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A fixed point theory for S-contractions in generalized Kasahara spaces

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Dedicated to the memory of Professor Viorel Radu

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Abstract

The aim of this paper is to present a fixed point theory for S-contractions in generalized Kasahara spaces $(X, \to, d)$, where $d : X \times X \to s(\mathbb{R}_+)$ is not necessarily an $s(\mathbb{R}_+)$-metric.

Keywords: Fixed point, S-contraction, generalized Kasahara space, sequence of successive approximations, $s(\mathbb{R}_+)$-metric, Neumann matrix, Ulam-Hyers stability.
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1. Introduction

In the mathematical literature, there are several papers in which fixed point results for S-contractions defined on $s(\mathbb{R}_+)$-metric spaces are given: N. Gheorghiu [8], P.P. Zabrejko and T.A. Makarevich [20], V.G. Angelov [1], M. Frigon [7], I.A. Rus [15]. On the other hand, there are papers containing fixed point theorems for generalized contractions defined in more general settings. For example, we mention the case of generalized Kasahara spaces, see K. Iséki [11], S. Kasahara [12] and [13], I.A. Rus [17], A.-D. Filip [5].

The aim of this paper is to present a fixed point theory for S-contractions in generalized Kasahara spaces $(X, \to, d)$, where $d : X \times X \to s(\mathbb{R}_+)$ is not necessarily an $s(\mathbb{R}_+)$-metric.

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2. Basic notions and notations

Let X be a nonempty set.

We consider the set of all sequences of X denoted by

$$s(X) := \big\{(x_n)_{n\in\mathbb{N}^*} \mid x_n \in X,\ n \in \mathbb{N}^*\big\}.$$

If $(X, \leq)$ is an ordered set, we define the order relation $\leq_s$ as follows: for all $(x_n)_{n\in\mathbb{N}^*}, (y_n)_{n\in\mathbb{N}^*} \in s(\mathbb{R}_+)$, $(x_n)_{n\in\mathbb{N}^*} \leq_s (y_n)_{n\in\mathbb{N}^*}$ if and only if $x_n \leq y_n$, for all $n \in \mathbb{N}^*$. In addition, by $(x_n)_{n\in\mathbb{N}^*} <_s (y_n)_{n\in\mathbb{N}^*}$ we understand that $(x_n)_{n\in\mathbb{N}^*} \leq_s (y_n)_{n\in\mathbb{N}^*}$ and $x_n \neq y_n$, for all $n \in \mathbb{N}^*$.

A functional $\rho : X \times X \to s(\mathbb{R}_+)$ defined by $(x, y) \mapsto (\rho_m(x, y))_{m\in\mathbb{N}^*}$ is called $s(\mathbb{R}_+)$-metric if the following conditions are satisfied:

(i) $\rho(x, y) = 0$ if and only if $x = y$, for all $x, y \in X$;

(ii) $\rho(x, y) = \rho(y, x)$, for all $x, y \in X$;

(iii) $\rho(x, z) \leq_s \rho(x, y) + \rho(y, z)$, for all $x, y, z \in X$.

The couple (X, ρ), where X is a nonempty set and ρ is an $s(\mathbb{R}_+)$-metric on X, is called $s(\mathbb{R}_+)$-metric space.

If (X, ρ) is an $s(\mathbb{R}_+)$-metric space, then

- $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in X if and only if for all $\varepsilon := (\varepsilon_m)_{m\in\mathbb{N}^*} \in s(\mathbb{R}^*_+)$, there exists $n_\varepsilon \in \mathbb{N}$ such that for all $n, p \in \mathbb{N}$ with $n \geq n_\varepsilon$ we have $\rho(x_n, x_{n+p}) \leq_s \varepsilon$, i.e., $\rho_m(x_n, x_{n+p}) \leq \varepsilon_m$, for all $m \in \mathbb{N}^*$.
- we denote by $\stackrel{\rho}{\to}$ the convergence structure induced by ρ on X, defined as follows: for all $(x_n)_{n\in\mathbb{N}} \subset X$,

$$x_n \stackrel{\rho}{\to} x \in X \text{ as } n \to \infty \text{ if and only if } \rho(x_n, x) \to 0 \in s(\mathbb{R}_+) \text{ as } n \to \infty,$$
$$\text{i.e., } \rho_m(x_n, x) \to 0 \in \mathbb{R} \text{ as } n \to \infty, \text{ for all } m \in \mathbb{N}^*.$$

- the couple $(X, \stackrel{\rho}{\to})$ is an L-space (more considerations on L-spaces can be found in the work of M. Fréchet [6], I.A. Rus [14] and [17], I.A. Rus, A. Petruşel and G. Petruşel [18](p.77)).

An $s(\mathbb{R}_+)$-metric space (X, ρ) is complete (in the Weierstrass' sense) if

$$\text{for all } (x_n)_{n\in\mathbb{N}} \subset X,\ \sum_{n\in\mathbb{N}} \rho(x_n, x_{n+1}) <_s +\infty \Rightarrow (x_n)_{n\in\mathbb{N}} \text{ converges in } X.$$

The notion of generalized Kasahara space was introduced by I.A. Rus in [17] as follows:

Definition 2.1. Let $(X, \to)$ be an L-space, $(G, +, \leq, \stackrel{G}{\to})$ be an L-space ordered semigroup with unity, 0 be the least element in $(G, \leq)$ and $d : X \times X \to G$ be an operator. The triple $(X, \to, d)$ is a generalized Kasahara space if and only if we have the following compatibility condition between $\to$ and d:

$$x_n \in X,\ \sum_{n\in\mathbb{N}} d(x_n, x_{n+1}) < +\infty \ \Rightarrow (x_n)_{n\in\mathbb{N}} \textit{ converges in } (X, \to). \tag{2.1}$$

Notice that by the inequality with the symbol $+\infty$ in the compatibility condition (2.1), we mean that the series $\sum_{n\in\mathbb{N}} d(x_n, x_{n+1})$ is convergent in $(G, +, \stackrel{G}{\to})$.

We present bellow examples of generalized Kasahara spaces, where the functional d takes values in $s(\mathbb{R}_+)$.

Example 2.2. Let (X, ρ) be a complete $s(\mathbb{R}_+)$-metric space and $d : X \times X \to s(\mathbb{R}_+)$ be a functional. If there exists a real constant $c > 0$ such that $\rho(x, y) \leq_s c \cdot d(x, y)$, for all $x, y \in X$, then $(X, \xrightarrow{\rho}, d)$ is a generalized Kasahara space.

Indeed, let us consider the sequence $(x_n)_{n\in\mathbb{N}} \subset X$ such that

$$\sum_{n\in\mathbb{N}} d(x_n, x_{n+1}) <_s +\infty, \ i.e., \ \sum_{n\in\mathbb{N}} d_m(x_n, x_{n+1}) < +\infty, \text{ for all } m \in \mathbb{N}^*.$$

Since there exists a real constant $c > 0$ such that $\rho(x, y) \leq_s c \cdot d(x, y)$, for all $x, y \in X$, we have the following estimations:

$$\sum_{n\in\mathbb{N}} \rho_m(x_n, x_{n+1}) \leq c \sum_{n\in\mathbb{N}} d_m(x_n, x_{n+1}) < +\infty, \text{ for all } m \in \mathbb{N}^*,$$

i.e., the series $\sum_{n\in\mathbb{N}} \rho_m(x_n, x_{n+1})$ is convergent in $\mathbb{R}_+$, for all $m \in \mathbb{N}^*$.

It follows that, for all $n, p \in \mathbb{N}$, we have

$$\rho_m(x_n, x_{n+p}) \leq \sum_{k=n}^{n+p-1} \rho_m(x_k, x_{k+1}) \to 0 \text{ as } n \to \infty, \text{ for all } m \in \mathbb{N}^*$$

which implies further that $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in (X, ρ).

Since (X, ρ) is complete, it follows that $(x_n)_{n\in\mathbb{N}}$ is convergent in $(X, \xrightarrow{\rho})$.

Example 2.3. Let $X := [0, 1]$ and $\rho : X \times X \to s(\mathbb{R}_+)$ be a complete generalized metric on X, defined by $\rho(x, y) = (|x - y|, 0, 0, \ldots)$, for all $x, y \in X$.

We consider the functional $d : X \times X \to s(\mathbb{R}_+)$ defined by

$$d(x, y) = \begin{cases} \rho(x, y), & x \neq 0 \text{ and } y \neq 0 \\ (1, 0, 0, \ldots), & x = 0 \text{ or } y = 0 \end{cases}$$

for all $x, y \in X$. Then $(X, \xrightarrow{\rho}, d)$ is a generalized Kasahara space.

Let $(X, \to, d)$ be a generalized Kasahara space and $f : X \to X$ be an operator. Then the set of all fixed points of f will be denoted by

$$F_f := \{x \in X \mid x = f(x)\}.$$

Concerning the Ulam-Hyers stability of the fixed point equation $x = f(x)$, we have the following definition:

Definition 2.4. Let $(X, \to, d)$ be a generalized Kasahara space, where $d : X \times X \to s(\mathbb{R}_+)$ is a generalized quasimetric on X and let $f : X \to X$ be an operator. Then the fixed point equation

$$x = f(x) \tag{2.2}$$

is said to be generalized Ulam-Hyers stable if there exists an increasing function $\psi : s(\mathbb{R}_+) \to s(\mathbb{R}_+)$, continuous at 0, with $\psi(0) = 0$ such that for any $\varepsilon \in s(\mathbb{R}_+)$ with $\varepsilon_m > 0$ for all $m \in \mathbb{N}^*$ and any solution $y^* \in X$ of the inequation $d(f(y), y) \leq_s \varepsilon$, there exists a solution x^* of (2.2) such that $d(x^*, y^*) \leq_s \psi(\varepsilon)$.

In particular, if $\psi(t) = C \cdot t^\tau$, for all $t \in s(\mathbb{R}_+)$ and $C \in M(\mathbb{R})$, then the fixed point equation (2.2) is called Ulam-Hyers stable.

Remark 2.5. More consideration regarding Ulam-Hyers stability can be found in [19], [9], [10] and [16].

We present next the notions and notations concerning infinite matrices which will be used in the following section of this paper. More considerations on infinite matrices can be found in the work of R.G. Cooke [4].

We denote the set of all infinite matrices by

$$M(\mathbb{R}) := \left\{(a_{ij})_1^\infty \mid a_{ij} \in \mathbb{R},\ i,j \in \mathbb{N}^*\right\},$$

where

$$(a_{ij})_1^\infty := \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots \\ a_{21} & a_{22} & a_{23} & \dots \\ a_{31} & a_{32} & a_{33} & \dots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

is an infinite matrix of real values.

We consider the functional

$$\|\cdot\| : M(\mathbb{R}) \to \mathbb{R}_+ \cup \{+\infty\}$$

defined by

$$\|A\| := \sup_{1\le i\le\infty} \sum_{j\in\mathbb{N}^*} |a_{ij}|, \text{ for all } A \in M(\mathbb{R}).$$

It can be proved that $\|\cdot\|$ is a generalized norm on $M(\mathbb{R})$.

An infinite matrix $A \in M(\mathbb{R})$ is called:

- ◇ *row-column-finite* if there exists only a finite number of nonzero elements in each row and column of the matrix.
- ◇ *Neumann matrix* if we can define A^n for all $n \in \mathbb{N}$ and the series $\sum_{n\in\mathbb{N}} A^n$ is termwise convergent.

We recall also the following result:

Theorem 2.6 (R.G. Cooke [4]; I.A. Rus, A. Petruşel and G. Petruşel [18] p.82)**.** *Let $A \in M(\mathbb{R})$ be an infinite matrix. If A is row-column-finite and $\|A\| < 1$ then:*

(*j*) *A is a Neumann matrix;*

(*jj*) $(I - A)^{-1} = \sum_{n\in\mathbb{N}} A^n$. *(Here, I denotes the identity infinite matrix).*

Finally, if $A \in M(\mathbb{R})$ is an infinite matrix, then by A^τ we understand the transpose matrix of A.

3. A fixed point theory for S-contractions

In 1922 S. Banach [2] and in 1930 R. Caccioppoli [3] have given the well-known contraction principle for α-contractions in complete metric spaces.

A similar result for S-contractions in complete $s(\mathbb{R}_+)$-metric spaces was given by I.A. Rus in [15]. Our aim is to give a fixed point theory for S-contractions in generalized Kasahara spaces $(X, \xrightarrow{\rho}, d)$, where $\rho : X \times X \to s(\mathbb{R}_+)$ is a complete generalized metric on X and $d : X \times X \to s(\mathbb{R}_+)$ is a functional.

Definition 3.1 (S-contraction)**.** Let $(X, \xrightarrow{\rho}, d)$ be a generalized Kasahara space, where $\rho : X \times X \to s(\mathbb{R}_+)$ is a complete generalized metric on X and $d : X \times X \to s(\mathbb{R}_+)$ is a functional. Let $f : X \to X$ be an operator and $S \in M(\mathbb{R})$ be an infinite matrix. The operator f is called S-contraction if the following conditions are satisfied:

(1) S is row-column-finite;

(2) S is a Neumann matrix;

(3) $\sum_{n\in\mathbb{N}} S^n d(x,y)$ converges, for all $x, y \in X$;

(4) $d(f(x), f(y))^\tau \leq_s S \cdot d(x,y)^\tau$, for all $x, y \in X$.

Definition 3.2 (Operator with closed graph)**.** Let $(X, \xrightarrow{\rho}, d)$ be a generalized Kasahara space, where $\rho : X \times X \to s(\mathbb{R}_+)$ is a complete generalized metric on X and $d : X \times X \to s(\mathbb{R}_+)$ is a functional. Let $f : X \to X$ be an operator. Then f has closed graph if for any sequence $(x_n)_{n\in\mathbb{N}} \subset X$ and $x^*, y^* \in X$ the following implication holds:

$$x_n \xrightarrow{\rho} x^* \text{ and } f(x_n) \xrightarrow{\rho} y^* \Rightarrow f(x^*) = y^*.$$

Our main result is the following one.

Theorem 3.3. *Let $(X, \xrightarrow{\rho}, d)$ be a generalized Kasahara space, where $\rho : X \times X \to s(\mathbb{R}_+)$ is a complete generalized metric on X and $d : X \times X \to s(\mathbb{R}_+)$ is a functional. Let $f : X \to X$ be an operator and $S \in M(\mathbb{R})$ be an infinite matrix. We assume that:*

(i) $f : (X, \xrightarrow{\rho}) \to (X, \xrightarrow{\rho})$ has closed graph;

(ii) $f : (X, d) \to (X, d)$ is an S-contraction.

Then the following statements hold:

(1) $F_f \neq \emptyset$;

(2) $f^n(x) \to x_f^* \in F_f$ as $n \to \infty$, for all $x \in X$;

(3) Let $x_f^* \in F_f$. If the functional d is a quasimetric (i.e., $d(x,y) = d(y,x) = 0 \Leftrightarrow x = y$ for all $x, y \in X$ and d satisfies the triangle inequality) and $\|S\| < 1$ then:

(3.1) $d(x, x_f^*)^\tau \leq_s (I - S)^{-1} d(x, f(x))^\tau$, for all $x \in X$;

(3.2) $d(x_f^*, x)^\tau \leq_s (I - S)^{-1} d(f(x), x)^\tau$, for all $x \in X$;

(3.3) $d(f^n(x), x_f^*)^\tau \leq_s (I - S)^{-1} S^n d(x, f(x))^\tau$, for all $x \in X$;

(3.4) $d(x_f^*, f^n(x))^\tau \leq_s (I - S)^{-1} S^n d(f(x), x)^\tau$, for all $x \in X$;

(3.5) if $(z_n)_{n\in\mathbb{N}} \subset X$ is such that $d(z_n, f(z_n)) \xrightarrow{s(\mathbb{R}_+)} 0$ as $n \to \infty$ then $d(z_n, x_f^*) \xrightarrow{s(\mathbb{R}_+)} 0$ as $n \to \infty$, i.e., the fixed point problem for the operator f is well-posed with respect to d;

(3.6) if $g : X \to X$ has the property that there exists $\eta := (\eta_m)_{m\in\mathbb{N}^*} \in s(\mathbb{R}_+^*)$ for which $d(g(x), f(x)) \leq_s \eta$, for all $x \in X$, then

$$x_g^* \in F_g \text{ implies } d(x_g^*, x_f^*)^\tau \leq_s (I - S)^{-1}\eta^\tau;$$

(3.7) the fixed point equation $x = f(x)$, for all $x \in X$ is Ulam-Hyers stable.

Proof. (1) & (2). Let $x_0 \in X$. We construct the sequence of successive approximations for f starting from x_0. Let $(x_n)_{n\in\mathbb{N}}$ be this sequence. Hence $x_n = f^n(x_0)$ for all $n \in \mathbb{N}$.

Since f is an S-contraction, we have the following estimations:

$$\begin{aligned} d(f(x_0), f^2(x_0))^\tau &\leq_s S \cdot d(x_0, f(x_0))^\tau \\ d(f^2(x_0), f^3(x_0))^\tau &\leq_s S \cdot d(f(x_0), f^2(x_0))^\tau \\ &\cdots \\ d(f^n(x_0), f^{n+1}(x_0))^\tau &\leq_s S \cdot d(f^{n-1}(x_0), f^n(x_0))^\tau. \end{aligned}$$

Hence, we can write for all $n \in \mathbb{N}$ that

$$d(f^n(x_0), f^{n+1}(x_0))^\tau \leq_s S \cdot d(f^{n-1}(x_0), f^n(x_0))^\tau \leq_s S^2 d(f^{n-2}(x_0), f^{n-1}(x_0))^\tau \\ \leq_s \ldots \leq_s S^n d(x_0, f(x_0))^\tau.$$

Next we estimate

$$\sum_{n\in\mathbb{N}} d(x_n, x_{n+1})^\tau = \sum_{n\in\mathbb{N}} d(f^n(x_0), f^{n+1}(x_0))^\tau \leq_s \sum_{n\in\mathbb{N}} S^n \cdot d(x_0, f(x_0))^\tau <_s +\infty.$$

Since $(X, \xrightarrow{\rho}, d)$ is a Kasahara space, we get that the sequence $(x_n)_{n\in\mathbb{N}}$ is convergent in $(X, \xrightarrow{\rho})$. Hence, there exists an element $x_f^* \in X$ such that $x_n \xrightarrow{\rho} x_f^*$ as $n \to \infty$. Using the fact that $f : (X, \xrightarrow{\rho}) \to (X, \xrightarrow{\rho})$ has closed graph, we obtain that $x_f^* \in F_f$.

(3). Let $x \in X$. Since d satisfies the triangle inequality, we have

$$d(x, x_f^*)^\tau \leq_s d(x, f(x))^\tau + d(f(x), f(x_f^*))^\tau \leq_s d(x, f(x))^\tau + S d(x, x_f^*)^\tau$$

and hence

$$d(x, x_f^*)^\tau \leq_s (I - S)^{-1} d(x, f(x))^\tau, \text{ for all } x \in X,$$

so (3.1) holds. Similarly we get (3.2).

We prove next (3.3). By taking $x := f^n(x)$ in (3.1), we have the following estimation

$$d(f^n(x), x_f^*)^\tau \leq_s (I - S)^{-1} d(f^n(x), f^{n+1}(x))^\tau, \text{ for all } x \in X. \tag{3.1}$$

On the other hand we have

$$d(f^n(x), f^{n+1}(x))^\tau \leq_s S \cdot d(f^{n-1}(x), f^n(x))^\tau \leq_s S^2 d(f^{n-2}(x), f^{n-1}(x))^\tau \\ \leq_s \ldots \leq_s S^n d(x, f(x))^\tau, \text{ for all } x \in X. \tag{3.2}$$

By (3.1) and (3.2) we obtain

$$d(f^n(x), x_f^*)^\tau \leq_s (I - S)^{-1} S^n d(x, f(x))^\tau, \text{ for all } x \in X,$$

so (3.3) holds. By a similar procedure we obtain (3.4).

We prove next (3.5). Let $(z_n)_{n\in\mathbb{N}} \subset X$ such that $d(z_n, f(z_n)) \xrightarrow{s(\mathbb{R}_+)} 0$ as $n \to \infty$. By (3.1) we have

$$d(z_n, x_f^*)^\tau \leq_s (I - S)^{-1} d(z_n, f(z_n))^\tau \xrightarrow{s(\mathbb{R}_+)} 0^\tau \text{ as } n \to \infty$$

so (3.5) holds.

We show now (3.6). Let $x_g^* \in F_g$. By (3.1) we have that

$$d(x_g^*, x_f^*)^\tau \leq_s (I - S)^{-1} d(x_g^*, f(x_g^*))^\tau = (I - S)^{-1} d(g(x_g^*), f(x_g^*))^\tau \\ \leq_s (I - S)^{-1} \eta^\tau.$$

Finally, we prove (3.7). Let $\varepsilon := (\varepsilon_1, \varepsilon_2, \ldots) \in s(\mathbb{R}_+)$ such that $\varepsilon_m > 0$, for all $m \in \mathbb{N}^*$. Let $y^* \in X$ be a solution of the inequation $d(f(y), y) \leq_s \varepsilon$. Then

$$d(x^*, y^*)^\tau = d(f(x^*), y^*)^\tau \leq_s d(f(x^*), f(y^*))^\tau + d(f(y^*), y^*)^\tau \\ \leq_s S \cdot d(x^*, y^*)^\tau + \varepsilon^\tau.$$

It follows that $d(x^*, y^*)^\tau \leq_s (I - S)^{-1} \varepsilon^\tau$. □

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