J. Nonlinear Sci. Appl. 3 (2010), no. 3, 157–163

The Journal of Nonlinear Sciences and Applications
http://www.tjnsa.com

FUZZY MINIMAL SEPARATION AXIOMS

M. ALIMOHAMMADY[1], E. EKICI[2,*], S. JAFARI[3], M. ROOHI[1]

Abstract. In this paper, we deal with some separation axioms in the context of fuzzy minimal structures.

1. Introduction

Zadeh introduced the concept of a fuzzy set in [14]. Subsequently, many attempts have been made to extend many science notions to the fuzzy setting, for example [8, 10, 11]. Fuzzy minimal structure and fuzzy minimal space introduced and investigated in [1–7]. For easy understanding of the material incorporated in this paper, we recall some basic definitions and results. For details on the following notions we refer to [1–14] and the references cited therein.

A *fuzzy set* in(on) a universe set X is a function with domain X and values in $I = [0, 1]$. The class of all fuzzy sets on X will be denoted by I^X and symbols A,B,... is used for fuzzy sets on X. 01_X is called *empty fuzzy set* where 1_X is the characteristic function on X. A family $\mathcal{M}$ of fuzzy sets in X is said to be a *fuzzy minimal structure in Chang's sense* on X if $\{01_X, 1_X\} \subseteq \mathcal{M}$. In this case $(X, \mathcal{M})$ is called a *fuzzy minimal space* [2]. A fuzzy set $A \in I^X$ is said to be *fuzzy m-open* if $A \in \mathcal{M}$. $B \in I_X$ is called a *fuzzy m-closed set* if $B^c \in \mathcal{M}$. Let

$$m - Int(A) = \bigvee\{U : U \leq A, U \in \mathcal{M}\} \text{ and } \tag{1.1}$$

$$m - Cl(A) = \bigwedge\{F : A \leq F, F^c \in \mathcal{M}\}. \tag{1.2}$$

A fuzzy set in X is called a *fuzzy point* if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is α $(0 < \alpha \leq 1)$, we denote this fuzzy point by x_α, where the point x is called its *support* [12, 13]. For any fuzzy point

Date: Revised: 03, May, 2010.
* Corresponding author

Key words and phrases. Fuzzy sets, fuzzy topology, fuzzy separation axiom.

x_α and any fuzzy set A, $x_\alpha \in A$ if and only if $\alpha \leq A(x)$. A fuzzy point x_α is called *quasi-coincident* with a fuzzy set B, denoted by $x_\alpha q B$, if $\alpha + B(x) > 1$. A fuzzy set A is called *quasi-coincident* with a fuzzy set B, denoted by AqB, if there exists a $x \in X$ such that $A(x) + B(x) > 1$ [12, 13]. When they are not quasi-coincident, it will denoted by $A \not{q} B$. Throughout in this paper we assume that all fuzzy minimal spaces are in the sense of Chang.

Proposition 1.1. [2] *For any two fuzzy sets A and B in a fuzzy minimal space $(X, \mathcal{M})$*

(1) $m - Int(A) \leq A$ *and* $m - Int(A) = A$ *if A is a fuzzy m-open set.*

(2) $A \leq m - Cl(A)$ *and* $A = m - Cl(A)$ *if A is a fuzzy m-closed set.*

(3) $m - Int(A) \leq m - Int(B)$ *and* $m - Cl(A) \leq m - Cl(B)$ *if* $A \leq B$.

(4) $(m - Int(A)) \wedge (m - Int(B)) \leq m - Int(A \wedge B)$ *and* $(m - Int(A)) \vee (m - Int(B)) \leq m - Int(A \vee B)$.

(5) $(m - Cl(A)) \vee (m - Cl(B)) \leq m - Cl(A \vee B)$ *and* $m - Cl(A \wedge B) \leq (m - Cl(A)) \wedge (m - Cl(B))$.

(6) $m - Int(m - Int(A)) = m - Int(A)$ *and* $m - Cl(m - Cl(B)) = m - Cl(B)$.

(7) $(m - Cl(A))^c = m - Int(A^c)$ *and* $(m - Int(A))^c = m - Cl(A^c)$.

Definition 1.2. [2] A fuzzy minimal space $(X, \mathcal{M})$ enjoys the *property U* if arbitrary union of fuzzy m-open sets is fuzzy m-open.

Proposition 1.3. [1] *For a fuzzy minimal structure $\mathcal{M}$ on a set X, the following statements are equivalent.*

(1) $(X, \mathcal{M})$ *has the property U.*

(2) *If* $m - Int(A) = A$, *then* $A \in \mathcal{M}$.

(3) *If* $m - Cl(B) = B$, *then* $B^c \in \mathcal{M}$.

Fuzzy minimal continuous functions was introduced and studied in [3].

Definition 1.4. [3] Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two fuzzy minimal spaces. We say that a fuzzy function $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ is *fuzzy minimal continuous* (briefly fuzzy m-continuous) if $f^{-1}(B) \in \mathcal{M}$, for any $B \in \mathcal{N}$.

Theorem 1.5. [3] *Consider the following properties for a fuzzy function $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ between two fuzzy minimal spaces.*

(1) f *is a fuzzy m-continuous function.*

(2) $f^{-1}(B)$ *is a fuzzy m-closed set for each fuzzy m-closed set* $B \in I^Y$.

(3) $m - Cl(f^{-1}(B)) \leq f^{-1}(m - Cl(B))$ *for each* $B \in I^Y$.

(4) $f(m - Cl(A)) \leq m - Cl(f(A))$ *for any* $A \in I^X$.

(5) $f^{-1}(m - Int(B)) \leq m - Int(f^{-1}(B))$ *for each* $B \in I^Y$.

Then (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5). *Moreover, if $(X, \mathcal{M})$ satisfies in the property U, then all of the above statements are equivalent.*

2. **Fuzzy minimal separation axioms**

Definition 2.1. A fuzzy set N in a fuzzy minimal space $(X, \mathcal{M})$ is said to be a *fuzzy minimal neighborhood* of a fuzzy point x_α if there is a fuzzy m-open set μ in X with $x_\alpha \in \mu$ and $\mu \leq N$.

Definition 2.2. Suppose $(X, \mathcal{M})$ is a fuzzy minimal space. A fuzzy set N in X is said to be a *fuzzy minimal q-neighborhood* of a fuzzy point x_α if there is a fuzzy m-open set μ in X with $x_\alpha q \mu$ and $\mu \leq A$.

Definition 2.3. Suppose $(X, \mathcal{M})$ is a fuzzy minimal space. A fuzzy point x_α in X is said to be *fuzzy minimal cluster point* of a fuzzy set A if every fuzzy minimal q-neighborhood of x_α is q-coincident with A.

Theorem 2.4. *Suppose $(X, \mathcal{M})$ is a fuzzy minimal space. A fuzzy point x_α is a fuzzy minimal cluster point of a fuzzy set A if and only if $x_\alpha \in m - Cl(A)$.*

Proof. Suppose $x_\alpha \notin m - Cl(A)$. Then, one can easily see that there exists m-closed set F in X with $A \leq F$ and $F(x) < \alpha$. Therefore, $x_\alpha q F^c$ and $A \not{q} F^c$; i.e., x_α is not a fuzzy minimal cluster point of A. Conversely, suppose x_α is not a fuzzy minimal cluster point of A. There exists a fuzzy minimal q-neighborhood N of x_α for which $N \not{q} A$. Then there exists a fuzzy m-open set μ in X with $x_\alpha q \mu$ and $\mu \leq N$. Therefore, $\mu \not{q} A$ which implies that $A \leq \mu^c$. Since μ^c is m-closed, so (1.2) implies that $m - Cl(A) \leq \mu^c$. That $x_\alpha \notin m - Cl(A)$ follows from the fact that $x_\alpha \notin \mu^c$.

Definition 2.5. A fuzzy minimal space $(X, \mathcal{M})$ is said to be

(1) *fuzzy minimal* T_0 if for every pair of distinct fuzzy points x_α and x_β,

when $x \neq y$ either x_α has a fuzzy minimal neighborhood which is not q-coincident with y_β or y_β has a fuzzy minimal neighborhood which is not q-coincident with x_α,

when $x = y$ and $\alpha < \beta$ (say), there is a fuzzy minimal q-neighborhood of y_β which is not q-coincident with x_α,

(2) *fuzzy minimal* T_1 if for every pair of distinct fuzzy points x_α and x_β,

when $x \neq y$ there is a fuzzy minimal neighborhood μ of x_α and a fuzzy minimal neighborhood ν of y_β with $\mu \not{q} y_\beta$ and $x_\alpha \not{q} \nu$,

when $x = y$ and $\alpha < \beta$ (say), y_β has a fuzzy minimal q-neighborhood which is not q-coincident with x_α,

(3) *fuzzy minimal* T_2 if for every pair of distinct fuzzy points x_α and x_β,

when $x \neq y$, x_α and y_β have fuzzy minimal q-neighborhoods which are not q-coincident,

when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy minimal neighborhood μ and y_β has a fuzzy minimal q-neighborhood ν in which $\mu \not{q} \nu$.

In short fuzzy $m - T_i$(i=0,1,2) spaces are used for fuzzy minimal T_i spaces.

Theorem 2.6. *Every fuzzy $m - T_2$ space is a fuzzy $m - T_1$ space and also every fuzzy $m - T_1$ space is a fuzzy $m - T_0$ space.*

Proof. Obvious.

Theorem 2.7. *A fuzzy minimal space* $(X, \mathcal{M})$ *is fuzzy* $m-T_1$ *if every fuzzy point* x_α *is fuzzy* m*-closed in* X.

Proof. Suppose x_α and y_β are distinct fuzzy points in X, there are two cases

(i) $x \neq y$

(ii) $x = y$ and $\alpha < \beta$ (say).

Assume that $x \neq y$. By hypothesis x_α^c and y_β^c are fuzzy m-open sets. It is easy to see that $x_\alpha \in y_\beta^c$, $y_\beta \in x_\alpha^c$, $x_\alpha \not{q} x_\alpha^c$ and $y_\beta \not{q} y_\beta^c$. In case that $x = y$ and $\alpha < \beta$, one can deduce that x_α^c is a fuzzy m-open set with $y_\beta q x_\alpha^c$ and $x_\alpha \not{q} x_\alpha^c$ which implies that $(X, \mathcal{M})$ is fuzzy $m - T_1$.

Theorem 2.8. *Let* $(X, \mathcal{M})$ *be a fuzzy minimal space. Then* $(X, \mathcal{M})$ *is fuzzy minimal* T_1 *if for each* $x \in X$ *and each* $\alpha \in [0, 1]$ *there exists a fuzzy minimal open set* μ *such that* $\mu(x) = 1 - \alpha$ *and* $\mu(y) = 1$ *for* $y \neq x$.

Proof. Let x_α be an arbitrary fuzzy point of X. We shall show that the fuzzy point x_α is fuzzy minimal closed. By hypothesis, there exists a fuzzy minimal open set μ such that $\mu(x) = 1 - \alpha$ and $\mu(y) = 1$ for $y \neq x$. We have $\mu^c = x_\alpha$. Thus, the fuzzy point x_α is fuzzy minimal closed and hence by Theorem 2.7 the fuzzy minimal space X is fuzzy minimal T_1.

Theorem 2.9. *Suppose* $i = 0, 1, 2$. *A fuzzy minimal space* $(X, \mathcal{M})$ *is fuzzy* $m-T_i$ *if and only if for any pair of distinct fuzzy points* x_α *and* y_β *with distinct supports, there exists a fuzzy* m*-continuous mapping* f *from* X *into a fuzzy* $m - T_i$ *space* $(Y, \mathcal{N})$ *such that* $f(x) \neq f(y)$.

Proof. We only prove the case that $i = 2$ and others are similar. Suppose $(X, \mathcal{M})$ is fuzzy $m - T_2$ space. Let $(Y, \mathcal{N}) := (X, \mathcal{M})$ and $f := id_X$. Clearly, $(Y, \mathcal{N})$ and f have the required properties. Conversely, suppose x_α and y_β are distinct fuzzy points in X. There are two cases

(i) $x \neq y$

(ii) $x = y$ and $\alpha < \beta$ (say).

When $x \neq y$, by assumption there is fuzzy m-continuous mapping f from $(X, \mathcal{M})$ into a fuzzy $m - T_2$ space $(Y, \mathcal{N})$ with $f(x) \neq f(y)$. Since $(Y, \mathcal{N})$ is fuzzy $m - T_2$ space and $(f(x))_\alpha$ and $(f(y))_\beta$ are distinct fuzzy points in Y, so there are fuzzy minimal neighborhoods μ and ν of $(f(x))_\alpha$ and $(f(y))_\beta$ respectively for which $\mu \not{q} \nu$. It follows from m-continuity of f that $f^{-1}(\mu)$ and $f^{-1}(\nu)$ are fuzzy minimal neighborhoods of x_α and y_β respectively. Since $\mu \not{q} \nu$, so $f^{-1}(\mu) \not{q} f^{-1}(\nu)$. In case that $x = y$ and $\alpha < \beta$ (say), $(f(x))_\alpha$ and $(f(y))_\beta$ are fuzzy points in Y with $f(x) = f(y)$. Since $(Y, \mathcal{N})$ is fuzzy $m - T_2$ space, so $(f(x))_\alpha$ has a fuzzy minimal neighborhood μ and $(f(y))_\beta$ has a fuzzy minimal q-neighborhoods ν for which $\mu \not{q}\, \nu$. Then $f^{-1}(\mu)$ is a fuzzy minimal q-neighborhood of x_α and $f^{-1}(\nu)$ is a fuzzy minimal q-neighborhood of y_β with $f^{-1}(\mu) \not{q} f^{-1}(\nu)$. Therefore, $(X, \mathcal{M})$ is fuzzy $m - T_2$ space.

Corollary 2.10. *Suppose* $(X, \mathcal{M})$ *and* $(Y, \mathcal{N})$ *are fuzzy minimal spaces and* $f : X \longrightarrow Y$ *is injective and fuzzy* m*-continuous.* $(X, \mathcal{M})$ *is fuzzy* $m - T_i$ *space if* $(Y, \mathcal{N})$ *is fuzzy* $m - T_i$ *space.*

Proof. It is an immediate consequence of Theorem 2.9.

Theorem 2.11. *Let $(X, \mathcal{M})$ be a fuzzy minimal space. If $(X, \mathcal{M})$ is fuzzy minimal T_2, then for any two distinct fuzzy points x_α and y_β, the following properties hold:*

(1) *If $x \neq y$, then there exist fuzzy open neighborhoods μ and ν of x_α and y_β, respectively, such that $m - Cl(\nu) \leq \mu^c$ and $m - Cl(\mu) \leq \nu^c$,*

(2) *If $x = y$ and $\alpha < \beta$ (say), then there exists a fuzzy open neighborhood μ of x_α such that $y_\beta \notin m - Cl(\mu)$.*

Proof. (1) : Let $x \neq y$. Then there exist fuzzy m-open neighborhoods μ and ν of x_α and y_β, respectively, such that $\mu \not{q} \nu$. Since $\mu \not{q} \nu$, then $\mu(z) \leq 1 - \nu(z)$ and $\nu(z) \leq 1 - \mu(z)$ for all $z \in X$. Since μ^c and ν^c are fuzzy m-closed, then $m - Cl(\nu) \leq \mu^c$ and $m - Cl(\mu) \leq \nu^c$.

(2) : Let $x = y$. Then there exist a fuzzy minimal q-neighborhood λ of y_β and a fuzzy open neighborhood μ of x_α such that $\lambda \not{q} \mu$. Now, let ν be a fuzzy m-open set in X such that $y_\beta q \nu$ and $\nu \leq \lambda$. Since $\beta > 1 - \nu(y) = (m - Cl(\nu^c))(y)$, $\nu \leq \lambda$ and $\mu \leq \lambda^c$, then $\beta > m - Cl(\mu)(y)$ for all $y \in X$. Thus, $y_\beta \notin m - Cl(\mu)$.

Theorem 2.12. *Let $(X, \mathcal{M})$ be a fuzzy minimal space. Suppose that $(X, \mathcal{M})$ enjoys the property U. Then $(X, \mathcal{M})$ is fuzzy minimal T_2 if and only if for any two distinct fuzzy points x_α and y_β, the following properties hold:*

(1) *If $x \neq y$, then there exist fuzzy m-open neighborhoods μ and ν of x_α and y_β, respectively, such that $m - Cl(\nu) \leq \mu^c$ and $m - Cl(\mu) \leq \nu^c$,*

(2) *If $x = y$ and $\alpha < \beta$ (say), then there exists a fuzzy m-open neighborhood μ of x_α such that $y_\beta \notin m - Cl(\mu)$.*

Proof. $(\Rightarrow)$: It follows from Theorem 2.11.

$(\Leftarrow)$: Let x_α and y_β be distinct fuzzy points in X and let $x \neq y$. Then there exist fuzzy m-open neighborhoods μ and ν of x_α and y_β, respectively, such that $m - Cl(\nu) \leq \mu^c$. This implies that for all $z \in X$, $\mu(z) + \nu(z) \leq (m - Cl(\nu))(z) + \mu(z) \leq 1$. Hence, $\mu \not{q} \nu$. Now, let $x = y$ and $\alpha < \beta$. Then there exists a fuzzy m-open neighborhood μ of x_α such that $y_\beta \notin m - Cl(\mu)$. Let $\lambda = (m - Cl(\mu))^c$. Since for all $z \in X$, $\lambda(z) + \mu(z) \leq 1$, then $\lambda \ /q \mu$. On the other hand, λ is a fuzzy open set and $\beta + \lambda(y) > \alpha + \lambda(y) \geq 1$. Hence, λ is a fuzzy minimal q-neighborhood of y_β such that $\lambda \not{q} \mu$.

Theorem 2.13. *Let $(X, \mathcal{M})$ be a fuzzy minimal space. If $(X, \mathcal{M})$ is fuzzy minimal T_2, then the following hold:*

(1) *for every fuzzy point x_α in X, $x_\alpha = \bigwedge\{m - Cl(\nu) : \nu$ is a fuzzy minimal neighborhood of $x_\alpha\}$.*

(2) *for every x, $y \in X$ with $x \neq y$, there exist a fuzzy minimal neighborhood μ of x_1 such that $y \notin$ supp$(m - Cl(\mu))$.*

Proof. (1) : Let $y_\beta \notin x_\alpha$. We shall show the existence of a fuzzy minimal neighborhood of x_α such that $y_\beta \notin m - Cl(\nu)$.

Let $x \neq y$. Then there exist fuzzy minimal open sets μ and ν containing y_1 and x_α, respectively such that $\mu \not{q} \nu$. Then ν is fuzzy minimal neighborhood of x_α and μ is a fuzzy minimal q-neighborhood of y_β such that $\mu \not{q} \nu$. Hence, by using Theorem 2.4 we get $y_\beta \notin m - Cl(\nu)$.

Let $x = y$. Then $\alpha < \beta$ and there exist a fuzzy minimal q-neighborhood μ of y_β and fuzzy minimal neighborhood ν of x_α such that $\mu \not{q} \nu$. Thus, $y_\beta \notin m - Cl(\nu)$.

(2) : For every x, $y \in X$ with $x \neq y$, since $(X, \mathcal{M})$ is fuzzy minimal T_2, then there exist fuzzy minimal open sets μ and ν such that $x_1 \in \mu$, $y_1 \in \nu$ and $\mu \not{q} \nu$. Then $\nu^c(y) = 0$ and $\mu \leq \nu^c$. Since ν^c is fuzzy minimal closed, $m - Cl(\mu) \leq \nu^c$. Thus, $m - Cl(\mu)(y) = 0$ and hence, $y \notin supp(m - Cl(\mu))$.

Theorem 2.14. *Let $(X, \mathcal{M})$ be a fuzzy minimal space with property U. Then $(X, \mathcal{M})$ is fuzzy minimal T_2 if and only if*

(1) *for every fuzzy point x_α in X, $x_\alpha = \bigwedge\{m - Cl(\nu) : \nu$ is a fuzzy minimal neighborhood of $x_\alpha\}$.*

(2) *for every x, $y \in X$ with $x \neq y$, there exist a fuzzy minimal neighborhood μ of x_1 such that $y \notin supp(m - Cl(\mu))$.*

Proof. $(\Rightarrow)$: It follows from Theorem 2.13.

$(\Leftarrow)$: Let x_α and y_β be two distinct fuzzy points in X.

Let $x \neq y$. Suppose that $0 < \alpha < 1$. There exists a real number δ such that $0 < \alpha + \delta < 1$. By hypothesis, there exists a fuzzy minimal neighborhood μ of y_β such that $x_\delta \notin m - Cl(\mu)$. Then x_δ has a fuzzy minimal q-neighborhood ν such that $\mu \not{q} \nu$. On the other hand, $\delta + \nu(x) > 1$ and $\nu(x) > 1 - \delta > \alpha$ and hence ν is a fuzzy minimal neighborhood of x_α such that $\mu \not{q} \nu$, where μ is a fuzzy minimal neighborhood of y_β. If $\alpha = \beta = 1$, by hypothesis there exists a fuzzy minimal neighborhood μ of x_1 such that $m - Cl(\mu)(y) = 0$. Thus, $\nu = (m - Cl(\mu))^c$ is a fuzzy minimal neighborhood of y_1 such that $\mu \not{q} \nu$.

Let $x = y$ and $\alpha < \beta$. Then there exists a fuzzy minimal neighborhood of x_α such that $y_\beta \notin m - Cl(\mu)$. Thus, there exists a fuzzy minimal q-neighborhood ν of y_β such that $\mu \not{q} \nu$. Hence, $(X, \mathcal{M})$ is fuzzy minimal T_2.

3. Acknowledgements:

This paper is supported by the Research Center in Algebraic Hyperstructures and Fuzzy Mathematics, University of Mazandaran, Babolsar, Iran.

References

[1] M. Alimohammady, S. Jafari and M. Roohi, Fuzzy minimal connected sets, Bull. Kerala Math. Assoc., 5 (2008), 1–15.

[2] M. Alimohammady and M. Roohi, Compactness in fuzzy minimal spaces, Chaos, Solitons & Fractals, 28 (2006), 906–912.

[3] M. Alimohammady and M. Roohi, *Fuzzy minimal structure and fuzzy minimal vector spaces*, Chaos, Solitons & Fractals, 27 (2006), 599–605.

[4] M. Alimohammady and M. Roohi, Fuzzy transfer minimal closed multifunctions, Ital. J. Pure Appl. Math., 22 (2007), 67–74.

[5] M. Alimohammady and M. Roohi, Fuzzy U_m sets and fuzzy (U, m)-continuous functions, Chaos, Solitons, & Fractals 28 (2006), 20–25.

[6] M. Alimohammady and M. Roohi, Λ_m-Set in fuzzy minimal space, Journal of Basic Science, 3 (2006), 11–17.

[7] M. Alimohammady and M. Roohi, Separation of fuzzy sets in fuzzy minimal spaces, Chaos, Solitons & Fractals, 31 (2007), 155–161.

[8] A. Azam, M. Arshad and I. Beg, Common fixed point theorems in cone metric spaces, J. Nonlinear Sci. Appl., 2 (2009), 204–213.
[9] B. Ghosh, Semi-continuous and semi-closed mappings and semi-connectedness in fuzzy setting, Fuzzy sets syst. 35 (1990), 345–355.
[10] I. M. Hanafy, βS^*-compactness in L-fuzzy topological spaces, J. Nonlinear Sci. Appl., 2 (2009), 27–37.
[11] S. Jafari, K. Viswanathan, M. Rajamani and S. Krishnaprakash, On decomposition of fuzzy A-continuity, J. Nonlinear Sci. Appl., 1 (2008), 236–240.
[12] P. Pao-Ming and L. Ying-Ming, Fuzzy topology I. neighborhood structure of a fuzzy point and and Moore-Smith convergence, J. Math. Anal. Appl. 76 (1980), 571–599.
[13] P. Pao-Ming and L. Ying-Ming, Fuzzy Topology II. Product and quotient spaces, J. Math. Anal. Appl., 77 (1980), 20–37.
[14] L. A. Zadeh, Fuzzy sets, Inf. control, 8 (1965), 338–353.

[1] Department of Mathematics, Faculty of Basic Sciences, University of Mazandaran, Babolsar 47416 – 1468, Iran
E-mail address: amohsen@umz.ac.ir
E-mail address: mehdi.roohi@gmail.com

[2] Department of Mathematics, Canakkale Onsekiz Mart University, Terzioglu Campus, 17020 Canakkale, Turkey
E-mail address: eekici@comu.edu.tr

[3] Department of Economics, Copenhagen University, Oester Farimagsgade 5, building 26, 1353 Copenhagen K, Denmark
E-mail address: jafari@stofanet.dk