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Common fixed point theorems for two mappings in b-metric-like spaces

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Abstract

The concept of b-metric-like space is a generalization of the notions of partial metric space, metric-like space and b-metric space. In the present paper, we establish the existence and uniqueness of common fixed points in a b-metric-like space. Then we derive some common fixed point results in partial metric spaces, metric-like spaces, and b-metric spaces.

Keywords: Common fixed point, b-metric-like space, partial metric space.

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1. Introduction

Fixed point theory is an important and actual topic of nonlinear analysis. During the last four decades fixed point theorem has undergone various generalizations either by relaxing the condition on contractivity or withdrawing the requirement of completeness or sometimes even both, but a very interesting generalization was obtained by changing the structure of the space according to this argument. Matthews [12] introduced the notion of partial metric space. The concept of b-metric space was introduced and studied by Bakhtin [6] and Czerwik [8]. Since then several papers have dealt with fixed point theory for single-valued and multi-valued operators in b-metric spaces (see [5, 7–11, 14, 16, 17]). Amini-Harandi [3] introduced the notion of metric-like space, which is an interesting generalization of partial metric space and dislocated metric space. Alghamdi and et al. [1] introduced a new generalization of metric-like space and partial metric space, which is called a b-metric-like space. Their fixed point results in b-metric-like spaces have generalized and improved some well-known results in the literature. Also, in [15, 18, 19] have given other fixed point results in b-metric-like spaces.

In this paper, we prove the existence and uniqueness of common fixed point for two self-mappings in b-metric-like spaces. Then we derive some common fixed point results in partial metric spaces, metric-like spaces, and b-metric spaces. In order to do this, we present the necessary definitions and results in b-metric-like spaces, which will be useful for the rest of the paper. For more details, we refer to [1, 2].

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Definition 1.1. A mapping $P : X \times X \to \mathbb{R}$ is said to be a partial metric on X if for any $x, y, z \in X$, the following conditions hold:

(P_1) $x = y$ if and only if $P(x, x) = P(y, y) = P(x, y)$;
(P_2) $P(x, x) \leqslant P(x, y)$;
(P_3) $P(x, y) = P(y, x)$;
(P_4) $P(x, z) \leqslant P(x, y) + P(y, z) - P(y, y)$.

Then the pair (X, P) is called a partial metric space.

Definition 1.2. A mapping $\sigma : X \times X \to \mathbb{R}$ is said to be a metric-like on X if for any $x, y, z \in X$, the following conditions hold:

(σ_1) $\sigma(x, y) = 0 \Rightarrow x = y$;
(σ_2) $\sigma(x, y) = \sigma(y, x)$;
(σ_3) $\sigma(x, z) \leqslant \sigma(x, y) + \sigma(y, z)$.

Then the pair (X, σ) is called a metric-like space.

Example 1.3. Let $X = \{0, 1\}$ and $\sigma(x, x) = \begin{cases} 2, & x = y = 0, \\ 1, & \text{otherwise.} \end{cases}$

The concept of b-metric space was introduced by Czerwik [8].

Definition 1.4. A b-metric on X is a mapping $D : X \times X \to [0, +\infty)$ such that for all $x, y, z \in X$ and constant $K \geqslant 1$ the following conditions hold:

(D_1) $D(x, y) = 0 \Leftrightarrow x = y$;
(D_2) $D(x, y) = D(y, x)$;
(D_3) $D(x, y) \leqslant K\Big[D(x, z) + D(z, y)\Big]$.

Then the pair (X, D) is called a b-metric-space.

Definition 1.5. A b-metric-like on X is a function $D : X \times X \to [0, +\infty)$ such that for all $x, y, z \in X$ and an constant $K \geqslant 1$ the following conditions hold:

(D_1) $D(x, y) = 0 \Rightarrow x = y$;
(D_2) $D(x, y) = D(y, x)$;
(D_3) $D(x, y) \leqslant K[D(x, z) + D(z, y)]$.

Then the pair (X, D) is called a b-metric-like space.

Example 1.6. Let $X = [0, +\infty)$. Define the function $D : X^2 \to [0, +\infty)$ by $D(x, y) = (\sqrt{x} + \sqrt{y})^2$. Then $(X, D, 2)$ is a b-metric-like space with the constant $K = 2$.

Definition 1.7. Let (X, D) be a b-metric-like space and let $\{x_n\}$ be a sequence of X and $x \in X$. Then $\{x_n\}$ is said to be convergent to x and denote it by $x_n \to x$, if $\lim_{n \to +\infty} D(x, x_n) = D(x, x)$.

Before we state and prove the main result, we recall the following lemmas which are needed in the next section.

Lemma 1.8 ([1, Lemma 2.13]). *Let (X, D, K) be a b-metric-like space, and let $\{x_k\}_{k=0}^n \subseteq X$. Then*

$$D(x_n, x_0) \leqslant KD(x_0, x_1) + \cdots + K^{n-1}D(x_{n-2}, x_{n-1}) + K^{n-1}D(x_{n-1}, x_n).$$

Lemma 1.9 ([1, Proposition 2.10]). *Let (X, D, K)be a b-metric-like space, and let $\{x_n\}$ be a sequence in X such that $\lim_{n \to +\infty} D(x_n, x) = 0$. Then*

(i) x *is unique;*
(ii) $k^{-1}D(x,y) \leqslant \lim_{n\to+\infty} D(x_n,y) \leqslant KD(x,y)$, *for all* $y \in X$.

Lemma 1.10 ([13, Lemma 2.2]). *Let* $(\mathcal{X},d)$ *be a* b*-metric space with constant* $s \geqslant 1$. *Then, every sequence* $\{x_n\}_{n\in\mathbb{N}}$ *of elements from* $\mathcal{X}$ *for which there exists* $\gamma \in (0,1)$ *such that* $d(x_n,x_{n+1}) \leqslant \gamma d(x_{n-1},x_n)$ $(n \in \mathbb{N}^*)$, *is a Cauchy sequence. Moreover, the following estimation holds*

$$d(x_{n+1},x_{n+p}) \leqslant \frac{\gamma^n S}{1-\gamma} d(x_0,x_1), \quad (n,p \in \mathbb{N}),$$

where $S = \sum_{i=1}^{\infty} \gamma^{2i\log_\gamma s + 2^{i-1}}$.

Notation 1.11. Let (X,D,K) be a b-metric-like space. Define $D^s : X^2 \to [0,+\infty]$ by

$$D^s(x,y) = |2D(x,y) - D(x,x) - D(y,y)|.$$

Clearly, $D^s(x,x) = 0$, for all $x \in X$.

2. Main results

In this section, we prove some common fixed point results for two mappings on a b-metric-like space. In fact, by using some ideas of [1] we generalize fixed point results for two mappings in b-metric-like spaces.

Theorem 2.1. *Let* (X,D,K) *be a complete* b*-metric-like space. Assume that* $S,T : X \to X$ *are onto mappings such that*

$$D(Tx,Sy) \geqslant \Big[R + L\min\{D^s(x,Tx), D^s(y,Sy), D^s(x,Sy), D^s(y,Tx)\}\Big] D(x,y), \tag{2.1}$$

for all $x,y \in X$, *where* $R > K$ *and* $L \geqslant 0$. *Then* T *and* S *have a unique common fixed point.*

Proof. First we show that if T and S have a common fixed point, and then the fixed point is unique. Let x,y be two common fixed points that is $Tx = Sx = x$ and $Ty = Sy = y$. If $x \neq y$, then from (2.1) we have

$$D(x,y) = D(Tx,Sy)) \geqslant RD(x.y) > KD(x,y) \geqslant D(x,y),$$

which is a contradiction. Thus $x = y$.

Fix $x_0 \in X$. Since T,S are onto, so there exists $x_1 \in X$ such that $x_0 = Tx_1$ and there exists an $x_2 \in X$ such that $Sx_2 = x_1$. By continuing this process, we get $x_{2n} = Tx_{2n+1}$, $x_{2n+1} = Sx_{2n+2}$, for all $n \in \mathbb{N}\cup\{0\}$. In case $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N}\cup\{0\}$. Then it is clear that x_{n_0} is a fixed point of T or S. Now, assume that $x_n \neq x_{n+1}$, for all $n \in \mathbb{N}$. From (2.1) we have

$$\begin{aligned}
&D(x_{2n+1},x_{2n})\\
&\quad = D(Sx_{2n+2},Tx_{2n+1})\\
&\quad \geqslant \Big[R + L\min\{D^s(x_{2n+1},Tx_{2n+1}), D^s(x_{2n+2},Sx_{2n+2}), D^s(x_{2n+1},Sx_{2n+2}), D^s(x_{2n+2},Tx_{2n+1})\}\Big]\\
&\quad\quad \times D(x_{2n+1},x_{2n+2}).
\end{aligned}$$

Which implies that $D(x_{2n+1},x_{2n}) \geqslant RD(x_{2n+1},x_{2n+2})$. Similarly we get

$$D(x_{2n+2},x_{2n+1}) \geqslant RD(x_{2n},x_{2n+1}),$$

and so $D(x_n,x_{n+1}) \leqslant R^{-1}D(x_{n-1},x_n)$ for all n. Lemma (1.10) implies that $\{x_n\}$ is a Cauchy sequence. Since (X,D,K) is a complete b-metric-like space, so the sequence $\{x_n\}$ converges to any z in X, and $\{x_{2n}\},\{x_{2n+1}\}$ also converge to $z \in X$. Thus

$$\lim_{n\to+\infty} D(x_n,z) = \lim_{n\to+\infty} D(x_{2n},z) = \lim_{n\to+\infty} D(x_{2n+1},z) = \lim_{n\to+\infty} D(x_n,x_m) = 0.$$

Since T and S are onto, there exists a $w_1 \in X$ and a $w_2 \in X$ such that $Tw_1 = z$ and $Sw_2 = z$. From (2.1) we have

$$\begin{aligned}D(x_{2n}, z) &= D(Tx_{2n+1}, Sw_2)\\ &\geqslant \Big[R + L\min\{D^s(x_{2n+1}, Tx_{2n+1}), D^s(w_2, Sw_2), D^s(x_{2n+1}, Sw_2), D^s(w_2, Tx_{2n+1})\}\Big] D(x_{2n+1}, w_2).\end{aligned}$$

Taking limit as $n \to +\infty$ in the above inequality, we get

$$0 = \lim_{n\to+\infty} D(x_{2n}, z) \geqslant R \lim_{n\to+\infty} D(x_{2n+1}, w_2),$$

which implies that $\lim_{n\to+\infty} D(x_{2n+1}, w_2) = 0$. Then Lemma 1.10 implies that $w_2 = z$ or $Sz = z$. Similarly, we have

$$\begin{aligned}D(x_{2n+1}, z) &= D(Sx_{2n+2}, Tw_1)\\ &\geqslant \Big[R + L\min\{D^s(x_{2n+2}, Sx_{2n+2}), D^s(w_1, Tw_1), D^s(w_1, Sx_{2n+2}), D^s(x_{2n+2}, Tw_1)\}\Big] D(x_{2n+2}, w_1).\end{aligned}$$

Taking limit as $n \to +\infty$ in the above inequality, we get

$$0 = \lim_{n\to+\infty} D(x_{2n+1}, z) \geqslant R \lim_{n\to+\infty} D(x_{2n+2}, w_1),$$

which implies that $\lim_{n\to+\infty} D(x_{2n+2}, w_1) = 0$, then by Lemma (1.10), $w_1 = z$ that is $Tz = z$. □

If we put $L = 0$ in Theorem 2.1, then we get the following corollary.

Corollary 2.2. *Let (X, D, K) be a complete b-metric-like space. Assume that $S, T : X \to X$ are onto mappings, such that $D(Tx, Sy) \geqslant RD(x, y)$ for all $x, y \in X$, where $R > K$. Then T and S have a unique common fixed point.*

Example 2.3. Let $X = [0, +\infty)$. Define the function $D : X^2 \to [0, +\infty)$ by $D(x, y) = (\sqrt{x} + \sqrt{y})^2$. Then $(X, D, 2)$ is a b-metric-like space. Define $S, T : X \to X$ such that $Sy = y$ and $Tx = 9x$ and $R = 3$. Then if we get $x \geqslant y$,

$$D(Tx, Sy) = (\sqrt{9x} + \sqrt{y})^2 = 9x + y + 6\sqrt{xy} \geqslant 3x + 3y + 6\sqrt{xy} = 3(\sqrt{x} + \sqrt{y})^2.$$

Hence T and S have a unique common fixed point.

In the following, we suppose that $\varphi : (0, +\infty) \to (L^2, +\infty)$ is a function, which satisfies the condition $\varphi(t_n) \to (L^2)^+ \Rightarrow t_n \to 0$, where $L > 1$. An example of this function is $\varphi(t) = 4(1 + t)$ with $L = 2$.

Theorem 2.4. *Let (X, D, K) be a complete b-metric-like space, assume that $S, T : X \to X$ are onto mappings, such that*

$$D(Tx, Sy) \geqslant \varphi\left(D(x, y)\right) D(x, y), \tag{2.2}$$

for all $x, y \in X$. Then T and S have a unique common fixed point.

Proof. It is clear that if T and S have a common fixed point, then it is unique. Suppose to the contrary that $x_0 \in X$. Since T and S are onto, there exists $x_1 \in X$ such that $x_0 = Tx_1$ and there exists $x_2 \in X$ such that $Sx_2 = x_1$. By continuing this process, we get $x_{2n} = Tx_{2n+1}$, $x_{2n+1} = Sx_{2n+2}$ for all $n \in \mathbb{N} \cup \{0\}$. In case $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N} \cup \{0\}$. Then it is clear that x_{n_0} is a fixed point of T or S. Now assume that $x_n \neq x_{n+1}$ for all n, from (2.2) we get

$$\begin{aligned}D(x_{2n}, x_{2n+1}) = D(Tx_{2n+1}, Sx_{2n+2}) &\geqslant \varphi\Big(D(x_{2n+1}, x_{2n+2})\Big) D(x_{2n+1}, x_{2n+2})\\ &\geqslant K^2 D(x_{2n+1}, x_{2n+2}) \geqslant D(x_{2n+1}, x_{2n+2}),\end{aligned}$$

and also

$$\begin{aligned} D(x_{2n+1}, x_{2n+2}) = D(Sx_{2n+2}, Tx_{2n+3}) &\geqslant \varphi\Big(D(x_{2n+3}, x_{2n+2})\Big) D(x_{2n+3}, x_{2n+2}) \\ &\geqslant K^2 D(x_{2n+3}, x_{2n+2}) \geqslant D(x_{2n+3}, x_{2n+2}). \end{aligned}$$

Then the sequence $D(x_n, x_{n+1})$ is a decreasing sequence in $\mathbb{R}^+$ and so there exists $s \geqslant 0$ such that $\lim_{n\to+\infty} D(x_n, x_{n+1}) = s$. Now we prove $s = 0$. Suppose to the contrary that $s > 0$. By (2.2) we deduce

$$K^2 \frac{D(x_{2n}, x_{2n+1})}{D(x_{2n+1}, x_{2n+2})} \geqslant \frac{D(x_{2n}, x_{2n+1})}{D(x_{2n+1}, x_{2n+2})} \geqslant \varphi\Big(D(x_{2n+1}, x_{2n+2})\Big) \geqslant K^2.$$

By taking limit as $n \to +\infty$ in the above inequality, we get

$$\lim_{n\to+\infty} B\Big(D(x_{2n+1}, x_{2n+2})\Big) = k^2.$$

Hence, $s = \lim_{n\to+\infty} D(x_{2n+1}, x_{2n+2}) = 0$, and $s = \lim_{n\to+\infty} D(x_{2n+3}, x_{2n+2}) = 0$, which is a contradiction. Hence $s = 0$. We shall show that $\limsup_{n,m\to+\infty} D(x_n, x_m) = 0$. Suppose to the contrary that $\limsup_{n,m\to+\infty} D(x_n, x_m) > 0$. Thus we have

$$D(x_{2n}, x_{2m+1}) = D(Tx_{2n+1}, Sx_{2m+2}) \geqslant \varphi\Big(D(x_{2n+1}, x_{2m+2})\Big) D(x_{2n+1}, x_{2m+2}),$$

that is

$$\frac{D(x_{2n}, x_{2m+1})}{\varphi\Big(D(x_{2n+1}, x_{2m+2})\Big)} \geqslant D(x_{2n+1}, x_{2m+2}).$$

By (D_3) we get,

$$\begin{aligned} D(x_{2n}, x_{2m+1}) &\leqslant KD(x_{2n}, x_{2n+1}) + K^2 D(x_{2n+1}, x_{2m+2}) + K^2 D(x_{2m+2}, x_{2m+1}) \\ &\leqslant KD(x_{2n}, x_{2n+1}) + K^2 \frac{D(x_{2n}, x_{2m+1})}{\varphi\Big(D(x_{2n+1}, x_{2m})\Big)} + K^2 D(x_{2m+2}, x_{2m+1}). \end{aligned}$$

Therefore

$$D(x_{2n}, x_{2m+1}) \leqslant \left(1 - \frac{K^2}{\varphi\Big(D(x_{2n+1}, x_{2m+2})\Big)}\right)^{-1} \Big(KD(x_{2n}, x_{2n+1}) + K^2 D(x_{2m+2}, x_{2m+1})\Big).$$

By taking limit as $n, m \to +\infty$ in the above inequality, since $\limsup_{n,m\to+\infty} D(x_{2n}, x_{2m+1}) > 0$ and $\lim_{n\to+\infty} D(x_{2n}, x_{2n+1}) = 0$, we get

$$\limsup_{m,n\to+\infty} \left(1 - \frac{K^2}{\varphi\Big(D(x_{2n+1}, x_{2m+2})\Big)}\right)^{-1} = +\infty,$$

which implies that

$$\limsup_{m,n\to+\infty} \varphi\Big(D(x_{2n+1}, x_{2m+2})\Big) = K^{2^+},$$

and so

$$\limsup_{m,n\to+\infty} D(x_{2n+1}, x_{2m+2}) = 0,$$

which is a contradiction. Hence $\limsup_{m,n\to+\infty} D(x_n, x_m) = 0$. Since $\lim_{m,n\to+\infty} D(x_n, x_m) = 0$, so $\{x_n\}$ is Cauchy. (X, D, K) is a complete b-metric-like space, hence the sequence $\{x_n\}$ is convergent to z. Hence

$$\lim_{n\to+\infty} D(x_n, z) = \lim_{n\to+\infty} D(x_{2n}, z) = \lim_{n\to+\infty} D(x_{2n+1}, z) = D(z, z) = \lim_{n\to+\infty} D(x_n, x_m) = 0.$$

Since T, S are onto, there exists $w_1 \in X$ and $w_2 \in X$ such that $Tw_1 = z$ and $Sw_2 = z$. We prove that $w_1 = w_2 = z$. Suppose to the contrary that $z \neq w_1$ and $z \neq w_2$, then we have

$$D(x_{2n}, z) = D(Tx_{2n+1}, Sw_2) \geqslant \varphi\Big(D(x_{2n+1}, w_2)\Big)D(x_{2n+1}, w_2),$$
$$D(x_{2n+1}, z) = D(Sx_{2n+2}, Tw_1) \geqslant \varphi\Big(D(x_{2n+2}, w_1)\Big)D(x_{2n+2}, w_1).$$

By taking limit as $n \to +\infty$ in the above inequalities and applying Lemma 1.9 we have

$$\begin{aligned} 0 = \lim_{n\to+\infty} D(x_{2n}, z) &\geqslant \lim_{n\to+\infty} \varphi\Big(D(x_{2n+1}, w_2)\Big) \lim_{n\to+\infty} D(x_{2n+1}, w_2) \\ &\geqslant K^{-1} \lim_{n\to+\infty} \varphi\Big(D(x_{2n+1}, Z)\Big)D(Z, w_2), \end{aligned}$$

and

$$\begin{aligned} 0 = \lim_{n\to+\infty} D(x_{2n+1}, z) &\geqslant \lim_{n\to+\infty} \varphi\Big(D(x_{2n+2}, w_1)\Big) \lim_{n\to+\infty} D(x_{2n+2}, w_1) \\ &\geqslant K^{-1} \lim_{n\to+\infty} \varphi\Big(D(x_{2n+2}, Z)\Big)D(Z, w_1), \end{aligned}$$

and hence $\lim_{n\to+\infty} \varphi\Big(D(x_{2n+1}, z)\Big) = 0$ and $\lim_{n\to+\infty} \varphi\Big(D(x_{2n}, z)\Big) = 0$, which is a contradiction. Since $\varphi(t) > K^2$, for all $t \in [0, +\infty)$, so

$$\lim_{n\to+\infty} \varphi\Big(D(x_{2n}, z)\Big) \geqslant K^2 > 0 \quad \text{and} \quad \lim_{n\to+\infty} \varphi\Big(D(x_{2n+1}, z)\Big) \geqslant K^2 > 0.$$

Therefore $z = w_1 = w_2$ that is $z = Tw_1 = Tz = Sw_2 = Sz$. □

Example 2.5. Let $\mathcal{X} = [0, +\infty)$ and $D : \mathcal{X}^2 \to [0, +\infty)$ defined by $D(x, y) = (x + y)^2$. Then $(\mathcal{X}, D, 2)$ is a complete b-metric-like space. Let $T, S : \mathcal{X} \to \mathcal{X}$ be defined by

$$T(x) = \begin{cases} 6x, & x \in [0, 1), \\ 2x + 1, & x \in [1, 2), \\ 4x + 2, & x \in [2, +\infty), \end{cases} \qquad \text{and} \qquad S(x) = \begin{cases} 5x, & x \in [0, 1), \\ 4x + 1, & x \in [1, 2), \\ 3x + 4, & x \in [2, +\infty). \end{cases}$$

Also, define $\varphi : (0, +\infty) \to (0, 4)$ by $\varphi(t) = 4$. Clearly, T and S are onto mappings. Also, it is easy to see that T and S satisfy in (2.2). Therefore the conditions of Theorem 2.4 hold and T and S have a common fixed point.

We know that b-metric-like spaces are an extension of partial metric, metric-like, and b-metric spaces. Therefore we get the following results.

Corollary 2.6. *Let (X, P) be a complete partial metric space. Suppose $S, T : X \to X$ are onto mappings, such that $P(Tx, Sy) \geqslant \varphi(P(x, y))P(x, y)$ for all $x, y \in X$, then T and S have a unique common fixed point.*

Corollary 2.7. *Let (X, σ) be a complete metric-like space. Suppose $S, T : X \to X$ are onto mappings, such that $\sigma(Tx, Sy) \geqslant \varphi(\sigma(x, y))\sigma(x, y)$ for all $x, y \in X$, then Tand S have a common fixed point.*

Corollary 2.8. *Let (X, d, K) be a complete b-metric space. Assume that $S, T : X \to X$ are onto mappings, such that $D(Tx, Sy) \geqslant \varphi\Big(d(x, y)\Big)d(x, y)$ for all $x, y \in X$, then T and S have a common fixed point.*

3. Conclusion

In this paper, we study the existence and uniqueness of common fixed points for two mappings in complete b-metric-like spaces. Moreover, since b-metric-like spaces are an extension of partial metric, metric-like, and b-metric spaces, we get the same common fixed point results in these spaces. For the application of the results, we need to continue to study especially the existence of solutions of some integral equations and differential equations.

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