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A note on set valued maps on admissible extension type spaces

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Abstract

We present a number of fixed point results for general classes of maps defined on a variety of extension type and admissible type spaces.

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1. Introduction

In this paper we present fixed point results for classes of maps defined on Hausdorff topological spaces. These include the PK, BPK, and KKM types maps so in particular include the Kakutani maps, the acyclic maps, the O'Neill maps, the admissible maps of Gorniewicz, the approachable maps, the permissible maps of Dzedzej and others (see [9]). Our spaces include ES(compact), AES(compact), ES admissible, and AES admissible for a subclass of the KKM class. Our results are new and extend previously know results in the literature; see [3–5, 9, 10, 12–16] and the references therein.

Now we describe the maps considered in this paper. Let H be the Čech homology functor with compact carriers and coefficients in the field of rational numbers K from the category of Hausdorff topological spaces and continuous maps to the category of graded vector spaces and linear maps of degree zero. Thus $H(X) = \{H_q(X)\}$ (here X is a Hausdorff topological space) is a graded vector space, $H_q(X)$ being the q-dimensional Čech homology group with compact carriers of X. For a continuous map $f : X \to X$, $H(f)$ is the induced linear map $f_\star = \{f_{\star q}\}$, where $f_{\star q} : H_q(X) \to H_q(X)$. A space X is acyclic if X is nonempty, $H_q(X) = 0$ for every $q \geqslant 1$, and $H_0(X) \approx K$.

Let X, Y, and Γ be Hausdorff topological spaces. A continuous single valued map $p : \Gamma \to X$ is called a Vietoris map (written $p : \Gamma \Rightarrow X$) if the following two conditions are satisfied:

(i) for each $x \in X$, the set $p^{-1}(x)$ is acyclic;

(ii) p is a perfect map, i.e., p is closed and for every $x \in X$ the set $p^{-1}(x)$ is nonempty and compact.

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Let $\phi : X \to Y$ be a multivalued map (note for each $x \in X$ we assume $\phi(x)$ is a nonempty subset of Y). A pair (p, q) of single valued continuous maps of the form $X \overset{p}{\leftarrow} \Gamma \overset{q}{\rightarrow} Y$ is called a selected pair of ϕ (written $(p, q) \subset \phi$) if the following two conditions hold:

(i) p is a Vietoris map;
(ii) $q(p^{-1}(x)) \subset \phi(x)$ for any $x \in X$.

Now we define the admissible maps of Gorniewicz [9]. A upper semicontinuous map $\phi : X \to Y$ with compact values is said to be admissible (and we write $\phi \in Ad(X, Y)$) provided there exists a selected pair (p, q) of ϕ. An example of an admissible map is a Kakutani map. A upper semicontinuous map $\phi : X \to CK(Y)$ is said to be Kakutani (and we write $\phi \in Kak(X, Y)$); here Y is a Hausdorff topological vector space and $CK(Y)$ denotes the family of nonempty, convex, compact subsets of Y.

Now we consider a general class of maps, namely the PK maps of Park. Let X and Y be Hausdorff topological spaces. Given a class $\mathcal{X}$ of maps, $\mathcal{X}(X, Y)$ denotes the set of maps $F : X \to 2^Y$ (nonempty subsets of Y) belonging to $\mathcal{X}$, and $\mathcal{X}_c$ the set of finite compositions of maps in $\mathcal{X}$. We let

$$\mathcal{F}(\mathcal{X}) = \{Z : \text{Fix}(F) \neq \emptyset \text{ for all } F \in \mathcal{X}(Z, Z)\},$$

where $\text{Fix}(F)$ denotes the set of fixed points of F.

The class $\mathcal{U}$ of maps is defined by the following properties:

(i) $\mathcal{U}$ contains the class C of single valued continuous functions;
(ii) each $F \in \mathcal{U}_c$ is upper semicontinuous and compact valued; and
(iii) $B^n \in \mathcal{F}(\mathcal{U}_c)$ for all $n \in \{1, 2, \ldots\}$; here $B^n = \{x \in \mathbf{R}^n : \|x\| \leqslant 1\}$.

We say $F \in PK(X, Y)$ if for any compact subset K of X there is a $G \in \mathcal{U}_c(K, Y)$ with $G(x) \subseteq F(x)$ for each $x \in K$. Recall PK is closed under compositions [12, 13].

Next we describe a class of maps more general than the PK maps in our setting. Let X be a convex subset of a Hausdorff topological vector space and Y a Hausdorff topological space. If $S, T : X \to 2^Y$ are two set valued maps such that $T(co(A)) \subseteq S(A)$ for each finite subset A of X, then we call S a generalized KKM mapping w.r.t. T. Now the set valued map $T : X \to 2^Y$ is said to have the KKM property if for any generalized KKM map $S : X \to 2^Y$ w.r.t. T the family $\{\overline{S(x)} : x \in X\}$ has the finite intersection property (the intersection of each finite subfamily is nonempty). We let

$$KKM(X, Y) = \{T : X \to 2^Y | T \text{ has the KKM property}\}.$$

Note $PK(X, Y) \subset KKM(X, Y)$ (see [5]). Next we recall the following result [5].

Theorem 1.1. *Let X be a convex subset of a Hausdorff topological vector space and Y, Z be Hausdorff topological spaces.*

(i) $T \in KKM(X, Y)$ *iff* $T|_\triangle \in KKM(\triangle, Y)$ *for each polytope* $\triangle$ *in* X;
(ii) *if* $T \in KKM(X, Y)$ *and* $f \in C(Y, Z)$, *then* $fT \in KKM(X, Z)$;
(iii) *if Y is a normal space, $\triangle$ a polytope of X and if $T : \triangle \to 2^Y$ is a set valued map such that for each $f \in C(Y, \triangle)$ we have that fT has a fixed point in $\triangle$, then $T \in KKM(\triangle, Y)$.*

Next we recall the following fixed point result for KKM maps. Recall a nonempty subset W of a Hausdorff topological vector space E is said to be admissible if for any nonempty compact subset K of W and every neighborhood V of 0 in E there exists a continuous map $h : K \to W$ with $x - h(x) \in V$ for all $x \in K$ and $h(K)$ is contained in a finite dimensional subspace of E (for example every nonempty convex subset of a locally convex space is admissible).

Theorem 1.2 ([4])**.** *Let X be an admissible convex set in a Hausdorff topological vector space and $T \in KKM(X, X)$ be a closed compact map. Then T has a fixed point in X.*

Next we will present an analogue of Theorem 1.1 (ii) for Tf (see [14]).

Theorem 1.3. *Let X be an admissible convex set in a Hausdorff topological vector space, Y a convex set in a Hausdorff topological vector space, and Y a normal space. If $T \in KKM(X,Y)$ is a upper semicontinuous map with compact values and $f \in C(Y,X)$, then $Tf \in KKM(Y,Y)$.*

Also we recall from [14] the following two properties. Let C and X be convex subsets of a Hausdorff topological vector space E with $C \subseteq X$ and Y a Hausdorff topological space.

(i) If $T \in KKM(X,Y)$, then $G \equiv T|_C \in KKM(C,Y)$.
(ii) If $T \in KKM(X,Y)$, $T(X) \subseteq Z \subseteq Y$ and Z is closed in Y, then $T \in KKM(X,Z)$.

Next we describe the better admissible class of maps BPK due to Park [16]. Let X be a convex subset of a Hausdorff topological vector space and Y a Hausdorff topological space. Now $F \in BPK(X,Y)$ if $F : X \to 2^Y$ and for any polytope P in X and any continuous map $f : F(P) \to P$ we have that $f(F|_P) : P \to 2^P$ has a fixed point.

From Theorem 1.1 (parts (i) and (iii)) note $BPK(X,Y) \subseteq KKM(X,Y)$ when Y is normal (the classes coincide for closed compact maps). We also note the following properties.

Let C and X be convex subsets of a Hausdorff topological vector space E with $C \subseteq X$ and Y a Hausdorff topological space.

(i) If $F \in BPK(X,Y)$, then $G \equiv F|_C \in BPK(C,Y)$.

Consider any polytope P in C and any continuous map $f : G(P) \to P$. Now since P is a polytope in X and (note $G(P) = F(P)$ since $P \subseteq C$) $f : F(p) \to P$ is a continuous map, then, since $F \in BPK(X,Y)$ there exists an $x \in P$ with $x \in fF|_P(x)$, i.e., $x \in fG(x)$, since $x \in P \subseteq C$. Thus $G = F|_C \in BPK(C,Y)$.

(ii) If $F \in BPK(X,Y)$ with $F(X) \subseteq Z \subseteq Y$, then $F \in BPK(X,Z)$.

Note $F : X \to 2^Y$ and let $G : X \to 2^Z$ be the map obtained by restricting the range of F. Consider any polytope P in X and any continuous map $f : G(P) \to P$. Now since $G(P) = F(P)$, then $f : F(P) \to P$ is continuous and since $F \in BPK(X,Y)$, there exists an $x \in P$ with $x \in fF|_P(x) = fG|_P(x)$. Thus $F \in BPK(X,Z)$.

(iii) If $F \in BPK(X,Y)$ and $f \in C(Y,X)$, then $fF \in BPK(X,X)$.

Note $fF : X \to 2^X$. Consider any polytope P in X and any continuous map $g : fF(P) \to P$. We must show there exists an $x \in P$ with $x \in g(fF)|_P(x)$. To see this note $gfF|_P = hF|_P : P \to 2^P$, where $h = gf : F(p) \to P$ is a continuous map (note $h(F(P)) = g(fF(P)) \subseteq P$). Since $F \in BPK(X,Y)$, there exists an $x \in P$ with $x \in hF|_P(x)$, i.e., $x \in (gf)F|_P(x) = g(fF)|_P(x)$. Thus $fF \in BPK(X,X)$.

Theorem 1.4 ([16])**.** *Let X be an admissible convex set in a Hausdorff topological vector space and $F \in BPK(X,X)$ be a closed compact map. Then F has a fixed point in X.*

Recall the Tychonoff cube T is the Cartesian product of copies of the unit interval and T lies in an appropriate locally convex topological vector space E [7, 8]. Note since any convex subset of a locally convex topological vector space is admissible, then T is a convex admissible subset of E. Now Theorem 1.2 (alternatively, Theorem 1.4) guarantees the following theorem.

Theorem 1.5. *Let $F \in KKM(T,T)$ (alternatively, $F \in BPK(T,T)$) be a closed map. Then F has a fixed point in T.*

For a subset K of a topological space X, we denote by $Cov_X(K)$ the directed set of all coverings of K by open sets of X (usually we write $Cov(K) = Cov_X(K)$). Given a map $F : X \to 2^X$ and $\alpha \in Cov(X)$, a point $x \in X$ is said to be an α-fixed point of F if there exists a member $U \in \alpha$ such that $x \in U$ and $F(x) \cap U \neq \emptyset$.

Given two maps $F, G : X \to 2^Y$ and $\alpha \in Cov(Y)$, F and G are said to be α-close if for any $x \in X$ there exists $U_x \in \alpha$, $y \in F(x) \cap U_x$ and $w \in G(x) \cap U_x$.

Theorem 1.6 ([2]). *Let X be a regular topological space, $F : X \to 2^X$ an upper semicontinuous map with closed values and suppose there exists a cofinal covering $\theta \subseteq Cov_X(\overline{F(X)})$ such that F has an α-fixed point for every $\alpha \in \theta$. Then F has a fixed point.*

Remark 1.7. From Theorem 1.6 in proving the existence of fixed points in uniform spaces for upper semicontinuous compact maps with closed values it suffices [3, page 298] to prove the existence of approximate fixed points (since open covers of a compact set A admit refinements of the form $\{U[x] : x \in A\}$ where U is a member of the uniformity [11, page 199] so such refinements form a cofinal family of open covers). Note also that uniform spaces are regular (in fact completely regular [6]). Also note in Theorem 1.6 if F is compact valued, then the assumption that X is regular can be removed. For convenience in this paper we apply Theorem 1.6 only when the space is uniform.

2. Fixed point results

By a space we mean a Hausdorff topological space. Let Q be a class of topological spaces. A space Y is an extension space for Q (written $Y \in ES(Q)$) if $\forall X \in Q$, $\forall K \subseteq X$ closed in X, any continuous function $f_0 : K \to Y$ extends to a continuous function $f : X \to Y$.

Suppose $X \in ES(\text{compact})$. We say $H \in GR(X, X)$ if $H : X \to 2^X$ satisfies the following property: if $Z = \overline{H(X)} \subseteq X$ is compact, $\forall \theta \in C(Z, T)$, $\forall \psi \in C(T, X)$ we have that $\theta H \psi \in KKM(T, T)$; here T is the Tychonoff cube.

Remark 2.1. Note one could replace $KKM(T, T)$ with $BPK(T, T)$ in the above if one wishes.

Remark 2.2.

(i) Note a PK map is an example of a GR map. To see this suppse $H \in PK(X, X)$, $Z = \overline{H(X)} \subseteq X$ is compact, $\theta \in C(Z, T)$ and $\psi \in C(T, X)$. Note $H \in PK(X, Z)$ and $\theta H \psi \in PK(T, T)$ since PK maps are closed under compositions [12, 13]. Now since $PK(T, T) \subseteq KKM(T, T)$, then $\theta H \psi \in KKM(T, T)$. Consequently $PK(X, Y) \subseteq GR(X, Y)$ (as a result the Kakutani maps, the acyclic maps, the admissibe maps of Gorniewicz, and the approachable maps are examples of GR maps).

(ii) Now let us consider the KKM maps when X is a convex subset of a Hausdorff topological vector space, $H \in KKM(X, X)$, $Z = \overline{H(X)} \subseteq X$ is compact, $\theta \in C(Z, T)$ and $\psi \in C(T, X)$. From Section 1 note $H \in KKM(X, Z)$ and (see Theorem 1.1) $\theta H \in KKM(X, T)$. We only have Theorem 1.3 so one cannot conclude anything about $\theta H \psi$ for a general H here.

Theorem 2.3. *Let $X \in ES(compact)$ and $F \in GR(X, X)$ a upper semicontinuous compact map with compact values. Then F has a fixed point.*

Proof. Recall from [10] that every compact space is homeomorphic to a closed subset of the Tychonoff cube T, so as a result $Z = \overline{F(X)}$ can be embedded as a closed subset $Z^\star$ of T; let $s : Z \to Z^\star$ be a homeomorphism. Also let $i : Z \hookrightarrow X$ and $j : Z^\star \hookrightarrow T$ be inclusions. Now since $X \in ES(\text{compact})$ and $is^{-1} : Z^\star \to X$, then is^{-1} extends to a continuous function $h : T \to X$. Let $G = jsFh$ and note $js \in C(Z, T)$ and $h \in C(T, X)$. Since $F \in GR(X, X)$, then $G \in KKM(T, T)$ and also note G is an upper semicontinuous (compact) map with compact values, so a closed map [1]. Now Theorem 1.5 guarantees an $x \in T$ with $x \in Gx$. Let $y = h(x)$, and so $y \in hjsF(y)$, i.e., $y = hjs(q)$ for some $q \in F(y)$. Since $hj(w) = is^{-1}(w)$ for $w \in Z^\star$, we have $hjs(q) = (hj)s(q) = i(q) = q$, and so $y \in F(y)$. □

A space Y is an approximate extension space for Q (written $Y \in AES(Q)$) if $\forall \alpha \in Cov(Y)$, $\forall X \in Q$, $\forall K \subseteq X$ closed in X, and any continuous function $f_0 : K \to Y$, there exists a continuous function $f : X \to Y$ such that $f|_K$ is α-close to f_0.

Theorem 2.4. *Let $X \in AES(compact)$ and $F \in GR(X, X)$ a upper semicontinuous compact map with compact values. Then for any $\alpha \in Cov_X(\overline{F(X)})$ we have that F has an α-fixed point.*

Proof. Let $\alpha \in Cov_X(Z)$, where $Z = \overline{F(X)}$. Now Z can be embedded as a closed subset $Z^\star$ of T; let $s : Z \to Z^\star$ be a homeomorphism. Let $i : Z \hookrightarrow X$ and $j : Z^\star \hookrightarrow T$ be inclusions. Next let $\alpha' = \alpha \cup \{X \backslash Z\}$ and note α' is an open covering of X. Since $X \in AES(compact)$ let the continuous map $h : T \to X$ be such that $h|_{Z^\star}$ and s^{-1} are α'-close. Thus (note $\alpha' = \alpha \cup \{X \backslash Z\}$) $hs : Z \to X$ and $i : Z \to X$ are α-close. Let $G = jsFh$ and since $F \in GR(X, X)$ we have $G \in KKM(T, T)$ and also note G is an upper semicontinuous (compact) map with compact values, so a closed map. Now Theorem 1.5 guarantees an $x \in T$ with $x \in Gx$. Let $y = h(x)$, and so $y \in hjsF(y)$, i.e., $y = hjs(q)$ for some $q \in F(y)$. Since hs and i are α-close there exists $U \in \alpha$ with $hs(q) \in U$ and $i(q) \in U$, i.e., $q \in U$ and $y = hjs(q) = hs(q) \in U$. Thus $q \in U$, $y \in U$, so $y \in U$ and $F(y) \cap U \neq \emptyset$ (since $q \in F(y)$). As a result F has an α-fixed point. □

Now Theorem 1.6, Remark 1.7, and Theorem 2.5 immediately yields the following result.

Theorem 2.5. *Let $X \in AES(compact)$ be a uniform space and $F \in GR(X, X)$ be a upper semicontinuous compact map with compact values. Then F has a fixed point.*

One could generalize the above results by considering ES admissible and AES admissible (generalization of admissible in Section 1) as defined below. Let W be a space.

Definition 2.6. We say W is ES admissible if for all compact subsets K of W and all $\alpha \in Cov_W(K)$, there exists a continuous function $\pi_\alpha : K \to W$ such that

(i) π_α and $i : K \hookrightarrow W$ are α-close;
(ii) $\pi_\alpha(K)$ is contained in a subset $C_\alpha \subseteq W$ and $C_\alpha \in ES(compact)$.

Definition 2.7. We say W is AES admissible if for all compact subsets K of W and all $\alpha \in Cov_W(K)$, there exists a continuous function $\pi_\alpha : K \to W$ such that

(i) π_α and $i : K \hookrightarrow W$ are α-close;
(ii) $\pi_\alpha(K)$ is contained in a subset $C_\alpha \subseteq W$, $C_\alpha \in AES(compact)$ and C_α is a uniform space.

Let W be ES admissible (respectively, AES admissible). We say $H \in GR_0(W, W)$ if $H : W \to 2^W$ satisfies the following property: if $K = \overline{H(W)} \subseteq W$ is compact, $\forall \alpha \in Cov_W(K)$ we have that $\pi_\alpha H \in GR(C_\alpha, C_\alpha)$; here π_α and C_α are as in Definition 2.6 (respectively, Definition 2.7).

Remark 2.8. The PK maps are an example of GR_0 maps. To see this suppse $H \in PK(W, W)$, $K = \overline{H(W)} \subseteq W$ is compact, $\alpha \in Cov_W(K)$ and $\pi_\alpha \in C(K, W)$; here π_α and C_α are as in Definition 2.6 (respectively, Definition 2.7). Note $H \in PK(C_\alpha, K)$ and $\pi_\alpha \in C(K, C_\alpha)$ so $\pi_\alpha H \in PK(C_\alpha, C_\alpha) \subseteq GR(C_\alpha, C_\alpha)$ since PK maps are closed under compositions and also note Remark 2.2 (i).

Theorem 2.9. *Let W be ES admissible and $F \in GR_0(W, W)$ be an upper semicontinuous compact map with compact values. Then for any $\alpha \in Cov_W(\overline{F(W)})$ we have that F has an α-fixed point.*

Proof. Let $\alpha \in Cov_W(K)$, where $K = \overline{F(W)}$. Since W is ES admissible, there exists a $\pi_\alpha \in C(K, W)$ and a $C_\alpha \in ES(compact)$ as described in Definition 2.6. Note $F \in GR_0(W, W)$ so $F_\alpha = \pi_\alpha F \in GR(C_\alpha, C_\alpha)$ and also note F_α is an upper semicontinuous compact map with compact values. Now Theorem 2.3 guarantees a $x \in C_\alpha$ with $x \in F_\alpha(x) = \pi_\alpha F(x)$, i.e., $x \in \pi_\alpha q$ for some $q \in F(x)$. Since π_α and i are α-close, there exists $U \in \alpha$ with $\pi_\alpha(q) \in U$ and $i(q) \in U$, i.e., $q \in U$ and $x \in U$. As a result $x \in U$ and $F(x) \cap U \neq \emptyset$ (since $q \in F(x)$). Thus F has an α-fixed point. □

Now Theorem 1.6, Remark 1.7, and Theorem 2.9 immediately yields the following result.

Theorem 2.10. *Let W be a uniform space, let W be ES admissible, and let $F \in GR_0(X, X)$ be an upper semicontinuous compact map with compact values. Then F has a fixed point.*

Theorem 2.11. *Let W be AES admissible and $F \in GR_0(W, W)$ be an upper semicontinuous compact map with compact values. Then for any $\alpha \in Cov_W(\overline{F(W)})$ we have that F has an α-fixed point.*

Proof. Let $\alpha \in Cov_W(K)$, where $K = \overline{F(W)}$. Since W is AES admissible there exists a $\pi_\alpha \in C(K, W)$ and a $C_\alpha \in AES(compact)$ with C_α a uniform space as described in Definition 2.7. Let $F_\alpha = \pi_\alpha F$ and note $F_\alpha \in GR(C_\alpha, C_\alpha)$ and also note F_α is an upper semicontinuous compact map with compact values. Theorem 2.5 guarantees an $x \in C_\alpha$ with $x \in F_\alpha(x) = \pi_\alpha F(x)$ and the same argument as in Theorem 2.9 concludes the proof. □

Theorem 2.12. *Let W be a uniform space, W be AES admissible, and $F \in GR_0(W, W)$ be an upper semicontinuous compact map with compact values. Then F has a fixed point.*

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