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Fixed point for fuzzy mappings in different generalized types of metric spaces

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Abstract

The aim of the paper is to establish some fixed point theorems for fuzzy mappings satisfying an implicit relation in left and right quasi-metric spaces. These theorems generalize the corresponding results in [S. Heilpern, J. Math. Anal. Appl., **83** (1981), 566–569], [V. Popa, Stud. Cercet. Ştiinţ. Ser. Mat. Univ. Bacău, **7** (1997), 127–133].

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1. Preliminaries

In this section, we mention some concepts and results for fuzzy mappings in metric spaces.

In [8], an element in any fuzzy set has a degree of belonging, a membership function may be used in order to introduce the value of degree of belonging for any element to a set, the value of degree of belonging takes real values on the whole closed interval $[0, 1]$. The *membership function* is

$$\mu_A : X \longrightarrow [0, 1].$$

Let (X, d) be a metric linear space. A *fuzzy set* in X is a function $A : X \longrightarrow [0, 1]$, i.e., it is an element of I^X where $I = [0, 1]$. If A is a fuzzy set and $x \in X$, then the function value $A(x)$ is called the grade of membership of $x \in A$. The collection of all fuzzy sets in X is denoted by $\mathfrak{F}(X)$. Let $A \in \mathfrak{F}(X)$ and α*-level* set of A, defined by

$$A_\alpha = \{x : A(x) \geqslant \alpha\} \text{ if } \alpha \in (0, 1]$$

and

$$A_0 = \overline{\{x : A(x) > 0\}},$$

whenever $\overline{\{\ \}}$ is the closure of set (non fuzzy) $\{\ \}$.

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Definition 1.1 ([5]). A fuzzy set A in X is an *approximate quantity* if its α-level set is a nonempty compact subset(non fuzzy) of X for each $\alpha \in [0,1]$. The set of all an approximate quantities denoted by $W^*(X)$, is a sub collection of $\mathfrak{F}(X)$.

Definition 1.2 ([5]). Let X be an arbitrary set and Y be a metric space. A mapping F is said to be a *fuzzy mapping* if F is a mapping from the set X into $W^*(Y)$, i.e., $F(x) \in W^*(Y)$ for each $x \in X$.

Definition 1.3 ([5]). Let (X,d) be a metric space and let $A, B \in W^*(X)$, $\alpha \in [0,1]$ and $CP(X)$ be a set of all nonempty compact subsets of X. Then it is defined the following

$$p_\alpha(A,B) = \inf\{d(x,y) : x \in A_\alpha, y \in B_\alpha\}, \qquad p(A,B) = \sup_\alpha p_\alpha(A,B),$$
$$D_\alpha(A,B) = H(A_\alpha, B_\alpha), \qquad D(A,B) = \sup\{D_\alpha(A,B) : \alpha \in [0,1]\},$$

where D_α is a α-distance, D is a distance between A and B and the Hausdorff distance H is defined on subsets A and B in the collection $CP(X)$ such that

$$H(A,B) = \max\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\}.$$

Proposition 1.4 ([12]). *If $A, B \in CP(X)$ and $a \in A$, then there exists $b \in B$ such that*

$$d(a,b) \leqslant H(A,B).$$

Lemma 1.5 ([5]). *Let (X,d) be a metric space and let $x \in X$, $A \in W^*(X)$. Then $x \in A_\alpha$ or $\{x\} \subseteq A$ if and only if*

$$p_\alpha(x,A) = 0.$$

Lemma 1.6 ([5]). *Let (X,d) be a metric space and let $A \in W^*(X)$. Then*

$$p_\alpha(x,A) \leqslant d(x,y) + p_\alpha(y,A),$$

for all $x, y \in X$ and $\alpha \in [0,1]$.

Lemma 1.7 ([5]). *Let (X,d) be a metric space and let $\{x_0\} \subseteq A$. Then*

$$p_\alpha(x_0,B) \leqslant D_\alpha(A,B),$$

for each $A, B \in W^*(X)$ and $\alpha \in [0,1]$.

Lemma 1.8 ([5]). *Let (X,d) be a metric space, $T : X \longrightarrow W^*(X)$ be fuzzy mapping and $x_0 \in X$. Then there exists $x_1 \in X$ such that $\{x_1\} \subset T(x_0)$.*

Following Popa [13], let G be the family of all continuous mappings $g : [0,\infty)^6 \longrightarrow [0,\infty)$ satisfying the following properties:

(g_1) g is non-decreasing in the 1st coordinate and g is non-increasing in 3rd, 4th, 5th, and 6th coordinate variables;

(g_1) there exists $k \in (0,1)$ such that for every $u, v \in [0,\infty)$, $g(u,v,u,v,u+v,0) \leqslant 0$ implies $u \leqslant kv$;

(g_1) if $u \in [0,\infty)$ such that $g(u,0,0,u,0,u) \leqslant 0$, then $u = 0$.

2. Fixed point theorems for fuzzy mappings in left quasi-metric spaces

First, we give the concept of left quasi-metric spaces (abbrev., lq-metric spaces) as follows.

Definition 2.1. Let X be a nonempty set. A map $d : X \times X \longrightarrow [0,\infty)$ is called a *distance* on X. A pair (X,d) is said to be a *distance space*. Now, we list some conditions as follows:

(i) $d(x,y) = 0$ or $d(y,x) = 0$ if and only if $x = y$;

(ii) $d(x,y) \leqslant d(z,x) + d(z,y)$,

for every $x, y, z \in X$. If d satisfies the conditions (i) and (ii), then (X, d) is called an lq-*metric space*.

Remark 2.2. Clearly, the concept of lq-metric spaces is a generalization of the concept of metric spaces.

Definition 2.3. Let (X, d) be an lq-metric space. We say that $\{x_n\}$ lq-*converges to* x if and only if $\lim_{n\to\infty} d(x_n, x) = 0$. In this case, x is called lq-limit of x_n.

Definition 2.4. Let (X, d) be an lq-metric space. We say that $\{x_n\}$ is *Cauchy* if and only if for every $\epsilon > 0$ there exists a positive integer $N = N(\epsilon)$ such that $d(x_n, x_m) < \epsilon$ for all $n \geqslant m > N$.

Definition 2.5. An lq-metric space (X, d) is *complete* if every Cauchy sequence in X is lq-convergent.

Now, we state the following lemmae without proof.

Lemma 2.6. *Let (X, d) be an lq-metric space and let $x \in X$, $A \in W^*(X)$. Then $x \in A_\alpha$ (i.e., $\{x\} \subseteq A$) if and only if*

$$p_\alpha(x, A) = 0.$$

Lemma 2.7. *Let (X, d) be an lq-metric space and let $\{x_0\} \subseteq A$. Then*

$$p_\alpha(x_0, B) \leqslant D_\alpha(A, B),$$

for each $A, B \in W^*(X)$ and $\alpha \in [0, 1]$.

Lemma 2.8. *Let (X, d) be an lq-metric space, $T : X \longrightarrow W^*(X)$ be a fuzzy mapping and $x_0 \in X$. Then there exists $x_1 \in X$ such that $\{x_1\} \subset T(x_0)$.*

Lemma 2.9. *Let (X, d) be an lq-metric space. Then any subsequence of lq-convergent sequence in X is lq-convergent.*

Also, we state and prove the following lemma.

Lemma 2.10. *Let (X, d) be an lq-metric space and let $A \in W^*(X)$ and A. Then*

$$p_\alpha(x, A) \leqslant d(y, x) + p_\alpha(y, A),$$

for each $A, B \in W^*(X)$ and $\alpha \in [0, 1]$.

Proof. We fined that

$$p_\alpha(x, A) = \inf_{z\in A_\alpha} d(x, z) \leqslant \inf_{z\in A_\alpha} (d(y, x) + d(y, z)) = d(y, x) + \inf_{z\in A_\alpha} d(y, z) = d(y, x) + p_\alpha(y, A).$$

□

Now, we are ready to state and prove our main theorem in the following way.

Theorem 2.11. *Let (X, d) be complete lq-metric space, $x_0 \in X$, and $T : X \longrightarrow W^*(X)$ be a fuzzy mapping. If there is a $g \in G$ such that, $\forall\, x, y \in X$*

$$g(D(Tx, Ty), d(x, y), p(Tx, x), p(Ty, y), p(y, Tx), p(x, Ty)) \leqslant 0,$$

then T has a fixed point.

Proof. Let x_0 be an arbitrary point in X, then by Lemma 2.8, then there exists $x_1 \in X$ such that $\{x_1\} \subset Tx_0$. Also, for $x_1 \in X$, there exists $x_2 \in X$ such that $\{x_2\} \subset Tx_1$. $(Tx_0)_1, (Tx_1)_1 \in CP(X)$, from Proposition 1.4, then $d(x_2, x_1) \leqslant D_1(Tx_1, Tx_0)$,

$$\begin{aligned} & g\left(d(x_2,x_1), d(x_1,x_0), d(x_2,x_1), d(x_1,x_0), d(x_1,x_0)+d(x_2,x_1), 0\right) \\ &\quad = g\left(d(x_2,x_1), d(x_1,x_0), d(x_2,x_1), d(x_1,x_0), d(x_1,x_0)+d(x_2,x_1)+d(x_2,x_2), 0\right) \\ &\quad \leqslant g\left(d(x_2,x_1), d(x_1,x_0), d(x_2,x_1), d(x_1,x_0), d(x_1,x_0)+d(x_1,x_2), 0\right) \\ &\quad \leqslant g\left(d(x_2,x_1), d(x_1,x_0), d(x_2,x_1), d(x_1,x_0), d(x_0,x_2), 0\right) \\ &\quad \leqslant g\left(D_1(Tx_1,Tx_0), d(x_1,x_0), p(Tx_1,x_1), p(Tx_0,x_0), p(x_0,Tx_1), p(x_1,Tx_0)\right) \\ &\quad \leqslant g\left(D(Tx_1,Tx_0), d(x_1,x_0), p(Tx_1,x_1), p(Tx_0,x_0), p(x_0,Tx_1), p(x_1,Tx_0)\right) \leqslant 0. \end{aligned}$$

From (g_2), there exists $k \in (0,1)$ such that $d(x_2, x_1) \leqslant kd(x_1, x_0)$. Continuing in this way we produce a sequence $\{x_n\}$ in X such that $x_n \subset T(x_{n-1})$ and

$$d(x_n, x_{n-1}) \leqslant k^{n-1} d(x_1, x_0).$$

It follows by induction that $d(x_n, x_{n-1}) \leqslant k^{n-1} d(x_1, x_0)$ for each $n \in \mathbb{N}$. Since

$$\begin{aligned} d(x_n, x_m) &\leqslant d(x_{n-1}, x_n) + d(x_{n-1}, x_m) \\ &\leqslant d(x_n, x_{n-1}) + d(x_n, x_n) + d(x_{n-2}, x_{n-1}) + d(x_{n-2}, x_m) \\ &\leqslant d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) + d(x_{n-1}, x_{n-1}) + \cdots + d(x_{m+1}, x_m) \\ &\leqslant [k^{n-1} + k^{n-2} + \cdots + k^m] d(x_1, x_0) \\ &\leqslant [k^{n-1} + k^{n-2} + \cdots] d(x_1, x_0). \end{aligned}$$

Furthermore, for $n \geqslant m$ we have

$$d(x_n, x_m) \leqslant \sum_{i=0}^{n-m-1} d(x_{n-i}, x_{n-i-1}) \leqslant \sum_{i=m}^{n-1} k^i d(x_1, x_0) \leqslant \frac{k^m}{(1-k)} d(x_1, x_0).$$

Then the sequence $\{x_n\}$ is Cauchy. Since (X, d) is complete, then there exists $x^* \in X$ such that $\lim_{n\to\infty} x_n = x^*$. By Lemma 2.9, we obtain the $\lim_{n\to\infty} x_{n+1} = x^*$. By Lemma 2.7 and Lemma 2.10, we get

$$p_\alpha(x^*, Tx^*) \leqslant d(x_{n+1}, x^*) + p_\alpha(x_{n+1}, Tx^*) \leqslant d(x_{n+1}, x^*) + D_\alpha(Tx_n, Tx^*) = d(x_{n+1}, x^*) + D_\alpha(x_{n+1}, Tx^*),$$

for each $\alpha \in [0, 1]$. Taking supremum on α in the last inequality, we obtain from the property (g_1) of G that

$$\begin{aligned} & g\left(p(x_{n+1}, Tx^*), d(x_n, x^*), d(x_{n+1}, x_n), p(x^*, Tx^*) + d(x^*, x^*), d(x_{n+1}, x^*) + d(x_{n+1}, x_{n+1}), p(x_n, Tx^*)\right) \\ &\quad \leqslant g\left(p(x_{n+1}, Tx^*), d(x_n, x^*), d(x_{n+1}, x_n), p(Tx^*, x^*), d(x^*, x_{n+1}), p(x_n, Tx^*)\right) \\ &\quad \leqslant g\left(D_1(Tx_n, Tx^*), d(x_n, x^*), p(Tx_n, x_n), p(Tx^*, x^*), p(x^*, Tx_n), p(x_n, Tx^*)\right) \\ &\quad \leqslant g\left(D(Tx_n, Tx^*), d(x_n, x^*), p(Tx_n, x_n), p(Tx^*, x^*), p(x^*, Tx_n), p(x_n, Tx^*)\right) \leqslant 0. \end{aligned}$$

As $n \to \infty$, we have that

$$g(p(x^*, Tx^*), 0, 0, p(x^*, Tx^*), 0, p(x^*, Tx^*)) \leqslant 0.$$

By the condition (g_3) of G, we have that $p(x^*, Tx^*) = 0$. So by Lemma 2.6 we conclude that T has a fixed point. □

Remark 2.12. Theorem 2.11 generalizes and improves Theorem 3.1 in [8], where X is complete lq-metric space instead of X is complete metric linear space.

Corollary 2.13. *Let* (X, d) *be complete* lq*-metric space,* $x_0 \in X$*, and* $T : X \longrightarrow W^*(X)$ *be a fuzzy mapping. If there exists* $k \in (0, 1)$ *such that*

$$D(Tx, Ty) \leqslant kd(x, y) \ \textit{for each} \ \ x, y \ \in X,$$

then T *has a fixed point.*

Proof. We consider the function $g : [0, \infty)^6 \longrightarrow [0, \infty)$ defined by

$$g(x_1, x_2, x_3, x_4, x_5, x_6) = x_1 - kx_2.$$

Since $g \in G$ we can apply Theorem 2.11 and obtain Corollary 2.13. □

3. Fixed point theorems for fuzzy mappings in right quasi-metric spaces

Now, we introduce the concept of right quasi-metric spaces (abbrev., rq-metric spaces) as follows.

Definition 3.1. Let X be a nonempty set. A map $d : X \times X \longrightarrow [0, \infty)$ is called a *distance* on X. A pair (X, d) is said to be a distance space. Now, we list some conditions as follows:

(i) $d(x, y) = 0$ or $d(y, x) = 0$ if and only if $x = y$;
(ii) $d(x, y) \leqslant d(x, z) + d(y, z)$,

for every $x, y, z \in X$. If d satisfies the conditions (i) and (ii), then (X, d) is called an rq*-metric space*.

Remark 3.2. Clearly, the concept of rq-metric spaces is different generalization of the concept of metric spaces.

Definition 3.3. Let (X, d) be an rq-metric space. We say that $\{x_n\}$ rq*-converges to* x if and only if $\lim_{n\to\infty} d(x, x_n) = 0$. In this case, x is called rq-limit of x_n.

Definition 3.4. Let (X, d) be an rq-metric space. We say that $\{x_n\}$ is Cauchy if and only if for every $\epsilon > 0$ there exists a positive integer $N = N(\epsilon)$ such that $d(x_n, x_m) < \epsilon$ for all $m \geqslant n > N$.

Definition 3.5. An rq-metric space (X, d) is complete if every Cauchy sequence in X is rq-convergent.

Now, we state the following lemmae without proof.

Lemma 3.6. *Let* (X, d) *be an* rq*-metric space and let* $x \in X$, $A \in W^*(X)$*. Then* $x \in A_\alpha$ *(i.e.,* $\{x\} \subseteq A$*) if and only if*

$$p_\alpha(A, x) = 0.$$

Lemma 3.7. *Let* (X, d) *be an* rq*-metric space and let* $\{x_0\} \subseteq A$*. Then*

$$p_\alpha(B, x_0) \leqslant D_\alpha(B, A),$$

for each $A, B \in W^*(X)$ *and* $\alpha \in [0, 1]$.

Lemma 3.8. *Let* (X, d) *be an* rq*-metric space,* $T : X \longrightarrow W^*(X)$ *be a fuzzy mapping and* $x_0 \in X$*. Then there exists* $x_1 \in X$ *such that* $\{x_1\} \subset T(x_0)$.

Lemma 3.9. *Let* (X, d) *be an* rq*-metric space. Then any subsequence of* rq*-convergent sequence in* X *is* rq*-convergent.*

Also, we state and prove the following lemma.

Lemma 3.10. *Let* (X, d) *be* rq*-metric space and let* $A \in W^*(X)$*. Then*

$$p_\alpha(A, x) \leqslant p_\alpha(A, y) + d(x, y),$$

for each $A, B \in W^*(X)$ *and* $\alpha \in [0, 1]$.

Proof. We fined that

$$p_\alpha(A,x) = \inf_{z\in A_\alpha} d(z,x) \leqslant \inf_{z\in A_\alpha} (d(z,y)+d(x,y)) = \inf_{z\in A_\alpha} d(z,y)+d(x,y) = p_\alpha(A,y)+d(x,y).$$

□

Now, we are ready to state and prove our main theorem in the following way.

Theorem 3.11. *Let (X,d) be complete rq-metric space, $x_0 \in X$, and $T : X \longrightarrow W^*(X)$ be a fuzzy mapping. If there is a $g \in G$ such that, $\forall\, x,y \in X$,*

$$g(D(Tx,Ty), d(x,y), p(x,Tx), p(y,Ty), p(y,Tx), p(Ty,x)) \leqslant 0,$$

then T has a fixed point.

Proof. Let x_0 be an arbitrary point in X, then by Lemma 3.8, then there exists $x_1 \in X$ such that $\{x_1\} \subset Tx_0$. Also, for $x_1 \in X$, there exists $x_2 \in X$ such that $\{x_2\} \subset Tx_1$. $(Tx_0)_1, (Tx_1)_1 \in CP(X)$, from Proposition 1.4, then $d(x_1,x_2) \leqslant D_1(Tx_0,Tx_1)$.

$$\begin{aligned}
& g\left(d(x_1,x_2), d(x_0,x_1), d(x_1,x_2), d(x_0,x_1), d(x_0,x_1)+d(x_1,x_2), 0\right)\\
&\quad = g\left(d(x_1,x_2), d(x_0,x_1), d(x_1,x_2), d(x_0,x_1), d(x_0,x_1)+d(x_2,x_2)+d(x_1,x_2), 0\right)\\
&\quad \leqslant g\left(d(x_1,x_2), d(x_0,x_1), d(x_1,x_2), d(x_0,x_1), d(x_0,x_1)+d(x_2,x_1), 0\right)\\
&\quad \leqslant g\left(d(x_1,x_2), d(x_0,x_1), d(x_1,x_2), d(x_0,x_1), d(x_0,x_2), 0\right)\\
&\quad \leqslant g\left(D_1(Tx_0,Tx_1), d(x_0,x_1), p(x_1,Tx_1), p(x_0,Tx_0), p(x_0,Tx_1), p(Tx_0,x_1)\right)\\
&\quad \leqslant g\left(D(Tx_0,Tx_1), d(x_0,x_1), p(x_1,Tx_1), p(x_0,Tx_0), p(x_0,Tx_1), p(Tx_0,x_1)\right) \leqslant 0.
\end{aligned}$$

From (g_2), there exists $k \in (0,1)$ such that $d(x_1,x_2) \leqslant kd(x_0,x_1)$. Continuing in this way we produce a sequence $\{x_n\}$ in X such that $x_{n+1} \subset T(x_n)$ and

$$d(x_n,x_{n+1}) \leqslant k^n d(x_0,x_1).$$

It follows by induction that $d(x_n,x_{n+1}) \leqslant k^n d(x_0,x_1)$ for each $n \in \mathbb{N}$. Since

$$\begin{aligned}
d(x_n,x_m) &\leqslant d(x_n,x_{n+1})+d(x_m,x_{n+1})\\
&\leqslant d(x_n,x_{n+1})+d(x_m,x_m)+d(x_{n+1},x_m)\\
&\leqslant d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})+d(x_m,x_{n+2})\\
&\leqslant d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})+d(x_m,x_m)+d(x_{n+2},x_{n+3})+\cdots+d(x_{m-1},x_m).
\end{aligned}$$

Then for $n < m$ we have

$$d(x_n,x_m) \leqslant \sum_{i=0}^{m-n-1} d(x_{n+i},x_{n+i+1}) \leqslant \sum_{i=n}^{m-1} k^i d(x_0,x_1) \leqslant \frac{k^n}{1-k} d(x_0,x_1).$$

Then the sequence $\{x_n\}$ is Cauchy. Since (X,d) is complete, then there exists $x^* \in X$ such that $\lim\limits_{n\to\infty} x_n = x^*$. By Lemma 3.9, we obtain the $\lim\limits_{n\to\infty} x_{n+1} = x^*$. By Lemma 3.10 and Lemma 3.7, we get

$$p_\alpha(Tx^*,x^*) \leqslant p_\alpha(Tx^*,x_{n+1})+d(x^*,x_{n+1}) \leqslant D_\alpha(Tx^*,Tx_n)+d(x^*,x_{n+1}) = D_\alpha(Tx^*,x_{n+1})+d(x^*,x_{n+1}),$$

for each $\alpha \in [0,1]$. Taking supremum on α in the last inequality, we obtain from the property (g_1) of G that

$$g\left(p(Tx^*,x_{n+1}), d(x^*,x_n), d(x_n,x_{n+1}), p(x^*,x^*)+d(Tx^*,x^*), d(x^*,x_{n+1}), p(Tx^*,x_n)\right)$$

$$\begin{aligned}&\leqslant g\left(p(Tx^*, x_{n+1}), d(x^*, x_n), d(x_n, x_{n+1}), p(x^*, Tx^*), d(x^*, x_{n+1}), p(Tx^*, x_n)\right)\\&\leqslant g\left(D_1(Tx^*, Tx_n), d(x^*, x_n), p(x_n, Tx_n), p(x^*, Tx^*), p(x^*, Tx_n), p(Tx^*, x_n)\right)\\&\leqslant g\left(D(Tx^*, Tx_n), d(x^*, x_n), p(x_n, Tx_n), p(x^*, Tx^*), p(x^*, Tx_n), p(Tx^*, x_n)\right) \leqslant 0.\end{aligned}$$

As $n \to \infty$, we have that

$$g(d(Tx^*, x^*), 0, 0, d(Tx^*, x^*), 0, d(Tx^*, x^*)) \leqslant 0.$$

By the condition (g_3) of G, then $d(Tx^*, x^*) = 0$. So by Lemma 3.6 we conclude that T has a fixed point. □

Remark 3.12. Theorem 3.11 generalizes and improves Theorem 3.1 in [8] where X is complete rq-metric space instead of X being complete metric linear space.

Corollary 3.13. *Let* (X, d) *be complete* rq*-metric space,* $x_0 \in X$*, and* $T : X \longrightarrow W^*(X)$ *be a fuzzy mapping. If there exists* $k \in (0, 1)$ *such that*

$$D(Tx, Ty) \leqslant kd(x, y) \quad \textit{for each} \quad x, y \in X,$$

then T *has a fixed point.*

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References

[1] M. Abbas, B. Damjanović, R. Lazović, *Fuzzy common fixed point theorems for generalized contractive mappings*, Appl. Math. Lett., **23** (2010), 1326–1330.
[2] H. M. Abu-Donia, *Common fixed points theorems for fuzzy mappings in metric spaces under φ-contraction condition*, Chaos Solitons Fractals, **34** (2007), 538–543.
[3] A. Azam, M. Arshad, I. Beg, *Fixed points of fuzzy contractive and fuzzy locally contractive maps*, Chaos Solitons Fractals, **42** (2009), 2836–2841.
[4] A. Azam, I. Beg, *Common fixed points of fuzzy maps*, Math. Comput. Model., **49** (2009), 1331–1336.
[5] I. Beg, M. A. Ahmed, *Fixed point for fuzzy contraction mappings satisfying an implicit relation*, Mat. Vesnik, **66** (2014), 351–356. 1.1, 1.2, 1.3, 1.5, 1.6, 1.7, 1.8
[6] R. K. Bose, D. Sahani, *Fuzzy mappings and fixed point theorems*, Fuzzy Sets and Systems, **21** (1987), 53–58.
[7] A. P. Farajzadeh, A. Kaewcharoen, S. Plubtieng, *An Application of Fixed Point Theory to a Nonlinear Differential Equation*, Abstr. Appl. Anal., **2014** (2014), 7 pages.
[8] S. Heilpern, *Fuzzy mappings and fixed point theorem*, J. Math. Anal. Appl., **83** (1981), 566–569. 1, 2.12, 3.12
[9] M. Imdad, J. Ali, *A general fixed point theorem in fuzzy metric spaces via an implicit function*, J. Appl. Math. Inform., **26** (2008), 591–603.
[10] E. Karapinar, B. Samet, *Generalized $\alpha - \psi$-contractive type mappings and related fixed point theorems with applications*, Abstr. Appl. Anal., **2012** (2012), 793–486.
[11] B. S. Lee, S. J. Cho, *A fixed point theorem for contractive type fuzzy mappings*, Fuzzy Sets and Systems, **61** (1994), 309–312.
[12] S. B. Nadler, *Multivalued contraction mappings*, Pacific J. Math., **30** (1969), 475–488. 1.4
[13] V. Popa, *Fixed point theorems for implicit contractive mappings*, Stud. Cercet. Ştiinţ. Ser. Mat. Univ. Bacău, **7** (1997), 127–133. 1
[14] R. A. Rashwan, M. A. Ahmed, *Common fixed point theorems for fuzzy mappings*, Arch. Math. (Brno), **38** (2002), 219–226.
[15] B. E. Rhoades, *A common fixed point theorem for sequence of fuzzy mappings*, Int. J. Math. Math. Sci., **8** (1995), 447–450.
[16] L. A. Zadeh, *Fuzzy sets*, Inform. Control, **8** (1965), 338–353.