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Fuzzy cone metric spaces

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Abstract

In this study, we define the fuzzy cone metric space, the topology induced by this space and some related results of them. Also we state and prove the fuzzy cone Banach contraction theorem.

Keywords:
Cone metric space, fuzzy metric space, fuzzy cone metric space, fuzzy cone contractive mapping
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1. Introduction and preliminaries

Huang and Zhang [5] introduced the notion of cone metric spaces by replacing real numbers with an ordering Banach space and proved some fixed point theorems for contractive mappings between these spaces. After the paper [5], series of articles about cone metric spaces started to appear.

On the other hand, after the theory of fuzzy sets which was introduced by Zadeh [4], there has been a great effort to obtain fuzzy analogues of classical theories. In particular, Kramosil and Michalek in [6] introduced the fuzzy metric space. Later on, George and Veeramani in [1] gave a stronger form of metric fuzziness.

In this paper, we introduce the notion of fuzzy cone metric space that generalize the corresponding notions of fuzzy metric space by George and Veeramani. Also we give the topology induced by this space and then give some properties about this topology such as Hausdorffness and first countability. Finally we give the fuzzy cone Banach contraction theorem. With the help of these results one can derive many properties of fuzzy cone metric spaces.

Throughout this paper E denotes a real Banach space and θ denotes the zero of E.

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Definition 1.1 ([5])**.** A subset P of E is called a cone if
1) P is closed, nonempty and $P \neq \{\theta\}$,
2) If $a, b \in \mathbb{R}$, $a, b \geq 0$ and $x, y \in P$, then $ax + by \in P$,
3) If both $x \in P$ and $-x \in P$ then $x = \theta$.

For a given cone, a partial ordering $\preceq$ on E via P is defined by $x \preceq y$ if only if $y - x \in P$. $x \prec y$ will stand for $x \preceq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in int\,(P)$. Throughout this paper, we assume that all cones has nonempty interior.

Proposition 1.2. *Let P be a cone of E. Then;*
1) [7] $int\,(P) + int\,(P) \subset int\,(P)$,
2) [7] $\lambda int\,(P) \subset int\,(P)$ for any $\lambda \in \mathbb{R}^+$,
3) [3] For each $\theta \preceq c_1$ and $\theta \preceq c_2$, there is an element $\theta \preceq c$ such that $c \preceq c_1$, $c \preceq c_2$.

The cone P is called normal if there exists a constant $K > 0$ such that for all $t, s \in E$, $\theta \preceq t \preceq s$ implies $\|t\| \leq K\|s\|$ and the least positive number K satisfying this properties is called normal constant of P [5]. Rezapour and Hamlbarani [7] showed that there are no cones with normal constant $K < 1$ and there exist cones of normal constant 1, and cones of normal constant $M > K$ for each $K > 1$.

Definition 1.3 ([5])**.** A cone metric space is an ordered (X, d), where X is any set and $d : X \times X \longrightarrow E$ is a mapping satisfying:
CM1) $\theta \preceq d\,(x, y)$ for all $x, y \in X$,
CM2) $d\,(x, y) = \theta$ if and only if $x = y$,
CM3) $d\,(x, y) = d\,(y, x)$ for all $x, y \in X$,
CM4) $d\,(x, z) \preceq d\,(x, y) + d\,(y, z)$ for all $x, y, z \in X$.

Definition 1.4 ([5])**.** Let (X, d) be a cone metric space, $x \in X$ and (x_n) be a sequence in X. Then
i) (x_n) is said to converge to x if for any $c \in E$ with $c \gg 0$ there exists a natural number n_0 such that $d\,(x_n, x) \ll c$ for all $n \geq n_0$. We denote this by $\lim_{n \longrightarrow \infty} x_n = x$ or $x_n \longrightarrow x$ as $n \to \infty$.
ii) (x_n) is said to be a Cauchy sequence if for any $c \in E$ with $c \gg \theta$ there exists a natural number n_0 such that $d\,(x_n, x_m) \ll c$ for all $n, m \geq n_0$.
iii) (X, d) is said to be a complete cone metric space if every Cauchy sequence is convergent.

In [3] Turkoglu, for $c \in E$ with $c \gg \theta$ and $x \in X$, define $B\,(x, c) = \{y \in X : d\,(x, y) \ll c\}$ and $\beta = \{B\,(x, c) : x \in X,\ c \in E \text{ with } c \gg \theta\}$, then show that

$$\tau_c = \{U \subset X : \forall x \in U, \exists B\,(x, c) \in \beta, x \in B\,(x, c) \subset U\}$$

is a topology on X.

Definition 1.5 ([2])**.** A binary operation $* : [0, 1] \times [0, 1] \longrightarrow [0, 1]$ is a continuous t-norm if $*$ satisfies the following conditions;
1) $*$ is associative and commutative,
2) $*$ is continuous,
3) $a * 1 = a$ for all $a \in [0, 1]$,
4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $a, b, c, d \in [0, 1]$.

Remark 1.6 ([1]). For any $r_1 > r_2$, we can find a r_3 such that $r_1 * r_3 \geq r_2$ and for any r_4 we can find a r_5 such that $r_5 * r_5 \geq r_4$. (r_1,r_2,r_3,r_4,$r_5 \in (0, 1)$)

Example 1.7. $a * b = ab$

Example 1.8. $a * b = \min\{a, b\}$

Definition 1.9 ([1])**.** A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions;
FM1) $M(x, y, t) > 0$,
FM2) $M(x, y, t) = 1$ if and only if $x = y$,
FM3) $M(x, y, t) = M(y, x, t)$,
FM4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
FM5) $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous,
$x, y, z \in X$ and $t, s > 0$.

Definition 1.10 ([1])**.** Let $(X, M, *)$ be a fuzzy metric space, $x \in X$ and (x_n) be a sequence in X. Then
i) (x_n) is said to converge to x if for any $t > 0$ and any $r \in (0, 1)$ there exists a natural number n_0 such that $M(x_n, x, t) > 1 - r$ for all $n \geq n_0$. We denote this by $\lim_{n \longrightarrow \infty} x_n = x$ or $x_n \longrightarrow x$ as $n \to \infty$.
ii) (x_n) is said to be a Cauchy sequence if for any $r \in (0, 1)$ and any $t > 0$ there exists a natural number n_0 such that $M(x_n, x_m, t) > 1 - r$ for all $n, m \geq n_0$.
iii) $(X, M, *)$ is said to be a complete metric space if every Cauchy sequence is convergent.

Remark 1.11 ([1]). Let $(X, M, *)$ be a fuzzy metric space. $\tau = \{A \subset X : x \in A$, if and only if there exists $t > 0$ and r, $r \in (0, 1)$ such that $B(x, r, t) \subset A\}$ is a topology on X.

2. Fuzzy cone metric spaces

In this section, we define the fuzzy cone metric space and the topology induced by this space. Then we give some properties.

Definition 2.1. A 3-tuple $(X, M, *)$ is said to be a fuzzy cone metric space if P is a cone of E, X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times int(P)$ satisfying the following conditions;
For all $x, y, z \in X$ and $t, s \in int(P)$ (that is $t \gg \theta$, $s \gg \theta$)
FCM1) $M(x, y, t) > 0$,
FCM2) $M(x, y, t) = 1$ if and only if $x = y$,
FCM3) $M(x, y, t) = M(y, x, t)$,
FCM4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
FCM5) $M(x, y, .) : int(P) \longrightarrow [0, 1]$ is continuous.

If we take $E = \mathbb{R}$, $P = [0, \infty)$ and $a * b = ab$, then every fuzzy metric spaces became a fuzzy cone metric spaces.

Example 2.2. Let $E = \mathbb{R}^2$. Then $P = \{(k_1, k_2) : k_1, k_2 \geq 0\} \subset E$ is a normal cone with normal constant $K = 1$ [5]. Let $X = \mathbb{R}$, $a * b = ab$ and $M : X^2 \times int(P) \longrightarrow [0, 1]$ defined by $M(x, y, t) = \frac{1}{e^{\frac{|x-y|}{\|t\|}}}$ for all $x, y \in X$ and $t \gg \theta$.
FCM1-2-3) are obvious. FCM4) We know that P is a normal cone with normal constant $K = 1$. Hence, $s \preceq t + s$ and $t \preceq t + s$ imply $\|s\| \leq \|t + s\|$ and $\|t\| \leq \|t + s\|$. Since $\frac{\|t+s\|}{\|s\|} \geq 1$ and $\frac{\|t+s\|}{\|t\|} \geq 1$, we can write

$$|x - z| \leq \frac{\|t + s\|}{\|t\|}|x - y| + \frac{\|t + s\|}{\|s\|}|y - z|,$$

i.e.

$$\frac{|x - z|}{\|t + s\|} \leq \frac{|x - y|}{\|t\|} + \frac{|y - z|}{\|s\|}.$$

Therefore

$$e^{\frac{|x-z|}{\|t+s\|}} \leq e^{\frac{|x-y|}{\|t\|}} e^{\frac{|y-z|}{\|s\|}}.$$

Thus $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$.
FCM5) If we define $n : int(P) \longrightarrow (0,\infty)$, $n(t) = \|t\| = \sqrt{k_1^2 + k_2^2}$ and $f : (0,\infty) \longrightarrow [0,1]$, $f(u) = e^{\frac{|x-y|}{u}}$, $M(x,y,.) : int(P) \longrightarrow [0,1]$ can be taught as composition of n and f. Since n and f are continuous, M is also continuous.
Hence $(X, M, *)$ is a fuzzy cone metric spaces.

Example 2.3. Let P be an any cone, $X = \mathbb{N}$, $a * b = ab$, $M : X^2 \times int(P) \longrightarrow [0,1]$ defined by $M(x,y,t) = \begin{cases} x/y & \text{if } x \leq y \\ y/x & \text{if } y \leq x \end{cases}$ for all $x, y \in X$ and $t \gg \theta$. Then $(X, M, *)$ is a fuzzy cone metric spaces.

Lemma 2.4. *$M(x,y,.) : int(P) \longrightarrow [0,1]$ is nondecreasing for all $x, y \in X$.*

Proof. Assume that $M(x,y,t) > M(x,y,s)$, for $s \gg t \gg 0$. Note that since $s \gg t$, $s - t$ in $int(P)$. By FCM4) and assumption, we have $M(x,y,t) * M(y,y,s-t) \leq M(x,y,s) < M(x,y,t)$. Since $M(y,y,s-t) = 1$ by FCM2), we have $M(x,y,t) < M(x,y,t)$ that is a contradiction. □

Definition 2.5. Let $(X, M, *)$ be a fuzzy cone metric space. For $t \gg \theta$, the open ball $B(x,r,t)$ with center x and radius $r \in (0,1)$ is defined by $B(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\}$.

Theorem 2.6. *Let $(X, M, *)$ be a fuzzy cone metric space. Define*

$$\tau_{fc} = \{A \subset X : x \in A \textit{ if and only if there exist } r \in (0,1), \textit{ and } t \gg \theta \textit{ such that } B(x,r,t) \subset A\},$$

then τ_{fc} is a topology on X.

Proof. 1. If $x \in \varnothing$, then $\varnothing = B(x,r,t) \subset \varnothing$. Hence $\varnothing \in \tau_{fc}$. Since for any $x \in X$, any $r \in (0,1)$ and any $t \gg \theta$, $B(x,r,t) \subset X$, then $X \in \tau_{fc}$.

2. Let $A, B \in \tau_{fc}$ and $x \in A \cap B$. Then $x \in A$ and $x \in B$, so there exist $t_1 \gg \theta, t_2 \gg \theta$ and r_1, $r_2 \in (0,1)$ such that $B(x,r_1,t_1) \subset A$ and $B(x,r_2,t_2) \subset B$. From Proposition 1.2 3), for $t_1 \gg \theta$, $t_2 \gg \theta$, there exists $t \gg \theta$ such that $t \ll t_1, t \ll t_2$ and take $r = \min\{r_1, r_2\}$. Then $B(x,r,t) \subset B(x,r_1,t_1) \cap B(x,r_2,t_2) \subset A \cap B$. Thus $A \cap B \in \tau_{fc}$.

3. Let $A_i \in \tau_{fc}$ for each $i \in I$ and $x \in \bigcup_{i \in I} A_i$. Then there exists $i_0 \in I$ such that $x \in A_{i_0}$. So, there exist $t \gg \theta$ and $r \in (0,1)$ such that $B(x,t,r) \subset A_{i_0}$. Since $A_{i_0} \subset \bigcup_{i \in I} A_i$, $B(x,r,t) \subset \bigcup_{i \in I} A_i$. Thus $\bigcup_{i \in I} A_i \in \tau_{fc}$. Hence, τ_{fc} is a topology on X. □

Theorem 2.7. *Let $(X, M, *)$ be a fuzzy cone metric space. Then (X, τ_{fc}) is Hausdorff.*

Proof. Let x, $y \in X$ such that $x \neq y$. From the definition of fuzzy metric, $1 > M(x,y,t) > 0$ say $M(x,y,t) = r$. From Remark 1.6, for all r_0 such that $1 > r_0 > r$ there exists $r_1 \in (0,1)$ such that $r_1 * r_1 > r_0$.

Now consider, the sets $B\left(x, 1 - r_1, \frac{t}{2}\right)$ and $B\left(y, 1 - r_1, \frac{t}{2}\right)$. We have to see

$$B\left(x, 1 - r_1, \frac{t}{2}\right) \cap B\left(y, 1 - r_1, \frac{t}{2}\right) = \varnothing.$$

Suppose that, $B\left(x, 1 - r_1, \frac{t}{2}\right) \cap B\left(y, 1 - r_1, \frac{t}{2}\right) \neq \varnothing$. Then there exists $z \in B\left(x, 1 - r_1, \frac{t}{2}\right) \cap B\left(y, 1 - r_1, \frac{t}{2}\right)$. Therefore, $M\left(x, z, \frac{t}{2}\right) > 1 - (1 - r_1) = r_1$ and $M\left(y, z, \frac{t}{2}\right) > 1 - (1 - r_1) = r_1$. From FCM4), $r = M(x,y,t) \geq M\left(x, z, \frac{t}{2}\right) * M\left(y, z, \frac{t}{2}\right)$. Then $r > r_1 * r_1$ so, $r > r_0 > r$. This is a contradiction. Hence $B\left(x, 1 - r_1, \frac{t}{2}\right) \cap B\left(y, 1 - r_1, \frac{t}{2}\right) = \varnothing$. □

Theorem 2.8. *Let $(X, M, *)$ be a fuzzy cone metric space. Then (X, τ_{fc}) is first countable.*

Proof. Let $t \gg \theta$, $x \in X$. We will show that $\beta_x = \left\{B\left(x, \frac{1}{n}, \frac{t}{n}\right) : n \in \mathbb{N}\right\}$ is a local basis for $x \in X$. Let $U \in \tau_{fc}$ and $x \in U$. Since U is open, then there exists $r \in (0,1)$ and $t \gg \theta$ such that $B(x,r,t) \subset U$. Choose $n \in \mathbb{N}$ such that $\frac{1}{n} < r$ and $\frac{t}{n} \ll t$. Now we just need to show $B\left(x, \frac{1}{n}, \frac{t}{n}\right) \subset B(x,r,t)$. Let $z \in B\left(x, \frac{1}{n}, \frac{t}{n}\right)$. Then $M\left(x, z, \frac{t}{n}\right) > 1 - \frac{1}{n} > 1 - r$. Since $\frac{t}{n} \preceq t$, by Lemma 2.4 we have $1 - r < M\left(x, z, \frac{t}{n}\right) \leq M(x,z,t)$. Hence $z \in B(x,r,t)$ which implies $B\left(x, \frac{1}{n}, \frac{t}{n}\right) \subset B(x,r,t) \subset U$. Consequently, β_x is countable local basis for x. Hence (X, τ_{fc}) is first countable topological space. □

Definition 2.9. Let $(X, M, *)$ be a fuzzy cone metric space, $x \in X$ and (x_n) be a sequence in X. Then (x_n) is said to converge to x if for any $t \gg \theta$ and any $r \in (0,1)$ there exists a natural number n_0 such that $M(x_n, x, t) > 1 - r$ for all $n \geq n_0$. We denote this by $\lim_{n \longrightarrow \infty} x_n = x$ or $x_n \longrightarrow x$ as $n \to \infty$.

Theorem 2.10. *Let $(X, M, *)$ be a fuzzy cone metric space, $x \in X$ and (x_n) be a sequence in X. (x_n) converges to x if and only if $M(x_n, x, t) \longrightarrow 1$ as $n \longrightarrow \infty$, for each $t \gg \theta$.*

Proof. ($\Rightarrow$:) Suppose that, $x_n \to x$. Then, for each $t \gg \theta$ and $r \in (0,1)$, there exists a natural number n_0 such that $M(x_n, x, t) > 1 - r$ for all $n \geq n_0$. We have $1 - M(x_n, x, t) < r$. Hence $M(x_n, x, t) \to 1$ as $n \to \infty$.

($\Leftarrow$:) Now, suppose that $M(x_n, x, t) \to 1$ as $n \to \infty$. Then, for each $t \gg \theta$ and $r \in (0,1)$, there exists a natural number n_0 such that $1 - M(x_n, x, t) < r$ for all $n \geq n_0$. In that case, $M(x_n, x, t) > 1 - r$. Hence $x_n \to x$ as $n \to \infty$. □

Definition 2.11. Let $(X, M, *)$ be a fuzzy cone metric space and (x_n) be a sequence in X. Then (x_n) is said to be a Cauchy sequence if for any $0 < \varepsilon < 1$ and any $t \gg \theta$ there exists a natural number n_0 such that $M(x_n, x_m, t) > 1 - \varepsilon$ for all $n, m \geq n_0$.

Definition 2.12. A fuzzy cone metric space is called complete if every Cauchy sequence is convergent.

Definition 2.13. Let $(X, M, *)$ be a fuzzy cone metric space. A subset A of X is said to be FC-bounded if there exists $t \gg \theta$ and $r \in (0,1)$ such that $M(x, y, t) > 1 - r$ for all $x, y \in A$.

Theorem 2.14. *In a fuzzy cone metric space, every compact set is closed and FC-bounded.*

Proof. Let A be a compact subset of X, $t \gg \theta$ and $r \in (0,1)$. Since $\{B(x, r, t) : x \in A\}$ is a open cover of A, there exist $x_1, x_2, \ldots, x_n \in A$ such that $A \subset \cup_{i=1}^{n} B(x_i, r, t)$. For any $x, y \in A$ there exist $1 \leq i, j \leq n$ such that $x \in B(x_i, r, t)$ and $y \in B(x_j, r, t)$. Hence we can write $M(x, x_i, t) > 1 - r$ and $M(x, x_j, t) > 1 - r$. Let $\alpha = \min\{M(x_i, x_j, t) : 1 \leq i, j \leq n\}$. Then we have

$$\begin{aligned} M(x, y, 3t) &\geq M(x, x_i, t) * M(x_i, x_j, t) * M(x_j, y, t) \\ &\geq (1 - r) * \alpha * (1 - r). \end{aligned}$$

Let $t' = 3t$, and choose $0 < s < 1$ such that $(1 - r) * \alpha * (1 - r) > 1 - s$. Hence for any $x, y \in A$, we have $M(x, y, t') > 1 - s$, and A is FC- bounded. On the other hand, since a fuzzy cone metric space is Hausdorff and every compact subset of a Hausdorff space is closed, A is closed. □

3. Fuzzy cone Banach contraction theorem

In [8], Gregori and Sapena gave the fuzzy Banach contraction theorem. Now we extent it for the complete fuzzy cone metric space.

Definition 3.1. Let $(X, M, *)$ be a fuzzy cone metric space and $f : X \longrightarrow X$ is a self mapping. Then f is said fuzzy cone contractive if there exists $k \in (0,1)$ such that

$$\frac{1}{M(f(x), f(y), t)} - 1 \leq k\left(\frac{1}{M(x, y, t)} - 1\right)$$

for each $x, y \in X$ and $t \gg \theta$. k is called the contractive constant of f.

Definition 3.2. Let $(X, M, *)$ be a fuzzy cone metric space and (x_n) be a sequence in X. Then (x_n) is said fuzzy cone contractive if there exists $k \in (0,1)$ such that

$$\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1 \leq k\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right)$$

for all $t \gg \theta$, $n \in \mathbb{N}$.

Theorem 3.3 (Fuzzy Cone Banach contraction theorem)**.** *Let $(X, M, *)$ be a complete fuzzy cone metric space in which fuzzy cone contractive sequences are Cauchy. Let $T : X \longrightarrow X$ be a fuzzy cone contractive mapping being k the contractive constant. Then T has a unique fixed point.*

Proof. Fix $x \in X$ and let $x_n = T^n(x)$, $n \in \mathbb{N}$. For $t \gg \theta$, we have

$$\frac{1}{M(T(x), T^2(x), t)} - 1 \leq k\left(\frac{1}{M(x, x_1, t)} - 1\right)$$

and by induction

$$\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1 \leq k\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right),$$

$n \in \mathbb{N}$. Then (x_n) is a fuzzy contractive sequence, by assumptions it is a Cauchy sequence and (x_n) converges to y, for some $y \in X$. By Theorem 2.10, we have

$$\frac{1}{M(T(y), T(x_n), t)} - 1 \leq k\left(\frac{1}{M(y, x_n, t)} - 1\right) \longrightarrow 0$$

as $n \longrightarrow \infty$. Then for each $t \gg \theta$, $\lim_{n\to\infty} M((T(y), T(x_n), t)) = 1$ and hence $\lim_{n\to\infty} T(x_n) = T(y)$, i.e., $\lim_{n\to\infty} x_{n+1} = T(y)$ and $T(y) = y$. Now we show uniqueness. Assume $T(z) = z$ for some $z \in Z$. For $t \gg \theta$, we have

$$\begin{aligned}
\frac{1}{M(y, z, t)} - 1 &= \frac{1}{M(T(y), T(z), t)} - 1 \\
&\leq k\left(\frac{1}{M(y, z, t)} - 1\right) \\
&= k\left(\frac{1}{M(T(y), T(z), t)} - 1\right) \\
&\leq k^2\left(\frac{1}{M(y, z, t)} - 1\right) \\
&\leq \cdots \leq k^n\left(\frac{1}{M(y, z, t)} - 1\right) \to 0
\end{aligned}$$

as $n \longrightarrow \infty$. Hence $M(y, z, t) = 1$ and $y = z$. □

4. Conclusion

We defined the notion of fuzzy cone metric space which is a generalization of fuzzy metric spaces and then the topology induced by this space. By using these definitions we gave some topological properties, such as Hausdorfness, first countability. The cone version of fuzzy Banach contraction theorem is also stated here. So one can study, by using these results, on the other fix point theorems, similar topological properties of this space and problems related to convergence of a sequence.

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