L^{\chi}m$.

(2) Let $L^{\chi}l = L^{\chi}m$, now,

$$
\begin{aligned}
L^{\chi}(lm^{-1}) &= \min\{L(m^{-1}l), \chi\}\\
&= \langle \min\{\alpha(lm^{-1}), \chi\}, \min\{\beta(lm^{-1}), \chi\}, \max\{(lm^{-1}), \chi\}\rangle\\
&= \langle (m\alpha^{\chi})(l), (m\beta^{\chi})(l), (m\gamma^{\chi})(l)\rangle = (mL^{\chi})(l) = (lL^{\chi})(l) = \min\{L(l^{-1}l), \chi\}\\
&= \langle \min\min\{\alpha(l^{-1}l), \chi\}, \min\{\beta(l^{-1}l), \chi\}, \max\{\gamma(x^{-1}l), \chi\}\rangle\\
&= \langle \min\{\alpha(e), \chi\}, \min\{\beta(e), \chi\}, \max\{\gamma(e), \chi\}\rangle\\
&= \min\{L(e), \chi\} = L^{\chi}(e).
\end{aligned}
$$

Thus $L(lm^{-1}) = L^{\chi}(e)$, so $lm^{-1} \in G_{L^{\chi}}$. Conversely, let $lm^{-1} \in G_{L^{\chi}}$, then $L^{\chi}(lm^{-1}) = L^{\chi}(e)$, so $\langle \alpha^{\chi}(lm^{-1}), \beta^{\chi}(lm^{-1}), \gamma^{\chi}(lm^{-1})\rangle =< \alpha^{\chi}(e), \beta^{\chi}(e), \gamma^{\chi}(e)\rangle$. Let $z \in G$ be any element, now,

$$
\begin{aligned}
mL^{\chi}(z) &= \min\{L(m^{-1}z), \chi\}\\
&= \langle \min\{\alpha(m^{-1}z), \chi\}, \min\{\beta(m^{-1}z), \chi\}, \max\{\gamma(m^{-1}z), \chi\}\rangle\\
&= \langle \alpha^{\chi}(m^{-1}z), \beta^{\chi}(m^{-1}z), \gamma^{\chi}(m^{-1}z)\rangle\\
&= L^{\chi}(m^{-1}z) = L^{\chi}((lm^{-1}), (l^{-1}z))\\
&\geqslant \min\{L^{\chi}(lm^{-1}), L^{\chi}(l^{-1}z)\}\\
&= \langle \min\{\alpha^{\chi}(lm^{-1}), \alpha^{\chi}(l^{-1}z)\}, \min\{\beta^{\chi}(lm^{-1}), \beta^{\chi}(l^{-1}z)\}, \max\{\gamma^{\chi}(lm^{-1}), \gamma^{\chi}(l^{-1}z)\}\rangle\\
&= \langle \min\{\alpha^{\chi}(e), \alpha^{\chi}(l^{-1}z)\}, \min\{\beta^{\chi}(e), \beta^{\chi}(l^{-1}z)\}, \max\{\gamma^{\chi}(e), \gamma^{\chi}(l^{-1}z)\}\rangle\\
&= \langle \min\{L^{\chi}(e), L^{\chi}(l^{-1}z)\}\\
&= L^{\chi}(l^{-1}z),\\
mL^{\chi}(z) &= lL^{\chi}(z).
\end{aligned}
$$

Interchanging the role of l and m, we get $mL^{\chi}(z) = lL^{\chi}(z)$, $\forall\ z \in G$. Hence $L^{\chi}l = L^{\chi}m$. □

Proposition 4.19. *Let* L *be* χ*-NSVNSG of a group* G *and* l, m, u, v *be any element in* G*.If* $lL^{\chi} = umL^{\chi}$ *and* $mL^{\chi} = vL^{\chi}$*, then,* $lmL^{\chi} = uvL^{\chi}$.

Proof. Since $lL^{\chi} = umL^{\chi}$ and $mL^{\chi} = vL^{\chi} \implies l^{-1}u, m^{-1}vG_{L^{\chi}}$, it means $l\langle \alpha^{\chi}, \beta^{\chi}, \gamma^{\chi}\rangle = u\langle \alpha^{\chi}, \beta^{\chi}, \gamma^{\chi}\rangle$ and $m\langle \alpha^{\chi}, \beta^{\chi}, \gamma^{\chi}\rangle = v\langle \alpha^{\chi}, \beta^{\chi}, \gamma^{\chi}\rangle l^{-1}u, m^{-1}v \in G_{L^{\chi}}$, where, $L = \{\langle l, \alpha\{l\}, \beta\{l\}, \gamma\{l\}\rangle,\ \ l \in G\}$. Now,

$$(lm)^{-1}(uv) = (m^{-1}l^{-1})(uv) = m^{-1}(l^{-1}u)v = m^{-1}(l^{-1}u)(mm^{-1})v = [m^{-1}(l^{-1}u)m](m^{-1}v) \in G_{L^{\chi}}.$$

As $G_{L^{\chi}}$ is a normal subgroup of G, so, $(lm)^{-1}(uv) \in G_{L^{\chi}} \implies lmL^{\chi} = uvL^{\chi} \implies lm\langle \alpha^{\chi}, \beta^{\chi}, \gamma^{\chi}\rangle = uv\langle \alpha^{\chi}, \beta^{\chi}, \gamma^{\chi}\rangle \implies lmL^{\chi} = uvL^{\chi}$. □

Definition 4.20. The group G/L^{χ} of all χ-NFCSGs of χ-NSVNSG, L of group G is called the factor group of G by L^{χ}.

Proposition 4.21. *Suppose* G/L^{χ} *is the collection of all* χ*-neutrosophic fuzzy cosets (*χ*-NFCs) of a group* G*. That is,* $G/L^{\chi} = \{L^{\chi}_{l} : l \in G\}$*, then the binary operation* $\bigotimes$ *on the set* G/L^{χ} *is as follows,* $L^{\chi}_{l} \bigotimes L^{\chi}_{m} = L^{t}_{lm}$ $\quad l, m \in G$.

Proof. Let $L^{\chi}_{l} = L^{\chi}_{l_1}$ and $L^{\chi}_{m} = L^{\chi}_{m_1}$ for some $l, m, l_1, m_1 \in G$. Taking $g \in G$ be any element, then

$$
\begin{aligned}
[L^{\chi}_{l} \bigotimes L^{\chi}_{m}](g) &= L^{\chi}_{lm}(g)\\
&= \min\{L(g(lm)^{-1}), \chi\}\\
&= \langle \min\{\alpha(g(lm)^{-1}), \chi\}, \min\{\beta(g(lm)^{-1}), \chi\}, \max\{\gamma(g(lm)^{-1}), \chi\}\rangle\\
&= \langle \min\{\alpha((gm^{-1})l^{-1}), \chi\}, \min\{\beta((gm^{-1})l^{-1}), \chi\}, \max\{\gamma((gm^{-1})l^{-1}), \chi\}\rangle\\
&= \{\alpha^{\chi}_{l}(gm^{-1}), \beta^{\chi}_{l}(gm^{-1}), \gamma^{\chi}_{l}(gm^{-1})\}\\
&= L^{\chi}_{l}(gm^{-1})\}\\
&= L^{\chi}_{l_1}(gm^{-1})\\
&= \min\{L((gm^{-1})l_1^{-1}), \chi\}\\
&= \langle \min\{\alpha((gm^{-1})l_1^{-1}), \chi\}, \min\{\beta((gm^{-1})l_1^{-1}), \chi\}, \max\{\gamma((gm^{-1})l_1^{-1}), \chi\}\rangle\\
&= \langle \min\{\alpha((gl_1^{-1})m^{-1}), \chi\}, \min\{\beta((gl_1^{-1})m^{-1}), \chi\}, \max\{\gamma((gl_1^{-1})m^{-1}), \chi\}\rangle\\
&= \{\alpha^{\chi}_{m}(l_1^{-1}g), \beta^{\chi}_{m}(l_1^{-1}g), \gamma^{\chi}_{m}(l_1^{-1}g)\}\\
&= L^{\chi}_{m}(l_1^{-1}g)\\
&= L^{\chi}_{m_1}(l_1^{-1}g)\\
&= \min\{L((gl_1^{-1})m_1^{-1}), \chi\}\\
&= \langle \min\{\alpha((l_1^{-1}g)m_1^{-1}), \chi\}, \min\{\beta((l_1^{-1}g)m_1^{-1}), \chi\}, \max\{\gamma((l_1^{-1})m_1^{-1}g), \chi\}\rangle\\
&= \langle \min\{\alpha(m_1^{-1}(l_1^{-1}g)), \chi\}, \min\{\beta(m_1^{-1}(l_1^{-1}g)), \chi\}, \max\{\gamma(m_1^{-1}(l_1^{-1}g)), \chi\}\rangle\\
&= \langle \min\{\alpha((m_1^{-1}l_1^{-1})g), \chi\}, \min\{\beta((m_1^{-1}l_1^{-1})g), \chi\}, \max\{\gamma((m_1^{-1}l_1^{-1})g), \chi\}\rangle\\
&= \langle \min\{\alpha((m_1l_1)^{-1}g), \chi\}, \min\{\beta((m_1l_1)^{-1}g), \chi\}, \max\{\gamma((m_1l_1)^{-1}g), \chi\}\rangle\\
&= \langle \min\{\alpha((m_1^{-1}l_1^{-1})g), \chi\}, \min\{\beta((m_1^{-1}l_1^{-1})g), \chi\}, \max\{\gamma((m_1^{-1}l_1^{-1})g), \chi\}\rangle\\
&= \langle \min\{\alpha(g(m_1l_1)^{-1}), \chi\}, \min\{\beta(g(m_1l_1)^{-1}), \chi\}, \max\{\gamma(g(m_1l_1)^{-1}), \chi\}\rangle\\
&= \min\{L(g(m_1l_1)^{-1}), \chi\}\\
&= L^{\chi}_{m_1l_1}(g).
\end{aligned}
$$

Therefore $\bigotimes$ is well define operation on G/L^{χ}. □

Proposition 4.22. *The set* G/L^{χ} *of all* χ-NFCs *of* χ*-NSVNSG,* L *of group* G*, from (form?) a group under the well define operation* $\bigotimes$*.*

Remark 4.23. It's simple to verify that the G/L^{χ} identity element is Le^{χ}, where identity of the group G is e, and $L^{\chi}_{l^{-1}}$ is inverse of an element L^{χ}_{l}.

5. Homomorphism

In this section, we will discuss homomorphism on proposed algebraic structure and several results given on it.

Theorem 5.1. *Define a homomorphism* $\hbar : G_1 \longrightarrow G_2$ *of group* G_1 *into a group* G_2*. Let* M *be* χ*-SVNSG of group* G_2*. Then* $\hbar^{-1}(M)$ *is* χ*-SVNSG of* G_1*.*

Proof. Let M be χ-SVNSG of group G_2. If $l_1, l_2 \in G_1$ be any elements, then

$$\begin{aligned}
(\hbar^{-1}(M))^{\chi}(l_1, l_2) &= \hbar^{-1}(M)(l_1, l_2)\\
&= M^{\chi}(\hbar(l_1, l_2))\\
&= M^{\chi}(\hbar(l_1), \hbar(l_2))\\
&\geqslant \min\{M^{\chi}(\hbar(l_1), M^{\chi}\hbar(l_2))\}\\
&= \min\{\hbar^{-1}M^{\chi}(l_1), \hbar^{-1}M^{\chi}(l_2)\}\\
&= \min\{(\hbar^{-1}(M))^{\chi}(l_1), (\hbar^{-1}(M))^{\chi}(l_2)\},\\
(\hbar^{-1}(M))^{\chi}(l_1, l_2) &\geqslant \min\{(\hbar^{-1}(M))^{\chi}(l_1), (\hbar^{-1}(M))^{\chi}(l_2)\},\\
(\hbar^{-1}(M))^{\chi}(l_1) &= \hbar^{-1}(M^{\chi})(l_1) = M^{\chi}(\hbar(l)^{-1}) = M^{\chi}(\hbar(l)) = \hbar^{-1}(M))^{\chi}(l),\\
(\hbar^{-1}(M))^{\chi}(l^{-1}) &= \hbar^{-1}(M)^{\chi}(l).
\end{aligned}$$

Hence, $\hbar^{-1}(M)$ is χ-SVNSG of group G_1. □

Theorem 5.2. *If $\hbar : G_1 \longrightarrow G_2$ be a homomorphism of group G_1 into a group G_2, let M be χ-NSVNSG of group G_2. Then $\hbar^{-1}(M)$ is χ-NSVNSG of G_1.*

Proof. Let M be χ-NSVNSG of group G_2. Let $l_1, l_2 \in G_1$ be any element, then

$$\begin{aligned}
[\hbar^{-1}(M)]^{\chi}(l_1 l_2) &= \hbar^{-1}(M)(l_1 l_2)\\
&= M^{\chi}(\hbar(l_1 l_2)) = M^{\chi}(\hbar(l_1)\hbar(l_2)) = M^{\chi}(\hbar(l_1 l_2)) = M^{\chi}(\hbar(l_2 l_1)) = \hbar^{-1}(M)^{\chi}(l_2 l_1),\\
\hbar^{-1}(M)^{\chi}(l_2 l_1) &= (\hbar^{-1}(M))^{\chi}(l_2 l_1).
\end{aligned}$$

Hence $\hbar^{-1}(M)$ is χ-NSVNSG of group G_1. □

Theorem 5.3. *A natural mapping $\hbar : G \longrightarrow G/L^{\chi}$, where G is a group and G/L^{χ} is a set of all χ-N fuzzy cosets of χ-NSVNSG, L of G defined by $\hbar(l) = L^{\chi}_{l}$ is an onto homomorphism with $\ker\hbar = G_{L^{\chi}}$.*

Proof. Suppose that $l, m \in G$, so $\hbar(lm) = L^{\chi}_{lm} = L^{\chi}_{l} L^{\chi}_{m} = \hbar(l)\hbar(m)$, therefore, $\hbar$ is a homomorphism. Clearly, $\hbar$ is surjective.

$$\ker\hbar = G_{L^{\chi}} = \{l \in G : L^{\chi}_{e} = L^{\chi}_{l}\} = \{l \in G : le^{-1} \in G\} = \{l \in G : l \in G\} = G_{L^{\chi}}.$$

□

Theorem 5.4. *Let L^{χ} be a χ-SVNSG of a group G_1 and $\hbar : G_1 \longrightarrow G_2$ be a bijective homomorphism, then $\hbar(L^{\chi})$ is χ-SVNSG of group G_2.*

Proof. Let L^{χ} be a χ-SVNSG of a group G_1. Assume that $m_1, m_2 \in G_2$, then there exist unique elements $l_1, l_2 \in G_1$ such that $\hbar(l_1) = m_1$ and $\hbar(l_2) = m_2$.

$$\begin{aligned}
(\hbar(L))^{\chi}(m_1 m_2) &= \min\{\hbar(L)(m_1 m_2, \chi\}\\
&= \min\{\hbar(L)(\hbar(l_1)\hbar(l_2)), \chi\}\\
&= \min\{\hbar(L)(\hbar(l_1 l_2)), \chi\}\\
&= \min\{L(l_1 l_2), \chi\}\\
&= L^{\chi}(l_1 l_2)\\
&= \{L^{\chi}(l_1), L^{\chi}(l_2)\}, \quad \forall \ l_1, l_2 \in G_2 \ \text{ such that } \ \hbar(l_1) = m_1 \text{ and } \hbar(l_2) = m_2\\
&\geqslant \min\{\forall\{L^{\chi}(l_1) : \hbar(l_1)\}, \forall\{L^{\chi}(l_2) : \hbar(l_2)\}\}\\
&= \min\{\hbar(L^{\chi})(m_1), \hbar(L^{\chi})(m_2)\} = \min\{(\hbar L)^{\chi}(m_1), (\hbar L)^{\chi}(m_2)\},
\end{aligned}$$

$$\hbar(L^{\chi})(m_1m_2) \geqslant \min\{(\hbar L)^{\chi}(m_1), (\hbar L)^{\chi}(m_2)\},$$
$$\hbar(L^{\chi})(m^{-1}) = \hbar(L)(m^{-1}) = \bigvee\{L^{\chi}(l^{-1}) : \hbar(l^{-1})m^{-1}\} = \bigvee\{L^{\chi}(l) : \hbar(l) = m\} = \hbar(L^{\chi})(m).$$

Hence $\hbar(L)$ is χ-SVNSG of G_2. □

Theorem 5.5. *Let $\hbar : G_1 \longrightarrow G_2$ be a bijective homomorphism and L^{χ} be a χ-NSVNSG of group G_1, then $\hbar(L^{\chi})$ is χ-NSVNSG of group G_2.*

Proof. Since L^{χ} is a χ-NSVNSG of a group G_1, let $m_1, m_2 \in G_2$. Then there exist unique elements $l_1, l_2 \in G_1$ such that $\hbar(l_1) = m_1$ and $\hbar(l_2) = m_2$,

$$\begin{aligned}(\hbar(L))^{\chi}(m_1m_2) &= \min\{\hbar(L)(m_1m_2, \chi\}\\ &= \min\{\hbar(L)(\hbar(l_1)\hbar(l_2)), \chi\}\\ &= \min\{\hbar(L)(\hbar(l_1l_2)), \chi\}\\ &= \min\{L(l_1l_2), \chi\}\\ &= L^{\chi}(l_1l_2) = L^{\chi}(l_2l_1) = \min\{L^{\chi}(l_2l_1)\} = \min\{\hbar(L)(\hbar(m_2m_1), \chi\} = (\hbar(L))^{\chi}(m_2m_1),\\ (\hbar(L))^{\chi}(m_1m_2) &= (\hbar(L))^{\chi}(m_2m_1).\end{aligned}$$

Hence, $\hbar(L^{\chi})$ is χ-NSVNSG of G_2. □

6. Conclusion

In this work, we introduce the notion of χ-single valued neutrosophic set and χ-single valued neutrosophic subgroups and then we discuss several operations and algebraic properties on them, respectively. In further work, researchers can extend this idea in topological spaces, rings, and fields.

Acknowledgment

The authors want to express their gratitude to the anonymous referees for the comprehensive reading of the paper and their pertinent comments and suggestions, which helped improve the manuscript.

References

[1] S. A. Akinleye, F. Smarandache, A. A. Agboola, *On neutrosophic quadruple algebraic structures*, Neutrosophic Sets and Systems, **12** (2016), 16 pages. 1
[2] M. I. Ali, F. Feng, X. Y. Liu, W. K. Min, M. Shabir, *On some new operations in soft set theory*, Comput. Math. Appl., **57** (2009), 1547–1553. 1
[3] S. Alkhazaleh, A. R. Salleh, N. Hassan, *Possibility fuzzy soft set*, Adv. Decis. Sci., **2011** (2011), 19 pages. 1
[4] M. Al-Tahan, *Some results on single valued neutrosophic (weak) polygroups*, Int. J. Neutro. Sci., **2** (2020), 38–46. 1
[5] M. Al-Tahan, B. Davvaz, M. Parimala, *A Note on Single Valued Neutrosophic Sets in Ordered Groupoids*, Int. J Neutro. Sci., **10** (2020), 73–83. 1
[6] I. Arockiarani, J. M. Jency, *More on fuzzy neutrosophic sets and fuzzy neutrosophic topological spaces*, Int. J. Inn. Res. Stud., **3** (2014), 643–652. 1
[7] I. Arockiarani, J. M. Jency, F. N. On, *Continuity in Fuzzy Neutrosophic Topological spaces*, Asian Acad. Res. J. Multidiscip., **1** (2014), 330–340.
[8] I. Arockiarani, I. R. Sumathi, *Fuzzy Neutrosophic Groups*, Adv. Fuzzy Math., **10** (2015), 117–122. 1
[9] K. T. Atanassov, *Intuitionistic fuzzy sets*, Central Sci. and Techn. Library, Bulg. Academy of Sciences, VII ITKRs Session, Sofia, (1984). 1
[10] K. T. Atanassov, S. Stoeva, L. A. Zadeh, *Intuitionistic fuzzy sets*, In Polish Symposium on Interval & Fuzzy Mathematics, **1983** (1983), 23–26. 1
[11] B. Banerjee, *Intuitionistic fuzzy subrings and ideals*, J. Fuzzy Math., **11** (2003), 139–155. 1
[12] S. Broumi, F. Smarandache, M. Dhar, *Rough neutrosophic sets*, Infinite Study, New York, (2014). 1
[13] M. Gulfam, M. K. Mahmood, F. Smarandache, S. Ali, *New Dombi Aggregation Operators on Bipolar Neutrosophic Set with Application in Multi-attribute Decision-making Problems*, J. Intell. Fuzzy Syst., **2021** (2021), 5043–5060. 1

[14] M. Gulzar, G. Abbas, F. Dilawar, *Algebraic Properties of ω-Q-fuzzy subgroup*, Int. J. Math. Comp. Sci. **15** (2020), 265–274.
[15] M. Gulzar, D. Alghazzawi, M. H. Mateen, *A note on complex fuzzy subfield*, Indon. J. Elec. Eng. Comput. Sci., **21** (2021), 1048–1056.
[16] M. Gulzar, D. Alghazzawi, M. H. Mateen, N. Kausar, *A certain class of t-intuitionistic fuzzy subgroups*, IEEE access, **8** (2020), 163260–163268.
[17] M. Gulzar, D. Alghazzawi, M. H. Mateen, M. Premkumar, *On some characterization of Q-complex fuzzy sub-rings*, J. Math. Comput. Sci., **22** (2020), 295–305.
[18] M. Gulzar, M. H. Mateen, D. Alghazzawi, N. Kausar, *A novel applications of complex intuitionistic fuzzy sets in group theory*, IEEE Access, **8** (2020), 196075–196085.
[19] M. Gulzar, M. H. Mateen, Y. M. Chu, D. Alghazzawi, G. Abbas, *Generalized Direct Product of Complex Intuitionistic Fuzzy Subrings*, Int. J. Comput. Intell. Syst., **2021** (2021), 14 pages.
[20] M. S. Hameed, S. Mukhtar, H. N. Khan, S. Ali, M. H. Mateen, M. Gulzar, *Pythagorean Fuzzy N-Soft Groups*, Indon. J. Elec. Eng. Comput. Sci., **2** (2021), 1030–1038. 1
[21] K. Hur, S. Y. Jang, H. W. Kang, *Intuitionistic fuzzy subgroupoids*, Int. J. Fuzzy Log. Intell. Syst., **3** (2003), 72–77. 1
[22] K. Hur, S. Y. Jang, H. W. Kang, *Intuitionistic fuzzy subgroups and cosets*, Honam Math. J., **26** (2004), 17–41.
[23] K. Hur, H. W. Kang, H. K. Song, *Intuitionistic fuzzy subgroups and subrings*, Honam Math. J., **25** (2003), 19–41. 1
[24] M. Ibrahim, A. Agboola, E. Adeleke, S. Akinleye, *Introduction to neutrosophic subtraction algebra and neutrosophic subtraction semigroup*, Int. J Neutro. Sci., **2** (2020), 47–62. 1
[25] J. M. Jency, I. Arockiarani, *Fuzzy neutrosophic subgroupoids*, Asian J. Appl. Sci., **2016** (2016), 13 pages. 1
[26] J. B. Liu, S. Ali, M. K. Mahmood, M. H. Mateen, *On m-polar Diophantine Fuzzy N-soft set with Applications*, Comb. Chem. High Through. Screen., **2020** (2020), 12 pages. 1
[27] M. K. Mahmood, S. Zeng, M. Gulfam, S. Ali, Y. Jin, *Bipolar neutrosophic dombi aggregation operators with application in multi-attribute decision making problems*, IEEE access, **8** (2020), 156600–156614. 1
[28] D. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl., **37** (1999), 19–31. 1
[29] X. D. Peng, Y. Yang, J. P. Song, Y. Jiang, *Pythagorean fuzzy soft set and its application*, Comp. Eng., **41** (2015), 224–229. 1
[30] X. Peng, H. Yuan, Y. Yang, *Pythagorean fuzzy information measures and their applications*, Int. J. Intell. Syst., **32** (2017), 991–1029. 1
[31] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl., **35** (1971), 512–517. 1
[32] S. Sebastian, T. V. Ramakrishnan, *Multi-fuzzy sets: An extension of fuzzy sets*, Fuzzy Inf. Eng., **3** (2011), 35–43. 1
[33] F. Smarandache, *A unifying field in logics: neutrosophic logic*, Mult.-Valued Log, **8** (2002), 385–438. 1
[34] F. Smarandache, *Neutrosophic set–a generalization of the intuitionistic fuzzy set*, Int. J. Pure Appl. Math., **24** (2005), 287–297.
[35] F. Smarandache, *(T, I, F)-Neutrosophic Structures*, Appl. Mech. Mater., **811** (2015), 104–109.
[36] H. Wang, P. Madiraju, Y. Zhang, R. Sunderrraman, *Interval neutrosophic sets*, Mathematics, **1** (2004), 274–277. 1
[37] H. Wang, F. Smarandache, R. Sunderraman, Y. Q. Zhang, *Interval neutrosophic sets and logic: theory and applications in computing*, Neutrosophic Book Series, Phoenix, (2005).
[38] H. Wang, F. Smarandache, Y. Q. Zhang, R. Sunderraman, *Single valued neutrosophic sets*, Multispace and Multi-structure, **4** (2010), 410–413. 1, 2.1, 2.2, 2.3, 2.4, 2.5
[39] Y. Yang, Y. Wang, Y. Zhang, D. Zhang, *Commentary on Generalized intuitionistic fuzzy soft sets with applications in decision-making*, Appl. Soft Comput., **37** (2015), 519–520. 1
[40] R. R. Yager, *Pythagorean fuzzy subsets*, In 2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS), **2013** (2013), 57–61. 1
[41] R. R. Yager, *Pythagorean membership grades in multicriteria decision making*, IEEE Trans. Fuzzy Syst., **22** (2013), 958–965.
[42] R. R. Yager, A. M. Abbasov, *Pythagorean membership grades, complex numbers, and decision making*, Int. J. Intell. Syst., **28** (2013), 436–452. 1
[43] L. A. Zadeh, *Fuzzy sets*, Inf. Control, **8** (1965), 338–353. 1
[44] L. A. Zadeh, *Fuzzy sets, fuzzy logic, and fuzzy systems*, in: Advances in Fuzzy Systems–Applications and Theory, **6** (1996), 394–432. 1
[45] S. Zeng, M. Shoaib, S. Ali, Q. Abbas, M. S. Nadeem, *Complex Vague Graphs and Their Application in Decision-Making Problems*, IEEE access, **8** (2020), 174094–174104. 1
[46] H. D. Zhang, L. G. Xiong, W. Y. Ma, *On interval-valued hesitant fuzzy soft sets*, Math. Probl. Eng., **2015** (2015), 17 pages. 1
[47] H. D. Zhang, L. Shu, *Dual hesitant fuzzy soft set and its properties*, Fuzzy Syst. Oper. Res. Manag., **2016** (2016), 171–182. 1