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On some new fixed point results in $b-$rectangular metric spaces

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Abstract

In this paper we consider, discuss, improve and generalize recent fixed point results for mappings in $b-$rectangular metric spaces. Thus, all our results are with much shorter proofs. Also, we prove Reich type theorem in the frame of $b-$metric space. The proofs of all our results are without using Hausdorff assumption. One example is given to support the result.

Keywords: Fixed point, common fixed point, weakly compatible, $b-$metric space, rectangular metric space, $b-$rectangular metric space
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1. Introduction and preliminaries

Fixed point theory is one of the traditional theory in mathematics and has a large number of applications in it and many branches of nonlinear analysis. It is well known that the Banach contraction principle [4] is a main result in the fixed point theory, which has been used and extended in many different directions. Also, there are some important generalizations of usual metric spaces. Three well known generalizations of (usual) metric spaces are $b-$metric spaces [3,6] or metric type spaces-MTS by some authors ([13,15,20,31]), generalized metric spaces (g.m.s.) [5] or rectangular metric spaces ([9,11,16,17,18,21,23,24], [27]–[32]) and rectangular $b-$metric space [10] or a $b-$generalized metric space ($b-$g.m.s.) [30].

The following definitions are introduced in [3,5,6,10] and [30], respectively:

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Definition 1.1. ([3,6]) Let X be a (nonempty) set and $s \geq 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is a $b-$metric on X if, for all $x, y, z \in X$, the following conditions hold:

(b1) $d(x, y) = 0$ if and only if $x = y$,
(b2) $d(x, y) = d(y, x)$,
(b3) $d(x, z) \leq s[d(x, y) + d(y, z)]$ ($b-$triangular inequality).

In this case, the pair (X, d) is called a $b-$metric space (metric type space).

Otherwise, for all definitions of notions as $b-$convergence, $b-$completeness, $b-$Cauchy in the frame of $b-$metric spaces see [1]–[3],[6],[13]—[15],[20,22], [25,26] and [30].

Definition 1.2. ([5]) Let X be a nonempty set, and let $d : X \times X \to [0, \infty)$ be a mapping such that for all $x, y \in X$ and all distinct points $u, v \in X$, each distinct from x and y :

(r1) $d(x, y) = 0$ if and only if $x = y$,
(r2) $d(x, y) = d(y, x)$,
(r3) $d(x, y) \leq d(x, u) + d(u, v) + d(v, y)$(rectangular inequality).

Then (X, d) is called a rectangular or generalized metric spaces (g.m.s.) or Branciari's space.
For all properties and definitions of notions in Branciari's spaces see [5,9,11,17,18,23,30].

Definition 1.3. ([10,30]) Let X be a nonempty set, $s \geq 1$ be a given real number and let $d : X \times X \to [0, \infty)$ be a mapping such that for all $x, y \in X$ and distinct points $u, v \in X$, each distinct from x and y :

(br1) $d(x, y) = 0$ if and only if $x = y$,
(br2) $d(x, y) = d(y, x)$,
(br3) $d(x, y) \leq s[d(x, u) + d(u, v) + d(v, y)]$ ($b-$rectangular inequality).

Then (X, d) is called a $b-$rectangular metric space or a $b-$generalized metric space ($b-$g.m.s.).

Note that every metric space is a rectangular metric space (g.m.s) and every rectangular metric space is a $b-$rectangular metric space (with coefficient $s = 1$). However the converse is not necessarily true ([10], Examples 2.4. and 2.5.). Also, every metric space is a $b-$metric space (metric type space) and every $b-$metric space is a $b-$rectangular metric space (not necessarily with the same coefficient).

Note also that every $b-$metric space with coefficient s is a $b-$rectangular metric space with coefficient s^2 but the converse is not necessarily true ([10], Examples 2.7).

Hence we have the following diagram where arrows stand for inclusions. The inverse inclusions do not hold.

$$\begin{array}{ccc} \text{metric space} & \to & b-\text{metric space} \\ \downarrow & & \downarrow \\ \text{rectangular metric space} & \to & b-\text{rectangular metric space} \end{array}$$

Note that limit of sequence in a $b-$rectangular metric space (the same as in a rectangular metric space (g.m.s)) is not necessarily unique and also every $b-$rectangular metric convergent sequence in a $b-$rectangular metric space is not necessarily $b-$rectangular metric-Cauchy ([10], Examples 2.7).

Here we will use the following (new, useful and very significant) result for the proofs of fixed point results in the frame of $b-$rectangular metric spaces. Our result is inspired by the proof of Theorem 5. from [30].

Lemma 1.4. *Let (X,d) be a b-rectangular metric space with $s \geq 1$ and let $\{x_n\}$ be a sequence in X such that*

$$x_n \neq x_m \text{ whenever } n \neq m \text{ and } \lim_{n\to\infty} d(x_n, x_{n+1}) = 0. \tag{1.1}$$

If $\{x_n\}$ is not a b-rectangular-Cauchy sequence, then there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that for the following sequences of real numbers

$$d\left(x_{m(k)}, x_{n(k)}\right),\ d\left(x_{m(k)+1}, x_{n(k)-1}\right) \text{ and } d\left(x_{m(k)}, x_{n(k)-2}\right) \tag{1.2}$$

hold:

$$d\left(x_{m(k)}, x_{n(k)}\right) \geq \varepsilon,\ \frac{\varepsilon}{s} \leq \overline{\lim}_{k\to\infty} d\left(x_{m(k)}, x_{n(k)-2}\right) \leq \varepsilon$$
$$\text{and } \frac{\varepsilon}{s} \leq \overline{\lim}_{k\to\infty} d\left(x_{m(k)+1}, x_{n(k)-1}\right) \tag{1.3}$$

Proof. If $\{x_n\}$ is not $b-$rectangular-Cauchy sequence, then there exist $\varepsilon > 0$ for which we can find two subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ such that $n(k)$ is the smallest index for which

$$n(k) > m(k) > k \text{ and } d\left(x_{m(k)}, x_{n(k)}\right) \geq \varepsilon. \tag{1.4}$$

This means that

$$d\left(x_{m(k)}, x_{n(k)-2}\right) < \varepsilon. \tag{1.5}$$

Now taking the upper limit as $k \to \infty$, we obtain

$$\overline{\lim}_{k\to\infty} d\left(x_{m(k)}, x_{n(k)-2}\right) \leq \varepsilon. \tag{1.6}$$

On the other hand, we have

$$\frac{1}{s} d\left(x_{m(k)}, x_{n(k)}\right) \leq d\left(x_{m(k)}, x_{m(k)+1}\right) + d\left(x_{m(k)+1,} x_{n(k)-1}\right) + d\left(x_{n(k)-1,} x_{n(k)}\right). \tag{1.7}$$

Using (1.1), (1.4) and taking the upper limit as $k \to \infty$, we get

$$\frac{\varepsilon}{s} \leq \overline{\lim}_{k\to\infty} d\left(x_{m(k)+1,} x_{n(k)-1}\right) \tag{1.8}$$

Using the $b-$rectangular inequality once again we have the following inequalities:

$$\frac{1}{s} d\left(x_{m(k)}, x_{n(k)}\right) \leq d\left(x_{m(k)}, x_{n(k)-2}\right) + d\left(x_{n(k)-2,} x_{n(k)-1}\right) + d\left(x_{n(k)-1,} x_{n(k)}\right). \tag{1.9}$$

Using (1.1), (1.4) and taking the upper limit as $k \to \infty$, we now get

$$\frac{\varepsilon}{s} \leq \overline{\lim}_{k\to\infty} d\left(x_{m(k),} x_{n(k)-2}\right). \tag{1.10}$$

The proof of Lemma 1.4. is complete. □

As well known, a sequence in a $b-$rectangular metric space may have two limits. However, there is a special situation where this is not possible, and this will be used in some proofs.

Lemma 1.5. ([30], Lemma 1) *Let (X,d) be a b-rectangular metric space with $s \geq 1$ and let $\{x_n\}$ be a b-rectangular-Cauchy sequence in X such that $x_n \neq x_m$ whenever $n \neq m$. Then $\{x_n\}$ can converge to at most one point.*

2. Main results

In our first result of this section, we generalize, complement and improve recent Banach and Kannan type results from ([10], Theorems 3.1. and 3.4.) for b-rectangular metric spaces. We also give positive answer to Open Problems 2)-Reich contraction in the frame of $b-$rectangular metric spaces.

In the following result we generalize and complement Theorems 3.1. and 3.4. from [10] with much shorter and nicer proofs.

Theorem 2.1. *Let (X,d) be a b-rectangular space with $s>1$, and let $f,g:X\to X$ be two self maps such that $f(X)\subseteq g(X)$, one of these two subsets of X being complete. If for some real numbers $a,b\geq 0$ with $a+2b<\frac{1}{s}$*

$$d(fx,fy)\leq ad(gx,gy)+b\left[d(gx,fx)+d(gy,fy)\right] \tag{2.1}$$

holds for all $x,y\in X$, then f and g have a unique point of coincidence ω. Moreover, for each $x_0\in X$, a corresponding Jungck sequence $\{y_n\}$ can be chosen such that $\lim_{n\to\infty}y_n=\omega$.

If, moreover, f and g are weakly compatible, then they have a unique common fixed point.

Proof. Let us prove that the point of coincidence of f and g is unique (if it exists). Suppose that ω_1 and ω_2 are distinct point of coincidence of f and g. From this follows that there exist two point u_1 and u_2 such that $fu_1=gu_1=\omega_1$ and $fu_2=gu_2=\omega_2$. Then (2.1) implies that

$$\begin{aligned} d(\omega_1,\omega_2) &= d(fu_1,fu_2)\\ &\leq ad(gu_1,gu_2)+b\left[d(gu_1,fu_1)+d(gu_2,fu_2)\right]\\ &= ad(\omega_1,\omega_2)+b\left[d(\omega_1,\omega_1)+d(\omega_2,\omega_2)\right]\\ &\qquad = ad(\omega_1,\omega_2)<d(\omega_1,\omega_2), \end{aligned} \tag{2.2}$$

which is a contradiction.

In order to prove that f and g have a point of coincidence, take an arbitrary point $x_0\in X$ and, using that $f(X)\subseteq g(X)$, choose sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_n=fx_n=gx_{n+1},\ \text{for } n=0,1,2,.... \tag{2.3}$$

If $y_k=y_{k+1}$ for some $k\in\mathbb{N}$, then $gx_{k+1}=y_k=y_{k+1}=fx_{k+1}$ and f and g have a point of coincidence. Suppose, further, that $y_n\neq y_{n+1}$ for all $n\in\mathbb{N}$. Putting $x=x_{n+1},y=x_n$ in (2.1) we obtain that

$$\begin{aligned} d(y_{n+1},y_n) &= d(fx_{n+1},fx_n)\\ &\leq ad(gx_{n+1},gx_n)+b\left[d(gx_{n+1},fx_{n+1})+d(gx_n,fx_n)\right]\\ &\qquad = ad(y_n,y_{n-1})+b\left[d(y_n,y_{n+1})+d(y_{n-1},y_n)\right], \end{aligned} \tag{2.4}$$

that is,

$$d(y_{n+1},y_n)\leq\lambda d(y_{n-1},y_n), \tag{2.5}$$

where $\lambda=\frac{a+b}{1-b}<\frac{1}{s}$ (because $a+2b<\frac{1}{s}$).

Now, by ([7], Lemma 1.8.) and (2.5) the sequence $\{y_n\}$ satisfies all conditions of Lemma 1.4.

Let us prove that $\{y_n\}$ is a $b-$rectangular-Cauchy sequence in $b-$rectangular metric space (X,d). If $\{x_n\}$ is not a $b-$rectangular-Cauchy sequence, then there exist $\varepsilon>0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that the following sequences of real numbers $d\left(x_{m(k)},x_{n(k)}\right)$, $d\left(x_{m(k)+1},x_{n(k)-1}\right)$ and $d\left(x_{m(k)},x_{n(k)-2}\right)$ satisfy (1.3). Putting $x=x_{m(k)+1},y=x_{n(k)-1}$ in (2.1) we have

$$d\left(y_{m(k)+1},y_{n(k)-1}\right)\leq ad\left(y_{m(k)},y_{n(k)-2}\right)+b\left[d\left(y_{m(k)},y_{m(k)+1}\right)+d\left(y_{n(k)-2},y_{n(k)-1}\right)\right]. \tag{2.6}$$

Taking the upper limit as $k\to\infty$ and using (1.1), (1.6) and (1.8) we get

$$\frac{\varepsilon}{s}\leq a\varepsilon+b\left[0+0\right],$$

that is $\frac{1}{s} \leq a$. A contradiction (because $a \leq a + 2b < \frac{1}{s}$). Hence, the sequence $\{y_n\} = \{fx_n\} = \{gx_{n+1}\}$ is a b-rectangular-Cauchy.

Suppose, e.g., that the subspace $g(X)$ is complete (the proof when $f(X)$ is complete is similar). Then $\{y_n\}$ tends to some $\omega \in g(X)$, where $\omega = gz$ for some $z \in X$. To prove that $fz = gz$, suppose that $fz \neq gz$. Then, by Lemma 1.5, it follows that $y_n = fx_n = gx_{n+1}$ differs from both fz and gz for n sufficiently large. Hence, we can apply $b-$rectangular inequality to obtain

$$\frac{1}{s} d(fz, gz) \leq d(fz, fx_n) + d(fx_n, fx_{n+1}) + d(fx_{n+1}, gz)$$

$$\leq ad(gz, gx_n) + b[d(gz, fz) + d(fx_n, gx_n)] + d(y_n, y_{n+1}) + d(y_{n+1}, gz)$$

$$= ad(y_{n-1}, gz) + bd(gz, fz) + bd(y_n, y_{n-1}) + d(y_n, y_{n+1}) + d(y_{n+1}, gz). \tag{2.7}$$

Taking the limit as $n \to \infty$, we get

$$\frac{1}{s} d(fz, gz) \leq a \cdot 0 + bd(gz, fz) + b \cdot 0+) + 0 = bd(gz, fz),$$

from which it follows that $d(gz, fz) = 0$ (because $b \leq a + 2b < \frac{1}{s}$), that is, $fz = gz$. Hence, $fz = gz = \omega$ is a unique point of coincidence of f, g.

If f, g are weakly compatible, a well-known Jungck's result implies that f and g have a unique fixed point (here it is ω). □

Taking $g = I_X$ (identity mapping of X) in Theorem 2.1. we obtain the following variant of Banach, Kannan and Reich-theorem in $b-$rectangular metric spaces.

Corollary 2.2. ([10], Theorem 3.1) *Let (X, d) be a complete b-rectangular metric space with coefficient $s > 1$ and $f : X \to X$ be a mapping satisfying:*

$$d(fx, fy) \leq \lambda d(x, y), \tag{2.8}$$

for all $x, y \in X$, where $\lambda \in [0, \frac{1}{s})$. Then f has a unique fixed point $z \in X$, and for each $x \in X$ the Picard sequence $\{f^n x\}$ converges to z in (X, d).

Corollary 2.3. ([10] Theorem 3.2) *Let (X, d) be a complete b-rectangular metric space with coefficient $s > 1$ and $f : X \to X$ be a mapping satisfying:*

$$d(fx, fy) \leq \lambda [d(x, fx) + d(y, fy)], \tag{2.9}$$

for all $x, y \in X$, where $\lambda \in [0, \frac{1}{s+1})$. Then f has a unique fixed point $z \in X$, and for each $x \in X$ the Picard sequence $\{f^n x\}$ converges to z in (X, d).

Corollary 2.4. *Let (X, d) be a complete b-rectangular metric space with coefficient $s > 1$ and $f : X \to X$ be a mapping satisfying:*

$$d(fx, fy) \leq Ad(x, y) + B[d(x, fx) + d(y, fy)], \tag{2.10}$$

for all $x, y \in X$, where $a, b \geq 0$ with $a + 2b < \frac{1}{s}$. Then f has a unique fixed point $z \in X$, and for each $x \in X$ the Picard sequence $\{f^n x\}$ converges to z in (X, d).

Corollary 2.5. *Let (X, d) be a complete b-rectangular metric space with coefficient $s > 1$ and $f, g : X \to X$ be two self maps such that $f(X) \subseteq g(X)$, one of these two subsets of X being complete. If f and g satisfy*

$$d(fx, fy) \leq \lambda [d(gx, gy) + d(gx, fx) + d(gy, fy)]$$

for all $x, y \in X$ and $\lambda \in [0, \frac{1}{2s+1})$. Then f and g have a unique common fixed point.

Remark 2.6. It is worth to notice that Corollary 2.2. holds for $\lambda \in [0, \frac{1}{s}]$. Hence, as in ([10], Theorem 3.1.). Indeed, if $d(fx, fy) \leq \frac{1}{s} d(x, y)$ then $d(f^2x, f^2y) \leq \frac{1}{s^2} d(x, y)$ and then (because $\frac{1}{s^2} < \frac{1}{s}$) we obtain by Theorem 2.1. that f^2 has unique fixed point. From this it follows that f has a unique fixed point.

Similar, we can prove using our approach that Corollary 2.3. holds if $\lambda \in [0, \frac{1}{s+1}]$, that is as in ([10], Theorem 3.2.).

Recall (see [21]) that a mapping $\psi : [0, +\infty) \to [0, +\infty)$ is called an altering distance function if:

(i) ψ is increasing and continuous,

(ii) $\psi(t) = 0$ if and only if $t = 0$.

The following theorem is a $b-$rectangular version of the main results from [21]. Also, it also extends Theorem 2 from [17] from rectangular to $b-$rectangular metric spaces.

Theorem 2.7. *Let (X, d) be a b-rectangular metric space with coefficient $s > 1$, and let $f, g : X \to X$ be two self maps such that $f(X) \subseteq g(X)$, one of these two subsets of X being complete. If, for some altering distance function ψ and some $c \in [0, 1)$,*

$$\psi(sd(fx, fy)) \leq c\psi(d(gx, gy)) \tag{2.11}$$

holds for all $x, y \in X$, then f and g have a unique point of coincidence. If, moreover, f and g are weakly compatible, then they have a unique common fixed point.

Proof. The proof follows the lines of proof of Theorem 2.1 except that the sequence $\{y_n\} = \{fx_n\} = \{gx_{n+1}\}$ is a b-rectangular-Cauchy. However, this follows from Lemma 1.4.. Indeed, putting $x = x_{m(k)+1}, y = x_{n(k)-1}$ in (2.11) we get

$$\psi\left(sd\left(fx_{m(k)+1}, fx_{n(k)-1}\right)\right) \leq c\psi\left(d\left(gx_{m(k)+1}, gx_{n(k)-1}\right)\right), \tag{2.12}$$

that is,

$$\psi\left(sd\left(y_{m(k)+1}, y_{n(k)-1}\right)\right) \leq c\psi\left(d\left(y_{m(k)}, y_{n(k)-2}\right)\right). \tag{2.13}$$

Now, using (1.6), (1.8), properties of the function ψ and taking the upper limit as $k \to \infty$, we get

$$\psi(\varepsilon) = \psi\left(s\frac{\varepsilon}{s}\right) \leq c\psi(\varepsilon) < \psi(\varepsilon), \tag{2.14}$$

which is a contradiction. The rest of the proof is omitted. □

Corollary 2.8. *Let (X, d) be a complete b-rectangular metric space with coefficient $s > 1$, and let $f : X \to X$ be a self map. If, for some altering distance function ψ and some $c \in [0, 1)$,*

$$\psi(sd(fx, fy)) \leq c\psi(d(x, y)) \tag{2.15}$$

holds for all $x, y \in X$, then f has a unique fixed point.

Remark 2.9. It is easy to see that the result of Theorem 2.7. remains valid if the inequality (2.11) is replaced by the following one

$$\psi(sd(fx, fy)) \leq c\psi(\max\{d(gx, gy), d(gx, fx), d(gy, fy)\}). \tag{2.16}$$

The following example support Corollary 2.8.

Example 2.10. Let $X = A \cup B$, where $A = \left\{\frac{1}{n} : n \in \{2, 3, 4, 5\}\right\}$ and $B = [1, 2]$. Define $d : X \times X \to [0, \infty)$ such that $d(x, y) = d(y, x)$ for all $x, y \in X$ and

$$\begin{aligned} d\left(\frac{1}{2}, \frac{1}{3}\right) &= d\left(\frac{1}{4}, \frac{1}{5}\right) = 0.03; d\left(\frac{1}{2}, \frac{1}{5}\right) = d\left(\frac{1}{3}, \frac{1}{4}\right) = 0.02; \\ d\left(\frac{1}{2}, \frac{1}{4}\right) &= d\left(\frac{1}{5}, \frac{1}{3}\right) = 0.06; d(x, y) = |x - y|^2 \text{ otherwise.} \end{aligned}$$

Then (X, d) is a $b-$rectangular metric space with coefficient $s = 4 > 1$. But (X, d) is neither a metric space nor a rectangular metric space. Let $f : X \to X$ be defined as:

$$f(x) = \begin{cases} \frac{1}{4} \text{ if } x \in A \\ \frac{1}{5} \text{ if } x \in B. \end{cases}$$

Then f satisfies all conditions of Corollary 2.8. with $c = \frac{12}{25}$ and $\psi(t) = \frac{1}{3}t$ and has a unique fixed point $x = \frac{1}{4}$. This example shows that Corollary 2.8. is genuine generalization of Corollary 2 from [17].

Corollary 2.11. *Let (X, d) be a b-rectangular metric space with coefficient $s > 1$,and let $f, g : X \to X$ be two self maps such that $f(X) \subseteq g(X)$, one of these two subsets of X being complete. If, for some altering distance function ψ and some $c \in [0, \frac{1}{s})$*

$$sd(fx, fy) \leq c\psi\left(\max\left\{d(gx, gy), d(gy, fx), \frac{1}{2}(d(gx, fx) + d(gy, fy))\right\}\right)$$

holds for all $x, y \in X$, then f and g have a unique point of coincidence. If, moreover, f and g are weakly compatible, then they have a unique common fixed point.

Finally, we will generalize one result from [19], that is Theorem 8.

Theorem 2.12. *Let (X, d) be a b-rectangular metric space with coefficient $s > 1$, and let $f, g : X \to X$ be two self maps such that $f(X) \subseteq g(X)$, one of these two subsets of X being complete. Assume that the following condition holds:*

$$sd(fx, fy) \leq \frac{1}{2}(d(gx, fx) + d(gy, fy)) - \phi(d(gx, fx), d(gy, fy)), \tag{2.17}$$

where $\phi : [0, \infty) \times [0, \infty) \to [0, \infty)$ is continuous, and $\phi(a, b) = 0$ if and only if $a = b = 0$. Then f and g have a unique point of coincidence say $\omega \in X$. Moreover, for each $x_0 \in X$, the corresponding Jungck sequence $\{y_n\}$ can be chosen such that $\lim_{n\to\infty} y_n = \omega$. in addition, if f and g are weakly compatible, then they have a unique common fixed point.

Proof. Since, the condition (2.17) becomes

$$d(fx, fy) \leq \frac{1}{s+1}(d(gx, fx) + d(gy, fy)), \tag{2.18}$$

for all $x, y \in X$ and arbitrary $\phi : [0, \infty) \times [0, \infty) \to [0, \infty)$, the rest of the proof is further as in Corollary 2.3. □

An open question:

Prove or disprove the following (**Meir-Keeler theorem**):

•*Let (X, d) be a b-rectangular metric space with coefficient $s > 1$, and let $f, g : X \to X$ be two self maps such that $f(X) \subseteq g(X)$, one of these two subsets of X being complete. Assume that the following condition holds:*

$$\text{for each } \varepsilon > 0 \text{ there exists } \delta > 0 \text{ such that}$$

$$\varepsilon \leq d(gx, gy) < \varepsilon + \delta \text{ implies } sd(fx, fy) < \varepsilon, \tag{2.19}$$

$$\text{and } fx = fy \text{ whenever } gx = gx. \tag{2.20}$$

Then f and g have a unique point of coincidence say $\omega \in X$. Moreover, for each $x_0 \in X$, the corresponding Jungck sequence $\{y_n\}$ can be chosen such that $\lim_{n\to\infty} y_n = \omega$. In addition, if f and g are weakly compatible, then they have a unique common fixed point.

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