J. Nonlinear Sci. Appl. 1 (2008), no. 4, 236–240

THE JOURNAL OF NONLINEAR SCIENCES AND APPLICATIONS
http://www.tjnsa.com

ON DECOMPOSITION OF FUZZY A-CONTINUITY

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ABSTRACT. In this paper, we introduce and study the notion of fuzzy $C-$sets and fuzzy $C-$continuity. We also prove a mapping $f : X \to Y$ is fuzzy $A-$continuous if and only if it is both fuzzy semi-continuous and fuzzy $C-$continuous.

1. INTRODUCTION

In the classical paper [10] of 1965, Zadeh generalized the usual notion of a set by introducing the important and useful notion of fuzzy sets. Since then, this notion has had tremendous effect on both pure and applied mathematics in different respects. Recently El-Naschie has shown in [4] and [5] that the notion of fuzzy topology may be relevant to quantum particle physics in connection with string theory and ε^{∞} theory. In 1986, Tong [9] introduced the notion of $A-$sets and $A-$continuous mappings in topological spaces and proved that a mapping is continuous if and only if it is both $\alpha-$continuous and $A-$continuous. In 1990, Ganster [7] established a decomposition of $A-$continuity: A mapping $f : X \to Y$ is $A-$continuous if and only if it is both semi-continuous and LC-continuous. Erguang and Pengfei [6] introduced the notion of $C-$sets and $C-$continuity and obtained another decomposition of $A-$continuity: A mapping $f : X \to Y$is $A-$continuous if and only if it is both semi-continuous and $C-$continuous. Recently, Rajamani and Ambika [8] introduced the notion of fuzzy $A-$sets and fuzzy $A-$continuity and obtained a decomposition of fuzzy continuity.
In this paper, we transform the notions of $C-$set and $C-$continuity to fuzzy topological settings and obtain a decomposition of fuzzy $A-$continuity.

Date: Received: 2 December 2008.
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2000 *Mathematics Subject Classification.* 54A05,54C05.
Key words and phrases. fuzzy A-set, fuzzy C-set, fuzzy A-continuity, fuzzy C-continuity.

2. PRELIMINARIES

Throughout this paper, X and Y denote fuzzy topological spaces (X,τ) and (Y,σ) respectively on which no separation axioms are assumed. Let λ be a fuzzy set in a fuzzy topological spaces X. The fuzzy interior of λ , fuzzy closure of λ and fuzzy preclosure of λ are denoted by $int(\lambda), cl(\lambda)$ and $pcl(\lambda)$ respectively.

Now, we recall some definitions and results which are used in this paper.

DEFINITION 2.1: A fuzzy set λ in a fuzzy topological space X is called

(1) fuzzy semi-open [1] if $\lambda \leq cl(int(\lambda))$;
(2) fuzzy pre-open [2] if $\lambda \leq int(cl(\lambda))$;
(3) fuzzy regular-open [1] if $\lambda = int(cl(\lambda))$.

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

DEFINITION 2.2: A fuzzy set λ in a fuzzy topological space X is called a fuzzy $A-$set [6] if $\lambda = \alpha \wedge \beta$, where α is a fuzzy open set and β is a fuzzy regular closed set.

DEFINITION 2.3: A map $f : X \to Y$ is said to be

(1) fuzzy continuous [3] if $f^{-1}(\mu)$ is fuzzy open in X, for every fuzzy open set μ in Y ;
(2) fuzzy semi-continuous [1] if $f^{-1}(\mu)$ is fuzzy semi-open in X, for every fuzzy open set μ in Y ;
(3) fuzzy pre-continuous [2] if $f^{-1}(\mu)$ is fuzzy pre-open in X, for every fuzzy open set μ in Y ;

The collection of all fuzzy $C-$sets and fuzzy semi-open sets in X will be denoted by FC(X,τ) and FSO(X,τ) respectively.

3. FUZZY $C-$SETS

DEFINITION 3.1: A fuzzy set λ in a fuzzy topological space X is called a fuzzy $C-$set if $\lambda = \alpha \wedge \beta$, where α is fuzzy open and β is fuzzy pre-closed in X.

PROPOSITION 3.2: Every fuzzy $A-$set is a fuzzy $C-$set.

REMARK 3.3: The converse of the Proposition 3.2. need not be true as seen from the following example.

EXAMPLE 3.4: Let $X = \{a, b, c\}$, Define $\alpha_1, \alpha_2, \alpha_3 : X \to [0, 1]$ by
α_1(a) = 0.3 α_2(a) = 0.4 α_3 (a) = 0.7
α_1(b) = 0.4 α_2(b) = 0.5 α_3 (b)= 0.6
α_1(c) = 0.4 α_2(c) = 0.5 α_3 (c) = 0.6
Let $\tau = \{0, 1, \alpha_1, \alpha_2\}$. Then (X,τ) is a fuzzy topological space. Now, α_3 is a

fuzzy $C-$set but not a fuzzy $A-$set.

REMARK 3.5: The concepts of fuzzy $C-$sets and fuzzy semi-open sets are independent as shown by the following examples.

EXAMPLE 3.6: Let $X = \{a, b, c\}$, Define $\alpha_1, \alpha_2, \alpha_3 : X \to [0, 1]$ by
α_1(a) = 0.2 α_2(a) = 0.3 α_3 (a) = 0.3
α_1(b) = 0.3 α_2(b) = 0.3 α_3 (b)= 0.3
α_1(c) = 0.3 α_2(c) = 0.4 α_3 (c)= 0.3
Let $\tau = \{0, 1, \alpha_1, \alpha_2\}$. Then (X, τ) is a fuzzy topological space. Now, α_3 is a fuzzy semi-open set but not a fuzzy $C-$set.

EXAMPLE 3.7: Let $X = \{a, b, c\}$, Define $\alpha_1, \alpha_2, \alpha_3 : X \to [0, 1]$ by
α_1(a) = 0.4 α_2(a) = 0.6 α_3 (a) = 0.5
α_1(b) = 0.5 α_2(b) = 0.7 α_3 (b)= 0.6
α_1(c) = 0.6 α_2(c) = 0.8 α_3 (c)= 0.7
Let $\tau = \{0, 1, \alpha_1, \alpha_2\}$. Then (X, τ) is a fuzzy topological space. Now, α_3 is a fuzzy $C-$set but not a fuzzy semi-open set.

LEMMA 3.8: Let α be a fuzzy set in a fuzzy topological space X. Then $\alpha \in FC(X, \tau)$ if and only if $\alpha = \lambda \wedge pcl(\alpha)$ for some fuzzy open set λ.

Proof: Let $\alpha \in FC(X, \tau)$. Then $\alpha = \lambda \wedge \mu$ where λ is fuzzy open and μ is fuzzy pre-closed. Now, $\alpha \leq \lambda$ and $\alpha \leq \mu$, we have $pcl(\alpha) \leq pcl(\mu) = \mu$, since μ is fuzzy pre-closed in X. Thus $pcl(\alpha) \leq \mu$. Therefore $\lambda \wedge pcl(\alpha) \leq (\lambda \wedge \mu) = \alpha \leq \lambda \wedge pcl(\alpha)$. (i.e.,) $\lambda \wedge pcl(\alpha) = \alpha$.
Converse part is obvious.

THEOREM 3.9: Let α be a fuzzy set in a fuzzy topological space X. Then $\alpha = \lambda \wedge cl(int(\alpha))$ for some fuzzy open set λ if and only if $\alpha \in FC(X, \tau) \wedge FSO(X, \tau)$.

Proof: Let $\alpha = \lambda \wedge cl(int(\alpha))$ for some fuzzy open set λ in X. Then $\alpha \leq cl(int(\alpha))$. So α is fuzzy semi open in X. Let $\beta = cl(int(\alpha))$, then β is fuzzy regular closed. Since every fuzzy regular closed set is fuzzy pre-closed, β is fuzzy pre-closed which implies α is fuzzy $C-$set. Thus $\alpha \in FC(X, \tau) \wedge FSO(X, \tau)$.
Conversely, let $\alpha \in FC(X, \tau) \wedge FSO(X, \tau)$. Then $\alpha \in FC(X, \tau)$ and $\alpha \in FSO(X, \tau)$. Since $\alpha \in FC(X, \tau), \alpha = \lambda \wedge pcl(\alpha)$, using Lemma 3.8. Thus $\alpha = \lambda \wedge cl(int(\alpha))$ for some fuzzy open set λ.

4. DECOMPOSITION OF FUZZY $A-$CONTINUITY

DEFINITION 4.1: A mapping $f : X \to Y$ is called fuzzy $A-$continuous if $f^{-1}(\mu)$ is a fuzzy $A-$ set in X, for every fuzzy open set μ in Y.

DEFINITION 4.2: A mapping $f : X \to Y$ is called fuzzy $C-$continuous if $f^{-1}(\mu)$ is a fuzzy $C-$set in X, for every fuzzy open set μ in Y.

PROPOSITION 4.3: Every fuzzy $A-$continuous function is fuzzy $C-$continuous.

REMARK 4.4: The converse of Proposition 4.3 need not be true as shown by the following example.

EXAMPLE 4.5: Let $X = \{a, b, c\}, Y = \{x, y, z\}$ and α_1, α_2 and α_3 are fuzzy sets defined as follows :
α_1(a) = 0.3 α_2(a) = 0.4 α_3 (a) = 0.7
α_1(b) = 0.4 α_2(b) = 0.5 α_3 (b)= 0.6
α_1(c) = 0.4 α_2(c) = 0.5 α_3 (c)= 0.6
Let $\tau_1 = \{0, 1, \alpha_1, \alpha_2\}, \tau_2 = \{0, 1, \alpha_3\}$. Then the mapping $f : (X, \tau_1) \to (Y, \tau_2)$ defined by f(a) = x, f(b) = y and f(c) = z is fuzzy $C-$continuous but not fuzzy $A-$continuous.

REMARK 4.6: The concepts of fuzzy $C-$continuity and fuzzy semi-continuity are independent as shown by the following examples.

THEOREM 4.7: Let $X = \{a, b, c\}, Y = \{x, y, z\}$ and α_1, α_2 and α_3 are fuzzy sets defined as follows :
α_1(a) = 0.2 α_2(a) = 0.3 α_3 (a) = 0.3
α_1(b) = 0.3 α_2(b) = 0.3 α_3 (b)= 0.3
α_1(c) = 0.3 α_2(c) = 0.4 α_3 (c)= 0.3
Let $\tau_1 = \{0, 1, \alpha_1, \alpha_2\}, \tau_2 = \{0, 1, \alpha_3\}$. Then the mapping $f : (X, \tau_1) \to (Y, \tau_2)$ defined by f(a) = x, f(b) = y and f(c) = z is fuzzy semi-continuous but not fuzzy $C-$continuous.

EXAMPLE 4.8: Let $X = \{a, b, c\}, Y = \{x, y, z\}$ and α_1, α_2 and α_3 are fuzzy sets defined as follows :
α_1(a) = 0.4 α_2(a) = 0.6 α_3 (a) = 0.5
α_1(b) = 0.5 α_2(b) = 0.7 α_3 (b)= 0.6
α_1(c) = 0.6 α_2(c) = 0.8 α_3 (c)= 0.7
Let $\tau_1 = \{0, 1, \alpha_1, \alpha_2\}, \tau_2 = \{0, 1, \alpha_3\}$. Then the mapping $f : (X, \tau_1) \to (Y, \tau_2)$ defined by f(a) = x, f(b) = y and f(c) = z is fuzzy $C-$continuous but not fuzzy semi-continuous.

THEOREM 4.9: A mapping $f : X \to Y$ is fuzzy $A-$continuous if and only if it is both fuzzy semi-continuous and fuzzy $C-$continuous.
Proof: Follows from Theorem 3.9.

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