Available online at www.tjnsa.com
J. Nonlinear Sci. Appl. 5 (2012), 252–258
Research Article

Journal of Nonlinear Science and Applications
Print: ISSN 2008-1898 Online: ISSN 2008-1901

# On Banach contraction principle in a cone metric space

Shobha Jain[a], Shishir Jain[b,*], Lal Bahadur Jain[c]

[a]*Quantum School of Technology, Roorkee (U.K), India.*
[b]*Shri Vaishnav Institute of Technology and Science, Indore (M.P.), India.*
[c]*Retd. Principal,Govt. Arts and Commerce College Indore (M.P.), India.*

`This paper is dedicated to Professor Ljubomir Ćirić`

`Communicated by Professor V. Berinde`

**Abstract**

The object of this paper is to establish a generalized form of Banach contraction principle for a cone metric space which is not necessarily normal. This happens to be a generalization of all different forms of Banach contraction Principle, which have been arrived at in L. G. Huang and X. Zhang [L. G. Huang and X. Zhang, J. Math. Anal. Appl 332 (2007), 1468–1476] and Sh. Rezapour, R. Hamlbarani [Sh. Rezapour, R. Hamlbarani, J. Math. Anal. Appl. 345 (2008) 719-724] and D. Ilic, V. Rakocevic [D. Ilic, V. Rakocevic, Applied Mathematics Letters **22** (2009), 728–731]. It also results that the theorem on quasi contraction of C̃irič [L. J. B. C̃irič, Proc. American Mathematical Society 45 (1974), 999–1006]. for a complete metric space also holds good in a complete cone metric space. All the results presented in this paper are new. 

*Keywords:* Cone metric space, common fixed points.

## 1. Introduction

There has been a number of generalizations of metric space. One such generalization is a cone metric space. In the second half of previous century a lot of work has been done in a K-metric space, which is in the setting of cone in a real normed linear space and variously defined notions of convergence and a Cauchy

*Corresponding author
*Email addresses:* shobajain1@yahoo.com ( Shobha Jain), jainshishir11@rediffmail.com (Shishir Jain), lalbahadurjain11@yahoo.com ( Lal Bahadur Jain)

*Received 2011-4-14*

sequence [13]. However, another school in U.S.S.R [7, 8, 9, 10] worked in K- metric space in the setting of a Banach space $B$ and a closed cone in it in the name of a generalized metric space or a SKS metric space. Recently, in [3] Huang and Zhang defined cone metric space in the same setting of a real Banach space E ordered with a closed cone P in it with $intP \neq \Phi$ defining convergence and a Cauchy sequence with respect to interior points of $P$. In this space they replaced the set of real numbers of a metric space by an ordered Banach Space and gave some fundamental results for a self map satisfying a contractive condition assuming the normality of cone metric space.

Recently, Rezapour and Hamlbarani [11] omitted the assumption of normality in cone metric space, which is a milestone in developing fixed point theory in cone metric space.In [5], the authors introduced the concept of a compatible pair of self maps in a cone metric space and established a basic result for a non-normal cone metric space with an example, while in [6] weakly compatible maps have been studied. In this paper we are proving a common fixed point theorem for a sequence of self maps satisfying a generalized contractive condition for a non-normal cone metric space. It results in a generalized form of Banach contraction principle in this space.

## 2. Preliminaries

**Definition 2.1.** [3] Let $E$ be a real Banach space and $P$ be a subset of $E$. $P$ is called a cone if
$(i)$ $P$ is a closed, nonempty and $P \neq \{0\}$;
$(ii)$ $a, b \in R, a, b \geq 0, x, y \in P$ imply $ax + by \in P$;
$(iii)$ $x \in Pand - x \in P$ imply $x = 0$.
Given a cone $P \subseteq E$, we define a partial ordering " $\leq$ " in $E$ by $x \leq y$ if $y - x \in P$. We write $x < y$ to denote $x \leq y$ but $x \neq y$ and $x << y$ to denote $y - x \in P^0$, where $P^0$ stands for the interior of $P$ .
$P$ is called normal if for some $M > 0$ for $x, y \in E, 0 \leq x \leq y$ implies
$\|x\| \leq \ M|y\|$.

**Proposition 2.2.** *Let $P$ be a cone in a real Banach space $E$ . If for $a \in P$ and $a \leq ka$, for some $k \in [0, 1)$ then $a = 0$.*

**Proof:** For $a \in P, k \in [0, 1)$ and $a \leq ka$ gives $(k - 1)a \in P$ implies $-(1 - k)a \in P$. Therefore by $(ii)$ we have $-a \in P$, as $1/(1 - k) > 0$. Hence $a = 0$, by $(iii)$.

**Proposition 2.3.** *[4] Let $P$ be a cone is a real Banach space $E$ with non-empty interior If for $a \in E$ and $a << c$, for all $c \in P^0$, then $a = 0$.*

*Remark* 2.4. [11] $\lambda P^0 \subseteq P^0$, for $\lambda > 0$ and $P^0 + P^0 \subseteq P^0$.

**Definition 2.5.** [3] Let $X$ be a nonempty set and $P$ be a cone in a real Banach space $E$. Suppose the mapping $d : X \times X \to E$ satisfies:
(a) $0 \leq d(x, y)$, for all $x, y \in X$ and $d(x, y) = 0$, if and only if $x = y$;
(b) $d(x, y) = d(y, x)$, for all $x, y \in X$;
(c) $d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.
Then $d$ is called a cone metric on $X$, and $(X, d)$ is called a cone metric space. If $P$ is normal , then $(X, d)$ is said to be a a normal cone metric space.

**Example 2.6.** [3] Let $E = R^2, P = \{(x, y) \in E : x \geq 0, y \geq 0\}$ and $X = R$ . For $x, y \in R$ define $d(x, y) = |x - y|(1, \alpha)$ where $\alpha \geq 0$ is some fixed constant. Then $(X, d)$ is a cone metric space.

**Example 2.7.** Let $E = C^2_R[0, 1]$ with the norm $\|f\| = \|f\|_\infty + \|f'\|_\infty$. Consider the cone $P = \{f \in E : f \geq 0\}$. Then $P$ is not a normal cone as shown in [11]. Taking $X = \{1, 1/2, 1/3 \ldots\}$ we define $d : X \times X \to P$ by $d(\frac{1}{m}, \frac{1}{n}) = f_{mn}$, where $f_{mn}(t) = |\frac{1}{m} - \frac{1}{n}|t$ , for all $t \in [0, 1]$ . Then $(X, d)$ is a non-normal cone metric space.$(X, d)$ is not a metric space as it is not normal.

**Definition 2.8.** [3] Let $(X, d)$ be a cone metric space with respect to a cone in a real Banach space $E$ with non-empty interior. Let $\{x_n\}$ be a sequence in $X$ and $x \in X$. If for every $c \in E$ with $0 << c$ there is a positive integer $N_c$ such that for all $n > N_c, d(x_n, x) << c$, then the sequence $\{x_n\}$ is said to converges to $x$, and $x$ is called limit of $\{x_n\}$ . We write $lim_{n\to\infty}x_n = x$ or $x_n \to x$, as $n \to \infty$.

**Definition 2.9.** [3] Let $(X, d)$ be a cone metric space with respect to a cone with nonempty interior in a real Banach space $E$. Let $\{x_n\}$ be a sequence in $X$. If for any $c \in E$ with $0 << c$ there is a positive integer $N_c$ such that for all $n, m > N_c, d(x_n, x_m) << c$, then the sequence $\{x_n\}$ is said to be a Cauchy sequence in $X$.

In the following $(X, d)$ will stand for a cone metric space with respect to a cone $P$ with $P^0 \neq \phi$ in a real Banach space $E$ and $\leq$ is partial ordering in $E$ with respect to $P$

*Remark* 2.10. It follows from above definitions that if $\{x_{2n}\}$ is a subsequence of a Cauchy sequence $\{x_n\}$ in a cone metric space $(X, d)$ and $x_{2n} \to z$ then $x_n \to z$.

**Definition 2.11.** [3] Let $(X, d)$ be a cone metric space. If every Cauchy sequence in $X$ is convergent in $X$, then $X$ is called a complete cone metric space.

**Proposition 2.12.** *Let $(X, d)$ be a cone metric space and $P$ be a cone in a real Banach space $E$. If $u \leq v, v << w$ then $u << w$.*

**Lemma 2.13.** *Let $(X, d)$ be a cone metric space and $P$ be a cone in a real Banach space $E$ and $k_1, k_2, k > 0$ are some fixed real numbers. If $x_n \to x, y_n \to y$ in $X$ and for some $a \in P$*
*(1.1) $ka \leq k_1 d(x_n, x) + k_2 d(y_n, y)$, for all $n > N$, for some integer $N$,*
*then $a = 0$.*

**Proof** As $x_n \to x$, and $y_n \to y$ for $c \in P^0$ there exists a positive integer $N_c$ such that
$\frac{c}{(k_1+k_2)} - d(x_n, x), \frac{c}{(k_1+k_2)} - d(y_n, y) \in P^0$, for all $n > N_c$.
Therefore by Remark 2.4, we have
$\frac{k_1 c}{(k_1+k_2)} - k_1 d(x_n, x), \frac{k_2 c}{(k_1+k_2)} - k_2 d(y_n, y) \in P^0$, for all $n > N_c$.
Again by adding and Remark 2.4, we have
$c - k_1 d(x_n, x) - k_2 d(y_n, y) \in P^0$ for all $n > \max\{N, N_c\}$.
From (1.1) and Proposition 2.12 we have $ka << c$, for each $c \in P^0$. By Proposition 2.3 , we have $a = 0$, as $k > 0$.

## 3. MAIN RESULTS

**Theorem 3.1.** *Let $(X, d)$ be a complete cone metric space with respect to a cone $P$ contained in a real Banach space $E$. Let $\{T_n\}$ be a sequence of self maps on $X$ satisfying:*
(3.1.1) *For some $\lambda, \mu, \delta, \alpha, \beta \in [0, 1)$ with $\lambda + \mu + \delta + 2\alpha < 1$, or else $\lambda + \mu + \delta + 2\beta < 1$, for all $x, y \in X$*
$d(T_i x, T_j y) \leq \lambda d(T_i x, x) + \mu d(T_j y, y) + \delta d(x, y) + \alpha d(x, T_j y) + \beta d(T_i x, y)$.
*For $x_0 \in X$, let $x_n = T_n x_{n-1}$, for all $n$. Then the sequence $\{x_n\}$ converges in $X$ and its limit $u$ is a common fixed point of all the maps of the sequence $\{T_n\}$. This fixed point is unique if $\delta + \alpha + \beta < 1$.*

*Proof.* We show that $\{x_n\}$ is a Cauchy sequence in $X$.
**Step I:** Taking $x = x_{n-1}, y = x_n$ and $i = n, j = n + 1$ in (3.1.1) we get,

$$\begin{aligned} d(T_n x_{n-1}, T_{n+1} x_n) &\leq \lambda d(T_n x_{n-1}, x_{n-1}) + \mu d(T_{n+1} x_n, x_n) + \delta(x_{n-1}, x_n) + \\ &\quad \alpha d(x_{n-1}, T_{n+1} x_n) + \beta d(T_n x_{n-1}, x_n). \end{aligned}$$

As $x_n = T_n x_{n-1}$, we have

$$\begin{aligned} d(x_n, x_{n+1}) &\leq \lambda d(x_n, x_{n-1}) + \mu d(x_{n+1}, x_n) + \delta(x_{n-1}, x_n) + \alpha d(x_{n-1}, x_{n+1}) + \beta d(x_n, x_n), \\ &\leq \lambda d(x_n, x_{n-1}) + \mu d(x_{n+1}, x_n) + \delta(x_{n-1}, x_n) + \alpha[d(x_{n-1}, x_n) + d(x_n, x_{n+1})]. \end{aligned}$$

Writing $d(x_n, x_{n+1}) = d_n$, we have
$d_n \leq \lambda d_{n-1} + \mu d_n + \delta d_{n-1} + \alpha[d_n + d_{n-1}]$,
i.e.
$(1 - \mu - \alpha)d_n = (\lambda + \delta + \alpha)d_{n-1}$,
which implies

$$d_n \leq h d_{n-1}, \tag{3.1}$$

if $h = \frac{(\lambda+\delta+\alpha)}{1-\mu-\alpha}$.
As $\lambda + \mu + \delta + 2\alpha < 1$ we obtain that $h < 1$.
Now
$d_n \leq h d_{n-1} \leq h^2 d_{n-2} \leq h^3 d_{n-3} \leq \ldots \leq h^n d_0$, where $d_0 = d(x_0, x_1)$.
Also
$d(x_{n+p}, x_n) \leq d(x_{n+p}, x_{n+p-1}) + d(x_{n+p-1}, x_{n+p-2}) + \ldots + d(x_{n+1}, x_n)$,
i. e.

$$\begin{aligned} d(x_{n+p}, x_n) &\leq d_{n+p-1} + d_{n+p-2} + \cdots + d_n. \\ &= d_n + d_{n+1} + \ldots d_{n+p-1} \\ &= h^n[1 + h + h^2 + h^3 + \ldots + h^{p-1}]d_0, \\ &\leq h^n d_0/(1-h), \end{aligned}$$

as $h < 1$ and $P$ is closed. Thus we obtain that

$$d(x_{n+p}, x_n) \leq h^n d_0/(1-h). \tag{3.2}$$

Now for $c \in P^0$, there exists $r > 0$ such that $c - y \in P^0$, if $||y|| < r$. Choose a positive integer $N_c$ such that for all $n \geq N_c$, $||h^n d_0/(1-h)|| < r$, which implies $c - h^n d_0/(1-h) \in P^0$ and $h^n d_0/(1-h) - d(x_{n+p}, x_n) \in P$, using (3.2).
So we have $c - d(x_{n+p}, x_n) \in P^0$, for all $n > N_c$ and for all $p$, by Proposition 2.12 . This implies $d(x_{n+p}, x_n) << c$, for all $n > N_c$, for all $p$. Hence $\{x_n\}$ is a Cauchy sequence in $X$, which is complete. Let $x_n \to u$.
**Step II:** For an arbitrary fixed m we show that $T_m u = u$.
Now,

$$\begin{aligned} d(T_m u, u) &\leq d(T_m u, T_n x_{n-1}) + d(T_n x_{n-1}, u), \\ &= d(x_n, u) + d(T_m u, T_n x_{n-1}). \end{aligned}$$

Using (3.1.1) with $x = x_{n-1}, y = u, i = n$ and $j = m$ we have

$$\begin{aligned} d(T_m u, u) \leq\ & d(x_n, u) + \lambda d(T_n x_{n-1}, x_{n-1}) + \mu d(T_m u, u) \\ & + \delta d(u, x_{n-1}) + \alpha d(T_m u, x_{n-1}) + \beta d(u, T_n x_{n-1}) \\ & = d(x_n, u) + \mu d(T_m u, u) + \lambda d(x_n, x_{n-1}) \\ & + \delta d(u, x_{n-1}) + \alpha d(T_m u, x_{n-1}) + \beta d(u, x_n), \\ & \leq d(x_n, u) + \mu d(T_m u, u) + \lambda[d(x_n, u) + d(u, x_{n-1})] \\ & + \delta d(u, x_{n-1}) + \alpha[d(T_m u, u) + d(u, x_{n-1})] + \beta[d(u, x_n). \end{aligned}$$

So

$$[1 - \mu - \alpha]d(T_m u, u) \leq\ [\mu + \delta + \alpha]d(x_{n-1}, u) + [1 + \lambda + \beta]d(u, x_n).$$

As $\{x_n\} \to u, \{x_{n-1}\} \to u$, and $1 - \mu - \alpha > 0$, using Lemma 2.13, we have $d(T_m u, u) = 0$, and we get $T_m u = u$. Thus $u$ is a common fixed point of all the maps of the sequence $\{T_n\}$.
**Step III (Uniqueness)**: Let $T_n z = z$, for all $n$, be another common fixed point of all the maps of the sequence $\{T_n\}$ . Now
$d(z, u) = d(T_n z, T_n u)$.
Taking $x = z$ and $y = u$ with $i = j = n$ in (3.1.1) we get
$d(z, u) \leq \lambda d(T_n z, z) + \mu d(T_n u, u) + \delta d(z, u) + \alpha d(z, T_n u) + \beta d(T_n z, u)$,
which gives
$d(z, u) \leq (\delta + \alpha + \beta)d(z, u)$.

As $\delta + \alpha + \beta < 1$, using Proposition 2.2, we have $d(z, u) = 0$ i. e. $u = z$. Thus u is the unique common fixed point of all the maps of the sequence $\{T_n\}$. To see the sufficiency of the alternate condition $\lambda + \mu + \delta + 2\beta < 1$, in step I we choose $x = u, y = x_{n-1}$ with $i = n + 1$ and $j = n$ in (3.1.1) to obtain $(1 - \lambda - \beta)d_n \leq (\mu + \delta + \beta)d_{n-1}$. Thus $d_n \leq h' d_{n-1}$, where $h' = \frac{(\mu+\delta+\beta)}{1-\lambda-\beta} < 1$.
Again in step II we choose $x = u, y = x_{n-1} i = m, j = n$ in (3.1.1) receiving $(1 - \lambda - \beta)d(T_m(u), u) \leq \ldots$ and we get $T_m u = u, \forall m$. □

**Theorem 3.2.** *Let $(X, d)$ be a complete cone metric space with respect to a cone $P$ contained in a real Banach space $E$. Let $\{A_n\}$ be a sequence of self maps in $X$ satisfying:*
(3.2.1) *For some $\lambda, \mu, \delta, \alpha, \beta \in [0, 1)$ with $\lambda + \mu + \delta + 2\alpha < 1$, or else $\lambda + \mu + \delta + 2\beta < 1$ and $\delta + \alpha + \beta < 1$, there exists positive integer $m_i$, for each i, such that for all $x, y \in X$*
$d(A_i^{m_i}x, A_j^{m_j}y) \leq \lambda d(A_i^{m_i}x, x) + \mu d(A_j^{m_j}y, y) + \delta(x, y) + \alpha d(x, A_j^{m_j}y) + \beta d(A_i^{m_i}x, y)$. *Then all the maps of the sequence $\{A_n\}$ have a unique common fixed point in $X$.*

*Proof.* In view of (3.2.1) and using Theorem 3.1 all the maps of the sequence $\{A_i^{m_i}\}$ have a unique common fixed point, say z. Hence $A_i^{m_i}z = z$, for all i. Now $A_1^{m_1}z = z$, implies $A_1^{m_1}A_1 z = A_1 z$. Taking $x = A_1 z, y = z, i = 1$ and $j = 2$ in (3.2.1) we have $A_1 z = z$. Continuing in similar way it follows that $A_i z = z$, for all i.Thus z is a common fixed point of all the maps of the sequence $\{A_i\}$. Its uniqueness follows from the fact that $A_i z = z$, implies $A_i^{m_i}z = z$, for all i. □

**Example 3.3.** (of Theorem 3.2) Let $X = [0, 1], E = R^2, P = \{(x, y) \in R^2 : x \geq 0, y \geq 0\} \subseteq R^2$, be a cone in $E$. Fix a real number $\gamma > 0$. We define $d : X \times X \to E$ by $d(x, y) = |x - y|(1, \gamma)$. Then $(X, d)$ is a complete cone metric space. Define $\{A_n\}$ on $X$ as follows:

$$A_n(x) = \begin{cases} 0, & if x \in [0, \frac{1}{n+2}] \\ \frac{1}{n+3}, otherwise. \end{cases}$$

Taking $m_i = 2$, for all i. Then the maps $A_1^2, A_2^2, A_3^2, \ldots$ satisfy the condition (3.2.1) for $\lambda = \mu = \delta = \frac{1}{15}$ and $\alpha = \beta = \frac{1}{10}$. Hence by Theorem 3.2, all the maps of the sequence $\{A_n\}$ have a unique common fixed point $(u = 0)$ in X.

Taking $T_1 = T_2 = T_3 = \cdots = T_{n-1} = T_n = \cdots = A$ in Theorem 3.1, we get the following general form of Banach contraction principal in a cone metric space which is not necessarily normal

**Theorem 3.4.** *Let $(X, d)$ be a complete cone metric space with respect to a cone $P$ contained in a real Banach space $E$ and $A$ be a self map in $X$ satisfying:*
(3.4.1) *For some $\lambda, \mu, \delta, \alpha, \beta \in [0, 1)$ with $\lambda + \mu + \delta + 2\alpha < 1$, or else $\lambda + \mu + \delta + 2\beta < 1$ , for all $x, y \in X$*
$d(Ax, Ay) \leq \lambda d(Ax, x) + \mu d(Ay, y) + \delta(x, y) + \alpha d(x, Ay) + \beta d(Ax, y)$.
*Then for each $x$ in $X$ the sequence $\{A^n x\}$ converges in $X$ and its limit $u$ is a fixed point of $A$.This fixed point is unique if $\delta + \alpha + \beta < 1$.*

In [3] L. G. Huang , X. Zhang and in [11] Sh. Rezapour, R. Hamlbarani proved following various forms of Banach contraction Principle in a normal Cone metric space and in a cone metric space respectively :
**Theorem 1[3] and Theorem 2.3[11] :** Let $(X, d)$ be a complete cone metric space, Suppose the mapping $T : X \times X \to X$ satisfies the contractive condition
$d(Tx, Ty) \leq kd(x, y)$, for all $x, y \in X$,
where $k \in [0, 1)$ is a constant. Then $T$ has a unique fixed point in $X$. For each $x \in X$, the iterative sequence $\{T^n x\}$ converges to the fixed point.
**Theorem 3[3] and Theorem 2.6 [11]:** Let $(X, d)$ be a complete cone metric space. Suppose the mapping $T : X \times X \to X$ satisfies the contractive condition
$d(Tx, Ty) \leq k[d(Tx, x) + d(Ty, y)]$ for all $x, y \in X$,
where $k \in [0, 1/2)$ is a constant. Then $T$ has a unique fixed point in $X$. And for $x \in X$, the iterative sequence $\{T^n x\}$ converges to the fixed point.

**Theorem 4 [3] and Theorem 2.7 [11]:** Let $(X,d)$ be a complete cone metric space. Suppose the mapping $T : X \times X \to X$ satisfies the contractive condition
$d(Tx,Ty) \leq k[d(Tx,y) + d(Ty,x)]$ for all $x, y \in X$,
where $k \in [0, 1/2)$ is a constant. Then $T$ has a unique fixed point in $X$. For each $x \in X$, the iterative sequence $\{T^n x\}$ converges to the fixed point.
**Theorem 2.8 [11]:** Let $(X,d)$ be a complete cone metric space. Suppose the mapping $T : X \times X \to X$ satisfies the contractive condition
$d(Tx,Ty) \leq kd(x,y) + ld(y,Tx)$ for all $x, y \in X$,
where $k, l \in [0, 1)$ are constants. Then $T$ has a fixed point in $X$. Also the fixed point of $T$ is unique whenever $k + l < 1$.

*Remark* 3.5. Above Theorems of [3] and [11] follow from Theorem 3.4 of this paper by taking :
(a) $\lambda = \mu = \alpha = \beta = 0$ and $\delta = k$,
(b) $\lambda = \mu = k$ and $\delta = \alpha = \beta = 0$,
(c) $\lambda = \mu = \delta = 0$ and $\alpha = \beta = k$, and
(d) $\lambda = \mu = \alpha = 0, \delta = k$, and $\beta = l$
respectively in it.
Precisely,Theorem 3.4 synthesizes and generalizes all the results of [3]and [11] for a non-normal cone metric space. Theorem 3.1 is a general form of Banach contraction principle in a complete cone metric space which is not necessarily normal.

**Definition 3.6.** [4] (Quasi contraction)A self-map f on a cone metric space $(X,d)$ is said to be a quasi contraction if for a fixed $\lambda \in (0,1), d(fx,fy) \leq \lambda u$ for every $x, y \in X$, where

$$u \in \{d(x,y), d(x,fx), d(y,fy), d(x,fy), d(y,fx)\}.$$

**Theorem 2.1 [4]:** Let $(X,d)$ be a complete cone metric space and P be a normal cone. Then a quasi contraction f has a unique fixed point in $X$ and for each $x \in X$ the iterative sequence $\{f^n(x)\}$ converges to the fixed point.

*Remark* 3.7. Keeping one of the constants $\{\alpha, \beta, \gamma, \delta, \mu\}$ non-zero and all others equal to zero in Theorem 3.4 , it follows that the above result of [4] is true even for non-normal complete cone metric space.

*Remark* 3.8. It has been established in L. J. B. C̃irič [2] that a quasi contraction has a unique fixed point in a complete metric space. It follows from the above Remark that the result of [2] is also true for a complete cone metric space even if it is non-normal.

## References


[1] V. Berinde, *Itrative approximation of fixed points*, Springer Verlag, 2007.

[2] L. J. B. C̃irič,*A generalization of Banach contraction princple*, Proc. American Mathematical Society **45** (1974), 999–1006. 3.8

[3] L. G. Huang, X. Zhang, *Cone metric spaces and fixed point theorems of contractive mappings*, J. Math. Anal. Appl. **332** (2007), 1468–1476. 1, 2.1, 2.5, 2.6, 2.8, 2.9, 2.11, 3, 3.5

[4] D. Ĩlic, V. Rakocevic, *Quasi-contraction on a cone metric space*, Applied Mathematics Letters **22** (2009), 728–731. 2.3, 3.6, 3, 3.7

[5] Sh. Jain, Sh. Jain and L.B. Jain, *Compatibilty and weak compatibility for four self maps in a cone metric space*, Bulletin of Mathematical analysis and application **1** (2010), 1–18. 1

[6] Sh. Jain, Sh. Jain and L.B. Jain, *Weakly compatibile maps in a cone metric space*, Rendiconti Del Seminario Matematica **68** (2010), 115–225. 1

[7] B. V. Kvedaras, A.V. Kibenko and A. I. Perov, *On some boundary value problems*, Litov. matem. sbornik **5** (1965), 69–84. 1

[8] E. M. Mukhamadiev and V.J. Stetsenko, *Fixed point principle in generalized metric space*, Izvestija AN Tadzh. SSR, fiz.-mat. i geol.-chem. nauki. **10** (1969), 8–19 [Russian]. 1

[9] A.I. Perov, *The Cauchy problem for systems of ordinary differential equations. In* , Approximate methods of solving differential equations, Kiev, Naukova Dumka, **12** (1964), 115–134 [Russian]. 1

[10] A.I. Perov and A.V. Kibenko, *An approach to studying boundary value problems*, Izvestija AN SSSR, ser. math. **30** (1966), 249–264. [Russian] 1

[11] Sh. Rezapour, R. Hamlbarani, *Some notes on the paper "Cone metric spaces and fixed point theorems of contractive mappings"*, J. Math. Anal. Appl. **345** (2008), 719–724. 1, 2.4, 2.7, 3, 3.5

[12] R. Vasuki, *A Fixed Point Theorem for a sequence of Maps satisfying a new contractive type contraction in Menger Space*, Math Japonica **35** (1990), 1099–1102.

[13] P.P. Zabrejko, *K-metric and K-normed linear spaces* Survey Collect. Math. **48** (1997), 825–859. 1