J. Nonlinear Sci. Appl. 1 (2008), no. 2, 91–101

THE JOURNAL OF NONLINEAR SCIENCES AND APPLICATIONS
http://www.tjnsa.com

BLOW-UP TIME OF SOME NONLINEAR WAVE EQUATIONS

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ABSTRACT. In this paper, we consider the following initial-boundary value problem

$$\begin{cases} u_{tt}(x,t) = \varepsilon Lu(x,t) + b(t)f(u(x,t)) \quad \text{in} \quad \Omega\times(0,T), \\ u(x,t)=0 \quad \text{on} \quad \partial\Omega\times(0,T), \\ u(x,0)=0 \quad \text{in} \quad \Omega, \\ u_t(x,0)=0 \quad \text{in} \quad \Omega, \end{cases}$$

where ε is a positive parameter, $b \in C^1(\mathbb{R}_+)$, $b(t)>0$, $b^{'}(t)\geq 0$, $t\in\mathbb{R}_+$, $f(s)$ is a positive, increasing and convex function for nonnegative values of s. Under some assumptions, we show that, if ε is small enough, then the solution u of the above problem blows up in a finite time, and its blow-up time tends to that of the solution of the following differential equation

$$\begin{cases} \alpha^{'}(t) = b(t)f(\alpha(t)), \quad t>0, \\ \alpha(0)=0, \quad \alpha^{'}(0)=0. \end{cases}$$

Finally, we give some numerical results to illustrate our analysis.

1. INTRODUCTION

Let Ω be a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. Consider the following initial-boundary value problem

$$u_{tt}(x,t) = \varepsilon Lu(x,t) + b(t)f(u(x,t)) \quad \text{in} \quad \Omega\times(0,T), \tag{1.1}$$

$$u(x,t)=0 \quad \text{on} \quad \partial\Omega\times(0,T), \tag{1.2}$$

$$u(x,0)=0 \quad \text{in} \quad \Omega, \tag{1.3}$$

Date: Received: May 2008; Accepted: 23 July 2008.

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2000 *Mathematics Subject Classification.* Primary 35B40, 35B50; Secondary 35K60.

Key words and phrases. Nonlinear wave equation, blow-up, convergence, numerical blow-up time.

$$u_t(x,0)=0 \quad \text{in} \quad \Omega, \tag{1.4}$$

where ε is a positive parameter, $b \in C^1(\mathbb{R}_+)$, $b(t)>0$, $b^{'}(t)\geq 0$, $t\in\mathbb{R}_+$. The operator L is defined as follows

$$Lu=\sum_{i,j=1}^{N}\frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right),$$

where $a_{ij}:\overline{\Omega}\to\mathbb{R}$, $a_{ij}\in C^1(\overline{\Omega})$, $a_{ij}=a_{ji}$, $1\leq i,j\leq N$, and there exists a constant $C>0$ such that

$$\sum_{i,j=1}^{N}a_{ij}(x)\xi_i\xi_j\geq C\|\xi\|^2 \quad \forall x\in\overline{\Omega} \quad \forall\xi=(\xi_1,...,\xi_N)\in\mathbb{R}^N,$$

where $\|\cdot\|$ stands for the Euclidean norm of $\mathbb{R}^N$. Here $(0,T)$ is the maximal time interval of existence of the solution u. The time T may be finite or infinite. When T is infinite, we say that the solution u exists globally. When T is finite, then the solution u develops a singularity in a finite time, namely,

$$\lim_{t\to T}\|u(\cdot,t)\|_\infty=\infty,$$

where $\|u(\cdot,t)\|_\infty=\sup_{x\in\Omega}|u(x,t)|$. In this last case, we say that the solution u blows up in a finite time, and the time T is called the blow-up time of the solution u.

Solutions of nonlinear wave equations which blow up in a finite time have been the subject of investigation of many authors (see [5], [7]–[9], [11]–[13], [17]–[19], and the references cited therein).

By standard methods, local existence, uniqueness, blow-up and global existence have been treated. In this paper, we are interested in the asymptotic behavior of the blow-up time when ε is small enough. Our work was motivated by the paper of Friedman and Lacey in [6], where they have considered the following initial-boundary value problem

$$\begin{cases} u_t(x,t)=\varepsilon\Delta u(x,t)+f(u(x,t)) \quad \text{in} \quad \Omega\times(0,T),\\ u(x,t)=0 \quad \text{on} \quad \partial\Omega\times(0,T),\\ u(x,0)=u_0(x) \quad \text{in} \quad \Omega,\end{cases}$$

where Δ is the Laplacian, $f:[0,\infty)\to(0,\infty)$ is a C^1 convex, increasing function, $\int_0^\infty\frac{ds}{f(s)}<\infty$, $u_0(x)$ is a continuous function in Ω. Under some additional conditions on the initial data, they have shown that the solution of the above problem blows up in a finite time, and its blow-up time tends to that of the solution $\lambda(t)$ of the following differential equation

$$\lambda^{'}(t)=f(\lambda(t)), \quad \lambda(0)=M, \tag{1.5}$$

as ε goes to zero, where $M=\sup_{x\in\Omega}u_0(x)$.

The proof developed in [6] is based on the construction of upper and lower solutions, and it is difficult to extend the method in [6] to the problem described in (1.1)–(1.4). In this paper, we prove similar results. More precisely, firstly, we show that when ε is small enough, then the solution u of (1.1)–(1.4) blows up in a finite time, and its blow-up time tends to that of the solution $\alpha(t)$ of the following differential equation

$$\alpha''(t) = b(t)f(\alpha(t)), \quad \alpha(0) = 0, \quad \alpha'(0) = 0. \tag{1.6}$$

A similar result has been obtained by N'gohisse and Boni in [15] in the case of the phenomenon of quenching (we say that a solution quenches in a finite time if it reaches a finite singular value in a finite time). Our paper is written in the following manner. In the next section, under some assumptions, we show that the solution u of (1.1)–(1.4) blows up in a finite time, and its blow-up time goes to that of the solution $\alpha(t)$ of the differential equation defined in (1.6). Finally, in the last section, we give some numerical results to illustrate our analysis.

2. Blow-up times

In this section, under some assumptions, we show that the solution u of (1.1)–(1.4) blows up in a finite time, and its blow-up time goes to that of the solution of the differential equation defined in (1.6) when ε tends to zero. We also prove that the above result remains valid when ε is fixed, and the domain Ω is large enough and is taken as parameter.

Before starting, let us recall a well known result. Consider the following eigenvalue problem

$$-L\varphi = \lambda\varphi \quad \text{in} \quad \Omega, \tag{2.1}$$

$$\varphi = 0 \quad \text{on} \quad \partial\Omega, \tag{2.2}$$

$$\varphi > 0 \quad \text{in} \quad \Omega. \tag{2.3}$$

The above problem admits a solution (φ, λ) with $\lambda > 0$. We can normalize φ so that $\int_\Omega \varphi dx = 1$.

Our first result is the following.

Theorem 2.1. *Let $A = \frac{\lambda}{b(0)} \int_0^\infty \frac{ds}{f(s)}$, and assume that the solution $\alpha(t)$ of the differential equation defined in (1.6) blows up at the time T_e. If $\varepsilon < A$, then the solution u of (1.1)–(1.4) blows up in a finite time, and its blow-up time T satisfies the following estimates*

$$0 \le T - T_e \le \frac{AT_e}{2} + o(\varepsilon). \tag{2.4}$$

Proof. Since $(0, T)$ is the maximal time interval on which the solution u exists, our aim is to show that T is finite and satisfies the above inequalities. Introduce the function $v(t)$ defined as follows

$$v(t) = \int_\Omega \varphi(x)u(x,t)dx \quad \text{for} \quad t \in [0, T).$$

Taking the derivative of v in t, and using (1.1), we find that

$$v''(t) = \varepsilon \int_\Omega \varphi(x)Lu(x,t)dx + b(t)\int_\Omega f(u(x,t))\varphi(x)dx \quad \text{for} \quad t \in (0, T).$$

According to Green's formula, and making use of (2.1), we note that

$$\int_\Omega Lu(x,t)\varphi(x)dx = \int_\Omega u(x,t)L\varphi(x)dx = -\lambda\int_\Omega \varphi(x)u(x,t)dx \quad \text{for} \quad t\in(0,T),$$

which implies that

$$v''(t) = -\lambda\varepsilon v(t) + b(t)\int_\Omega f(u(x,t))\varphi(x)dx \quad \text{for} \quad t\in(0,T).$$

Jensen's inequality renders

$$v''(t) \geq -\lambda\varepsilon v(t) + b(t)f(v(t)) \quad \text{for} \quad t\in(0,T).$$

This estimate may be rewritten in the following manner

$$v''(t) \geq b(t)f(v(t))\left(1 - \frac{\lambda\varepsilon v(t)}{b(t)f(v(t))}\right) \quad \text{for} \quad t\in(0,T).$$

We observe that $b(t) \geq b(0)$ for $t\in(0,T)$, and $\int_0^\infty \frac{d\sigma}{f(\sigma)} = \sup_{t\geq 0}\int_0^t \frac{d\sigma}{f(\sigma)} \geq \sup_{0\leq t\leq\infty}\frac{t}{f(t)}$, because $f(s)$ is an increasing function for nonnegative values of s. Using these observations, we arrive at

$$v''(t) \geq (1-\varepsilon A)b(t)f(v(t)) \quad \text{for} \quad t\in(0,T). \tag{2.5}$$

Set

$$w(t) = v\left(\frac{t}{\sqrt{1-\varepsilon A}}\right) \quad \text{for} \quad t\in[0,\sqrt{1-\varepsilon A}T).$$

A straightforward computation reveals that

$$w^{''}(t) \geq b\left(\frac{t}{\sqrt{1-\varepsilon A}}\right)f(w(t)) \quad \text{for} \quad t\in[0,\sqrt{1-\varepsilon A}T).$$

Since $b(s)$ is nondecreasing for nonnegative values of s, we discover that

$$w^{''}(t) \geq b(t)f(w(t)) \quad \text{for} \quad t\in[0,\sqrt{1-\varepsilon A}T). \tag{2.6}$$

It is not hard to check that

$$w(0) = 0 \quad \text{for} \quad w^{'}(0) = 0. \tag{2.7}$$

Integrate the inequality (2.6) over $(0,t)$ to obtain

$$w^{'}(t) \geq \int_0^t b(s)f(w(s))ds \quad \text{for} \quad t\in[0,\sqrt{1-\varepsilon A}T). \tag{2.8}$$

Recall that $\alpha(t)$ is the solution of the following differential equation

$$\alpha^{''}(t) = b(t)f(\alpha(t)), \quad t\in[0,T_e),$$

$$\alpha(0) = 0, \quad \alpha^{'}(0) = 0, \quad t\in[0,T_e),$$

which implies that

$$\alpha^{'}(t) = \int_0^t b(s)f(\alpha(s))ds \quad \text{for} \quad t\in[0,T_e). \tag{2.9}$$

Since $w(0) = v(0)$, thanks to (2.8) and (2.9), an application of the maximum principle gives

$$w(t) \geq \alpha(t) \quad \text{for} \quad t \in [0, T^*), \tag{2.10}$$

where $T_* = \min\{T_e, \sqrt{1-\varepsilon AT}\}$. We deduce that

$$T \leq \frac{T_e}{\sqrt{1-\varepsilon AT}}. \tag{2.11}$$

To prove this estimate, we argue by contradiction. Suppose that $T > \frac{T_e}{\sqrt{1-\varepsilon AT}} = T^{'}$. From (2.10), we observe that w blows up at the time T_e, because $w(T_e) \geq \alpha(T_e) = \infty$, which implies that

$$v(T^{'}) = v(\frac{T_e}{\sqrt{1-\varepsilon AT}}) = w(T_e) = \infty. \tag{2.12}$$

Let us notice that $\|u(\cdot,t)\|_\infty \geq v(t)$ for $t \in (0,T)$. Since v blows up at the time $T^{'}$, owing to the above estimate, it is easy to check that u also blows up at the time $T^{'}$. But, this contradicts the fact that $(0,T)$ is the maximal time interval of existence of the solution u.

Now, introduce the function $U(t)$ defined as follows

$$U(t) = \max_{x\in\Omega} u(x,t) \quad \text{for} \quad t \in [0,T). \tag{2.13}$$

We know that there exists $x_0 \in \Omega$ such that $U(t) = u(x_0,t)$ for $t \in (0,T)$. It is not hard to see that $Lu(x_0,t) \leq 0$ for $t \in (0,T)$. Making use of (1.1), we see that

$$U^{''}(t) \leq b(t)f(U(t)), \quad t \in (0,T). \tag{2.14}$$

Obviously, because of (1.3) and (1.4), we also have

$$U(0) = 0 \quad \text{and} \quad U^{'}(0) = 0. \tag{2.15}$$

Integrating the inequality (2.14) from 0 to t, we find that

$$U^{'}(t) \leq \int_0^t b(s)f(U(s))ds \quad \text{for} \quad t \in (0,T). \tag{2.16}$$

Since $U(0) = \alpha(0)$, using (2.9) and (2.16), an application of the maximum principle renders

$$U(t) \leq \alpha(t) \quad \text{for} \quad t \in (0,T_0), \tag{2.17}$$

where $T_0 = \min\{T, T_e\}$. We deduce that

$$T \geq T_e. \tag{2.18}$$

Indeed, assume that $T < T_e$. Taking into account (2.17), we observe that $U(T) \leq \alpha(T) < \infty$. But, this contradicts the fact that $(0,T)$ is the maximal time interval of existence of the solution u. Apply Taylor's expansion to obtain

$$\frac{1}{\sqrt{1-\varepsilon A}} = 1 + \frac{1}{2}\varepsilon A + o(\varepsilon). \tag{2.19}$$

Use (2.11), (2.18) and the above relation to complete the rest of the proof. □

Remark 2.2. If $b(t) = 1$, then the solution $\alpha(t)$ defined in (1.6) satisfies

$$\alpha^{''}(t) = f(\alpha(t)), \quad t \in (0, T_e), \tag{2.20}$$

$$\alpha(0) = 0, \quad \alpha^{'}(0) = 0. \tag{2.21}$$

Multiply both sides of (2.20) by $\alpha^{'}(t)$ to obtain

$$(\frac{(\alpha^{'}(t))^2}{2})^{'} = (F(\alpha(t)))_t \quad \text{for} \quad t \in (0, T_e), \tag{2.22}$$

where $F(s) = \int_0^s f(\sigma)d\sigma$. Integrating the equality (2.22) over $(0, t)$, we find that

$$\frac{(\alpha^{'}(t))^2}{2} = F(\alpha(t)) \quad \text{for} \quad t \in (0, T_e),$$

which implies that

$$\alpha^{'}(t) = \sqrt{2F(\alpha(t))} \quad \text{for} \quad t \in (0, T_e).$$

Let us notice that if the integral $\int_0^{\infty} \frac{d\sigma}{\sqrt{F(\sigma)}}$ is finite, then $\alpha(t)$ blows up at the time $T_e = \frac{1}{\sqrt{2}} \int_0^{\infty} \frac{d\sigma}{\sqrt{F(\sigma)}}$. In fact, we observe that

$$\frac{d\sigma}{\sqrt{F(\sigma)}} = \sqrt{2}dt \quad \text{for} \quad t \in (0, T_e).$$

Integrate the above equality over $(0, T_e)$ to arrive at

$$T_e = \frac{1}{\sqrt{2}} \int_0^{\infty} \frac{d\sigma}{\sqrt{F(\sigma)}}. \tag{2.23}$$

If $f(s) = e^s$, then $F(s) = e^s - 1$. In this case $T_e = \frac{1}{\sqrt{2}} \int_0^{\infty} \frac{d\sigma}{\sqrt{e^{\sigma}-1}}$, and its value is slightly equal 2.22.

Remark 2.3. Assume that Dirichlet boundary condition (1.2) is replaced by that of Robin, that is,

$$\frac{\partial u}{\partial \eta} + \beta(x)u = 0 \quad \text{for} \quad \partial\Omega \times (0, T), \tag{2.24}$$

where $\beta \in C^0(\partial\Omega)$, $\beta(x) > 0$ on $\partial\Omega$, $\frac{\partial u}{\partial \eta} = \sum_{i,j=1}^N a_{ij} \cos(\nu, x_i) \frac{\partial u}{\partial x_j}$, ν is the exterior normal unit vector on $\partial\Omega$.
Consider the following eigenvalue problem

$$-L\psi = \lambda\psi \quad \text{in} \quad \Omega, \tag{2.25}$$

$$\frac{\partial \psi}{\partial \eta} + \beta(x)\psi = 0 \quad \text{for} \quad \partial\Omega, \tag{2.26}$$

$$\psi(x) > 0 \quad \text{in} \quad \Omega. \tag{2.27}$$

We know that the above eigenvalue problem admits a solution (ψ, λ) with $\lambda > 0$, and we can normalize ψ so that $\int_\Omega \psi(x)dx = 1$. Introduce the function $v(t)$ defined as follows

$$v(t) = \int_\Omega u(x,t)\psi(x)dx \quad \text{in} \quad t \in [0,T).$$

Take twice the derivative of v in t and use (1.1), to obtain

$$v''(t) = \varepsilon \int_\Omega Lu(x,t)\psi(x)dx + b(t)\int_\Omega f(u(x,t))\psi(x)dx \quad for \quad t \in (0,T).$$

By Green's formula, we have

$$\int_\Omega Lu(x,t)\psi(x)dx = \int_\Omega u(x,t)L\psi(x)dx + \int_{\partial\Omega} \psi(x)\frac{\partial u(x,t)}{\partial\eta}ds - \int_{\partial\Omega} u(x,t)\frac{\partial\psi(x)}{\partial\eta}ds.$$

Using (2.24)–(2.26), we find that $\int_\Omega Lu(x,t)\psi(x)dx = -\lambda\int_\Omega u(x,t)\psi(x)dx$.
Now, reasoning as in the proof of Theorem 2.1, we see that the result of Theorem 2.1 remains valid.

Now, let us show that we can obtain a result as the one given in Theorem 2.1 if the parameter ε is fixed, and the domain Ω is large enough.
Assume that the domain Ω contains the ball $B(0,R) = \{x \in \mathbb{R}^N; \parallel x \parallel < R\}$. Since $\Omega \supset B(0,R)$, we know that the eigenvalue λ defined in (2.1) obeys the following estimates

$$0 < \lambda \le \lambda_R = \frac{D}{R^2}, \tag{2.28}$$

where D is a positive constant which depends only on the upper bound of the coefficients of the operator L and the dimension N.
At the moment, we may state our result in the case where the domain Ω is large enough.

Theorem 2.4. *Assume that $dist(0, \partial\Omega) > 0$. Suppose that the solution $\alpha(t)$ of the differential equation defined in (1.6) blows up at the time T_e and let $E = \frac{D}{b(0)}\int_0^\infty \frac{d\sigma}{f(\sigma)}$. If $dist(0,\Omega) > \sqrt{\varepsilon E}$, then the solution u of (1.1)–(1.4) blows up in a finite time, and its blow-up time T obeys the following estimates*

$$0 \le T - T_e \le \frac{\varepsilon E T_e}{2(dist(0,\partial\Omega))^2} + o(\frac{1}{(dist(0,\partial\Omega))^2}).$$

Proof. As in the proof of Theorem 2.1, it is not difficult to see that the solution u of (1.1)–(1.4) blows up in a finite time T which obeys the following estimates

$$T_e \le T \le \frac{T_e}{\sqrt{1-\varepsilon A}}, \tag{2.29}$$

where $A = \frac{\lambda}{b(0)}\int_0^\infty \frac{d\sigma}{f(\sigma)}$.
Thanks to (2.28), $\lambda \le \frac{D}{R^2}$, which implies that $A \le \frac{D}{b(0)R^2}\int_0^\infty \frac{d\sigma}{f(\sigma)} = \frac{E}{R^2}$, where $R = dist(0,\partial\Omega)$. We deduce from (2.29) that

$$T_e \le T \le \frac{T_e}{\sqrt{1-\frac{\varepsilon E}{R^2}}}. \tag{2.30}$$

Apply Taylor's expansion to obtain

$$\frac{1}{\sqrt{1-\frac{\varepsilon E}{R^2}}}=1+\frac{\varepsilon E}{2R^2}+o(\frac{1}{R^2}). \tag{2.31}$$

Use (2.30), and the above relation to complete the rest of the proof. □

A direct consequence of Theorem 2.4 is that, if the solution $\alpha(t)$ of the differential equation defined in (1.6) blows up at the time T_e, and $\Omega=\mathbb{R}^N$, then the solution u of (1.1)–(1.4) blows up at the time T, and the following relation holds

$$T=T_e.$$

3. Numerical results

In this section, we give some computational results to confirm the theory established in the previous section. We consider the radial symmetric solution of (1.1)–(1.4) when $\Omega=B(0,1)$, $L=\Delta$, $b(t)=1$, and $f(u)=e^u$. Hence, the problem (1.1)–(1.4) may be rewritten as follows

$$u_{tt}=\varepsilon(u_{rr}+\frac{N-1}{r}u_r)+e^u,\quad r\in(0,1),\quad t\in(0,T), \tag{3.1}$$

$$u_r(0,t)=0,\quad u(1,t)=0,\quad t\in(0,T), \tag{3.2}$$

$$u(r,0)=0,\quad u_t(r,0)=0,\quad r\in(0,1). \tag{3.3}$$

Let I be a positive integer and let $h=1/I$. Define the grid $x_i=ih$, $0\leq i\leq I$ and approximate the solution u of (3.1)–(3.3) by the solution $U_h^{(n)}=(U_0^{(n)},...,U_I^{(n)})^T$ of the following explicit scheme

$$\frac{U_0^{(n+1)}-2U_0^{(n)}+U_0^{(n-1)}}{\Delta t_n^2}=\varepsilon N\frac{2U_1^{(n)}-2U_0^{(n)}}{h^2}+e^{U_0^{(n)}},$$

$$\begin{aligned}\frac{U_i^{(n+1)}-2U_i^{(n)}+U_i^{(n-1)}}{\Delta t_n^2} &= \varepsilon(\frac{U_{i+1}^{(n)}-2U_i^{(n)}+U_{i-1}^{(n)}}{h^2}+\frac{(N-1)}{ih}\frac{U_{i+1}^{(n)}-U_{i-1}^{(n)}}{2h})\\ &\quad +e^{U_i^{(n)}},\quad 1\leq i\leq I-1,\end{aligned}$$

$$U_I^{(n)}=0,$$

$$U_i^{(0)}=0,\quad U_i^{(1)}=0,\quad 0\leq i\leq I.$$

In order to permit the discrete solution to reproduce the properties of the continuous one when the time t approaches the blow-up time T, we need to adapt the size of the time step so that we take $\Delta t_n=\min\{h^2,e^{-\frac{1}{2}\|U_h^{(n)}\|_\infty}\}$ with $\|U_h^{(n)}\|_\infty=\sup_{0\leq i\leq I}|U_i^{(n)}|$. We also approximate the solution u of (3.1)–(3.3) by the solution $U_h^{(n)}$ of the implicit scheme below

$$\frac{U_0^{(n+1)}-2U_0^{(n)}+U_0^{(n-1)}}{\Delta t_n^2}=\varepsilon N\frac{2U_1^{(n+1)}-2U_0^{(n+1)}}{h^2}+e^{U_0^{(n)}},$$

$$\frac{U_i^{(n+1)} - 2U_i^{(n)} + U_i^{(n-1)}}{\Delta t_n^2} = \varepsilon\Big(\frac{U_{i+1}^{(n+1)} - 2U_i^{(n+1)} + U_{i-1}^{(n+1)}}{h^2} + \frac{(N-1)}{ih}\frac{U_{i+1}^{(n+1)} - U_{i-1}^{(n+1)}}{2h}\Big)$$
$$+e^{U_i^{(n)}}, \quad 1 \le i \le I-1,$$

$$U_I^{(n+1)} = 0,$$

$$U_i^{(0)} = 0, \quad U_i^{(1)} = 0, \quad 0 \le i \le I.$$

As in the case of the explicit scheme, here, we also choose $\Delta t_n = \min\{h^2, e^{-\frac{1}{2}\|U_h^{(n)}\|_\infty}\}$. We need the following definition.

Definition 3.1. We say that the discrete solution $U_h^{(n)}$ of the explicit scheme or the implicit scheme blows up in a finite time if $\lim_{n\to\infty} \|U_h^{(n)}\|_\infty = \infty$, and the series $\sum_{n=0}^{\infty} \Delta t_n$ converges, where $\|U_h^{(n)}\|_\infty = \sup_{0\le i\le I} |U_i^{(n)}|$. The quantity $\sum_{n=0}^{\infty} \Delta t_n$ is called the numerical blow-up time of the discrete solution $U_h^{(n)}$.

In the tables 1 and 2, in rows, we present the numerical blow-up times, the numbers of iterations n, the CPU times and the orders of the approximations corresponding to meshes of 16, 32, 64, 128. We take for the numerical blow-up time $T^n = \sum_{j=0}^{n-1} \Delta t_j$ which is computed at the first time when

$$\Delta t_n = |T^{n+1} - T^n| \le 10^{-16}.$$

The order(s) of the method is computed from

$$s = \frac{\log((T_{4h} - T_{2h})/(T_{2h} - T_h))}{\log(2)}.$$

Numerical experiments for N=2; $\varepsilon = \frac{1}{50}$

Table 1. Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the explicit Euler method

I	T^n	n	CPU_t	s
16	2.220623	1143	3	–
32	2.221240	4555	21	–
64	2.221391	18203	169	2.03
128	2.221428	72767	1183	2.03

TABLE 2. Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the implicit Euler method

I	T^n	n	CPU_t	s
16	2.220623	1143	3	–
32	2.221240	4555	21	–
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