Available online at www.tjnsa.com
J. Nonlinear Sci. Appl. 9 (2016), 240–246
Research Article

Journal of Nonlinear Science and Applications
Print: ISSN 2008-1898 Online: ISSN 2008-1901

A unique fixed point result using generalized contractive conditions on cyclic mappings in partial metric spaces

W. Shamaila[a], M. Akram[b,c,*]

[a]Department of Mathematics, Kinnaird College for Women, Lahore, Pakistan
[b]College of Science, Department of Mathematics and Statistics, King Faisal University, Al-Ahsa, Saudi Arabia
[c]Department of Mathematics GC University, Lahore, Pakistan.

Communicated by Yeol Je Cho

Abstract

The purpose of this paper is to study fixed point result for generalized contractive condition on cyclic mappings in complete partial metric spaces. The effectiveness of the result is also illustrated through an example.

Keywords: Cyclic Mappings, generalized contractions, partial metric spaces, common fixed point.
2010 MSC: 26D15, 26A51, 41A55.

1. Introduction and preliminaries

The concept of partial metric space, a generalization of metric space, was introduced by Steve Matthews [18] in 1992 (see also [1, 3]). He proved that the Banach's contraction mapping principle [7] can be generalized to the partial metric context for applications in program verification (see also [2]). Later many researchers studied fixed point theorems in complete partial metric spaces. For more details, see [4]-[11].

Banach's contraction mapping principle is one of the most important results in nonlinear analysis. Generalization of this principle has been a very active field of research. Particularly, in 2003 Kirk, Srinivasan and Veeramani [12] introduced the notion of cyclic representation and characterized the Banach's contraction

*Corresponding author
Email addresses: shamailawaheed20@gmail.com (W. Shamaila), maahmed@kfu.edu.sa, dr.makram@gcu.edu.pk (M. Akram)

Received 2015-06-25

mapping principle in context of a cyclic mapping. In the last decade, many theorems for cyclic mappings have been obtained (see e.g. [13]-[20]).

We state some definitions and results needed in the sequel.

Definition 1.1 ([1, 3])**.** Let X be a non-empty set. A partial metric "p" on X is a function from $X \times X$ to R^+ such that for every element x, y and z of X it satisfies following axioms.

p_1 : $0 \leq p(x,x) \leq p(x,y)$;
p_2 : $p(x,x) = p(x,y) = p(y,y)$ if and only if $x = y$;
p_3 : $p(x,y) = p(y,x)$; (Symmetry)
p_4 : $p(x,z) \leq p(x,y) + p(y,z) - p(y,y)$. (Triangular inequality)

If "p" is a partial metric on X then (X,p) is called a partial metric space.

In partial metric space self distance of a point not necessarily zero. For a partial metric p on X, the function $d_p : X \times X \to \mathbb{R}^+$ defined by $d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ for all $x, y \in X$ is a metric on X.

Example 1.2 ([3, 20])**.** Let $X = [0,\infty)$ define the function $p : [0,\infty) \times [0,\infty) \to [0,\infty)$ by $p(x,y) = \max\{x,y\}$, for all $x, y \in X$. Then (X,p) is a partial metric space and the self-distance $p(x,x) = x$ for every point x of X.

Example 1.3 ([3, 20])**.** Let X be the collection of non-empty closed bounded interval in R, such that $X = \{[a,b] : a \leq b \;\&\; a, b \in \mathbb{R}\}$. Define the function $p : X \times X \to [0,\infty)$ by $p([a,b],[c,d]) = \max\{b,d\} - \min\{a,c\}$, for every element x, y of X. Then (X,p) is a partial metric space and the self-distance of any member of X is $p([a,b],[a,b]) = \max\{b,b\} - \min\{a,a\} = b - a$.

Each partial metric "p" on X generates a T_0 topology τ_p on X for which the collection

$$\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$$

of all open balls forms a base. Where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for each $\varepsilon > 0$ and $x \in X$.

Remark 1.4. It is obvious from the definition of partial metric that if $p(x,y) = 0$, then $x = y$. But if $x = y$, then $p(x,y)$ may not be zero.

Definition 1.5 ([1, 3, 22])**.**

1. A sequence $\{x_n\}$ in a partial metric space (X,p) converges to the limit $x \in X$ if and only if $\lim\limits_{n\to\infty} p(x,x_n) = p(x,x)$.
2. A sequence $\{x_n\}$ in a partial metric space (X,p) is called Cauchy if and only if $\lim\limits_{m,n\to\infty} p(x_m,x_n)$ exists and is finite.
3. A partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $\lim\limits_{n,m\to\infty} p(x_m,x_n) = p(x,x)$.

Lemma 1.6 ([1, 3, 22])**.**

1. *A sequence $\{x_n\}$ is a Cauchy sequence in a partial metric space (X,p) if and only if it is a Cauchy sequence in the metric space (X,d_p).*
2. *A partial metric space (X,p) is complete if and only if the metric space (X,d_p) is complete. Moreover, $\lim\limits_{n\to\infty} d_p(x,x_n) = 0$, if and only if $p(x,x) = \lim\limits_{n\to\infty} p(x,x_n) = \lim\limits_{n,m\to\infty} p(x_n,x_m)$, where x is the limit of $\{x_n\}$ in (X,d_p).*

3. *Let (X,p) be a complete partial metric space. Then*
   (a) If $p(x,y)=0$, then $x=y$.
   (b) If $x\neq y$, then $p(x,y)>0$.
4. *Let (X,p) be a partial metric space. Assume that the sequence $\{x_n\}$ is converging to z as $n\to\infty$. such that $p(z,z)=0$. Then $\lim\limits_{n\to\infty} p(x_n,y)=p(z,y)$ for all elements y of X.*

Definition 1.7 ([12])**.** Let A and B be non-empty subsets of a metric space (X,d) and $F: A\cup B\to A\cup B$. F is called cyclic map if $F(A)\subset B$ and $F(B)\subset A$.

In order to prove our main result we shall need the following lemma.

Lemma 1.8 ([23, 24])**.** *Let $\phi:[0,\infty)\to[0,\infty)$ be non-decreasing and let $t>0$. If $\lim\limits_{n\to\infty}\phi^n(t)=0$, then $\phi(t)<t$.*

2. Main Result

In this section we establish a fixed point result involving generalized contraction defined on cyclic mappings in setting of partial metric spaces.

Theorem 2.1. *Let A and B be non-empty closed subsets of a complete partial metric space (X,p). Suppose that $F: A\cup B\to A\cup B$ is a cyclic map and the condition*

$$p(Fx,Fy)\leq\phi(M(x,y)), \tag{2.1}$$

is satisfied for all $x\in A$ and $y\in B$, where

$$M(x,y)=\max\left\{p(x,y),p(x,Fx),p(y,Fy),\frac{1}{2}[p(x,Fy)+p(y,Fx)]\right\}, \tag{2.2}$$

and $\phi:[0,\infty)\to[0,\infty)$ is a non-decreasing function such that $\lim\limits_{n\to\infty}\phi^n(t)=0$ for all $t>0$. Then F has a unique fixed point in $A\cap B$.

Proof. Let $x_0\in A$ be an arbitrary point and define the sequence $\{x_n\}$ as $x_n=Fx_{n-1}$ for all $n\in N$. Since F is cyclic map so the subsequence

$$\{x_{2k}\}\subset A \text{ and } \{x_{2k+1}\}\subset B. \tag{2.3}$$

If $x_{l+1}=x_l$ for some natural number l then x_l is the required fixed point. Assume that $x_{n+1}\neq x_n$ for all $n\in\mathbb{N}$. Suppose that n is even that is $n=2k$. Substituting $x=x_{2k}$ and $y=x_{2k+1}$ in (2.1), we have

$$\begin{aligned} p(Fx_{2k},Fx_{2k+1})\leq\phi(\max\{&p(x_{2k},x_{2k+1}),p(x_{2k},Fx_{2k}),p(x_{2k+1},Fx_{2k+1}),\\ &\frac{1}{2}[p(x_{2k},Fx_{2k+1})+p(x_{2k+1},Fx_{2k})]\}) \end{aligned}$$

and

$$\begin{aligned} p(x_{2k+1},x_{2k+2})\leq\phi(\max\{&p(x_{2k},x_{2k+1}),p(x_{2k+1},x_{2k+2}),\\ &\frac{1}{2}[p(x_{2k},x_{2k+2})+p(x_{2k+1},x_{2k+1})]\}). \end{aligned} \tag{2.4}$$

From the triangular inequality

$$p(x_{2k},x_{2k+2})+p(x_{2k+1},x_{2k+1})\leq p(x_{2k},x_{2k+1})+p(x_{2k+1},x_{2k+2}),$$

so

$$\max\left\{p\left(x_{2k},x_{2k+1}\right),p\left(x_{2k+1},x_{2k+2}\right),\frac{1}{2}\left[p\left(x_{2k},x_{2k+2}\right)+p\left(x_{2k+1},x_{2k+1}\right)\right]\right\}$$
$$\leq\max\left\{p\left(x_{2k},x_{2k+1}\right),p\left(x_{2k+1},x_{2k+2}\right)\right\}.$$

Using this in (2.4) we have

$$p\left(x_{2k+1},x_{2k+2}\right)\leq\phi\left(\max\left\{p\left(x_{2k},x_{2k+1}\right),p\left(x_{2k+1},x_{2k+2}\right)\right\}\right). \tag{2.5}$$

If

$$\max\left\{p\left(x_{2k},x_{2k+1}\right),p\left(x_{2k+1},x_{2k+2}\right)\right\}=p\left(x_{2k+1},x_{2k+2}\right),$$

then inequality (2.5) becomes $p\left(x_{2k+1},x_{2k+2}\right)\leq\phi\left(p\left(x_{2k+1},x_{2k+2}\right)\right)<p\left(x_{2k+1},x_{2k+2}\right)$, (by lemma 1.8) which is a contradiction. Therefore

$$\max\left\{p\left(x_{2k},x_{2k+1}\right),p\left(x_{2k+1},x_{2k+2}\right)\right\}=p\left(x_{2k},x_{2k+1}\right)$$

and (2.5) becomes

$$p\left(x_{2k+1},x_{2k+2}\right)\leq\phi\left(p\left(x_{2k},x_{2k+1}\right)\right),$$

for all $k\in\mathbb{N}$, since ϕ is non-decreasing, we deduce that

$$p\left(x_{2k+1},x_{2k+2}\right)\leq\phi^{2k}\left(p\left(x_0,x_1\right)\right),\ \text{for all } k\in\mathbb{N}. \tag{2.6}$$

Now, assume that n is odd that is $n=2k+1$. Then the inequality (2.1) with $x=x_{2k+1}$ and $y=x_{2k+2}$ becomes

$$p\left(Fx_{2k+1},Fx_{2k+2}\right)\leq\phi\left(\max\left\{\begin{array}{l}p\left(x_{2k+1},x_{2k+2}\right),p\left(x_{2k+1},Fx_{2k+1}\right),p\left(x_{2k+2},Fx_{2k+2}\right)\\ ,\frac{1}{2}\left[p\left(x_{2k+1},Fx_{2k+2}\right)+p\left(x_{2k+2},Fx_{2k+1}\right)\right]\end{array}\right\}\right)$$

and

$$p\left(x_{2k+2},x_{2k+3}\right)\leq\phi\left(\max\left\{p\left(x_{2k+1},x_{2k+2}\right),p\left(x_{2k+2},x_{2k+3}\right),\frac{1}{2}\left[p\left(x_{2k+1},x_{2k+3}\right)+p\left(x_{2k+2},x_{2k+2}\right)\right]\right\}\right). \tag{2.7}$$

Again from the triangular inequality we have

$$p\left(x_{2k+1},x_{2k+3}\right)+p\left(x_{2k+2},x_{2k+2}\right)\leq p\left(x_{2k+1},x_{2k+2}\right)+p\left(x_{2k+2},x_{2k+3}\right).$$

Therefore

$$\max\left\{p\left(x_{2k+1},x_{2k+2}\right),p\left(x_{2k+2},x_{2k+3}\right),\frac{1}{2}\left[p\left(x_{2k+1},x_{2k+3}\right)+p\left(x_{2k+2},x_{2k+2}\right)\right]\right\}$$
$$\leq\max\left\{p\left(x_{2k+1},x_{2k+2}\right),p\left(x_{2k+2},x_{2k+3}\right)\right\}.$$

Using this in (2.7) we get

$$p\left(x_{2k+2},x_{2k+3}\right)\leq\phi\left(\max\left\{p\left(x_{2k+1},x_{2k+2}\right),p\left(x_{2k+2},x_{2k+3}\right)\right\}\right). \tag{2.8}$$

If $\max\left\{p\left(x_{2k+1},x_{2k+2}\right),p\left(x_{2k+2},x_{2k+3}\right)\right\}=p\left(x_{2k+2},x_{2k+3}\right)$, then (2.8) becomes

$$p\left(x_{2k+2},x_{2k+3}\right)\leq\phi\left(p\left(x_{2k+2},x_{2k+3}\right)\right)<p\left(x_{2k+2},x_{2k+3}\right),\ \text{(by lemma 1.8)}.$$

Which is a contradiction. Therefore,

$$\max\left\{p\left(x_{2k+1},x_{2k+2}\right),p\left(x_{2k+2},x_{2k+3}\right)\right\}=p\left(x_{2k+1},x_{2k+2}\right).$$

Using this in (2.8) we have $p(x_{2k+2}, x_{2k+3}) \leq \phi(p(x_{2k+1}, x_{2k+2}))$, for all $k \in \mathbb{N}$.

Then since ϕ is non-decreasing we obtain

$$p(x_{2k+2}, x_{2k+3}) \leq \phi^{2k+1}(p(x_0, x_1)), \tag{2.9}$$

for all $k \in \mathbb{N}$. Combining (2.6) and (2.9), we get

$$p(x_n, x_{n+1}) \leq \phi^n(p(x_0, x_1)), \text{for all } n \in \mathbb{N}. \tag{2.10}$$

Hence,

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = 0. \tag{2.11}$$

Also by p_1 and p_2

$$\lim_{n\to\infty} p(x_n, x_n) = 0. \tag{2.12}$$

Now we shall prove that $\{x_n\}$ is a Cauchy sequence in (X, d_p), for this firstly we show that $\{x_{2n}\}$ is Cauchy sequence in (X, d_p). By using the definition of d_p

$$\begin{aligned} d_p(x_{2n}, x_{2n+1}) &= 2p(x_{2n}, x_{2n+1}) - p(x_{2n}, x_{2n}) - p(x_{2n+1}, x_{2n+1}) \\ &\leq 2p(x_{2n}, x_{2n+1}) + p(x_{2n}, x_{2n}) + p(x_{2n+1}, x_{2n+1}) \\ &\leq 2p(x_{2n}, x_{2n+1}) + p(x_{2n}, x_{2n+1}) + p(x_{2n}, x_{2n+1}) \\ &= 4p(x_{2n}, x_{2n+1}). \end{aligned}$$

Hence,

$$d_p(x_{2n}, x_{2n+1}) \leq 4p(x_{2n}, x_{2n+1}) \leq 4\phi^{2n}(p(x_0, x_1)). \tag{2.13}$$

From the above inequality we have

$$\lim_{n\to\infty} d_p(x_{2n}, x_{2n+1}) = 0. \tag{2.14}$$

Now, we consider

$$\begin{aligned} d_p(x_{2n+k}, x_{2n}) &\leq d_p(x_{2n+k}, x_{2n+k-1}) + d_p(x_{2n+k-1}, x_{2n+k-2}) + \cdots + d_p(x_{2n-1}, x_{2n}) \\ &\leq 4\phi^{2n+k-1}(p(x_0, x_1)) + 4\phi^{2n+k-2}(p(x_0, x_1)) + ... + 4\phi^{2n-1}(p(x_0, x_1)). \end{aligned}$$

Since $\lim_{n\to\infty} \phi^n(t) = 0$ for all $t > 0$, thus from the above inequality we deduce that $\{x_{2n}\}$ is a cauchy sequence and hence $\{x_{2n}\} \subseteq A$ converges to a point $z \in A$. Using similar arguments we can prove that $\{x_{2n+1}\}$ is a Cauchy sequence in B. Therefore $\{x_{2n+1}\} \subseteq B$ converges to a point $y \in B$. Then,

$$\lim_{n\to\infty} d_p(x_{2n}, z) = 0, \text{ and } \lim_{n\to\infty} d_p(x_{2n+1}, y) = 0. \tag{2.15}$$

It is clear that

$$0 \leq d_p(z, y) \leq d_p(z, x_{2n}) + d_p(x_{2n}, x_{2n+1}) + d_p(x_{2n+1}, y)$$

by taking limit as $n \to \infty$, and using (2.14) and (2.15), we get $d_p(z, y) = 0$ which implies that $z = y$. Thus both sequences converge to the same limit z and moreover $\{x_{2n}\} \cup \{x_{2n+1}\} = \{x_n\}$. Hence, $\{x_n\} \in X$ converges to $z \in X$. By using lemma 1.6 (ii) we have $\lim_{n\to\infty} d_p(x_n, z) = 0$ if and only if

$$p(z, z) = \lim_{n\to\infty} p(z, x_n) = \lim_{m,n\to\infty} p(x_n, x_m). \tag{2.16}$$

Suppose that $p(z, z) \neq 0$, then $p(z, z) > 0$. Applying (2.1) with $x = x_n$ and $y = x_m$, we have

$$p(x_{n+1}, x_{m+1}) \leq \phi\Big(\max\Big\{ p(x_n, x_m), p(x_n, x_{n+1}), p(x_m, x_{m+1}),$$

$$\frac{1}{2}\left[p\left(x_n, x_{m+1}\right)+p\left(x_m, x_{n+1}\right)\right]\Big\}\Big).$$

Letting $n, m \rightarrow \infty$, using (2.11) and (2.16), we get

$$p(z, z) \leq \phi(p(z, z))<p(z, z),$$

which is a contradiction hence, $p(z, z)=0$. Also from (2.16), we have

$$p(z, z)=\lim_{n \rightarrow \infty} p\left(z, x_n\right)=\lim_{m, n \rightarrow \infty} p\left(x_n, x_m\right)=0. \tag{2.17}$$

Now, we show that z is the fixed point of F. Assume the contrary that $p(z, F z)>0$. It follows that there is $n_0 \in N$ such that for all $n>n_0$

$$\max \left\{p\left(x_{n-1}, z\right), p\left(x_{n-1}, x_n\right), p(z, F z), \frac{1}{2}\left[p\left(x_{n-1}, F z\right)+p\left(z, x_n\right)\right]\right\} \leq p(z, F z). \tag{2.18}$$

Consider (2.1) with $x=x_{n-1}$ and $y=z$, then we have

$$\begin{aligned} p\left(x_n, F z\right) & \leq \phi\left(\max \left\{p\left(x_{n-1}, z\right), p\left(x_{n-1}, x_n\right), p(z, F z), \frac{1}{2}\left[p\left(x_{n-1}, F z\right)+p\left(z, x_n\right)\right]\right\}\right) \\ & \leq \phi(p(z, F z))<p(z, F z). \end{aligned}$$

Taking limit as $n \rightarrow \infty$, in the above inequality we obtain

$$\lim_{n \rightarrow \infty} p\left(x_n, F z\right) \leq p(z, F z). \tag{2.19}$$

Also for each $n>n_0$,

$$p(z, F z) \leq p\left(z, x_n\right)+p\left(x_n, F z\right)-p\left(x_n, x_n\right) \leq p\left(z, x_n\right)+p\left(x_n, F z\right).$$

Taking limit as $n \rightarrow \infty$, in the above inequality, and taking into account (2.19), we get $p(z, F z) \leq \phi(p(z, F z))<p(z, F z)$, which forces $p(z, F z)=0$, according to lemma 1.6 (iii) $F z=z$, that is $z \in A \cap B$ is the fixed point of F. Now, assume that $z^* \in X$ is another fixed point of F such that $z \neq z^*$. Put $x=z$ and $y=z^*$ in (2.1), we have

$$p\left(F z, F z^*\right) \leq \phi\left(\max \left\{p\left(z, z^*\right), p(z, F z), p\left(z^*, F z^*\right), \frac{1}{2}\left[p\left(z, F z^*\right)+p\left(z^*, F z\right)\right]\right\}\right),$$

which gives $p\left(z, z^*\right) \leq \phi\left(p\left(z, z^*\right)\right)<p\left(z, z^*\right)$, and hence $p\left(z, z^*\right)=0$, by Lemma 1.6 (iii) $z=z^*$. Thus the fixed point of F is unique.

Now we give an example of cyclic map satisfying the conditions of Theorem 2.1. □

Example 2.2. Let $X=[0,1]$. Define the function $p: X \times X \rightarrow R^{+}$ by $p(x, y)=\max \{x, y\}$ then (X, p) is a complete partial metric space. Let $A=B=[0,1]$ and define the mapping $F: A \cup B \rightarrow A \cup B$ by $F x=\frac{x}{3}$ and $\phi:[0, \infty) \rightarrow[0, \infty)$ by $\phi(t)=\frac{t}{2}$. Let $x \in A$ and $y \in B$. Assume that $x \geq y$. $p(F x, F y)=p\left(\frac{x}{3}, \frac{y}{3}\right)=\frac{x}{3}$. Now consider

$$\begin{aligned} & \phi\left(\max \left\{p(x, y), p(x, F x), p(y, F y), \frac{1}{2}[p(x, F y)+p(y, F x)]\right\}\right) \\ & \quad=\phi\left(\max \left\{p(x, y), p\left(x, \frac{x}{3}\right), p\left(y, \frac{y}{3}\right), \frac{1}{2}\left[p\left(x, \frac{y}{3}\right)+p\left(y, \frac{x}{3}\right)\right]\right\}\right)=\phi(x)=\frac{x}{2}. \end{aligned}$$

Hence $\frac{x}{3} \leq \frac{x}{2}$, thus F satisfies all conditions of the Theorem 2.1 so F has a unique fixed point in $A \cap B$, namely ‘0’.

Corollary 2.3. *Let A and B be non-empty closed subsets of a complete partial metric space (X, p). Assume that $F : A \cup B \to A \cup B$ is a cyclic map satisfying*

$$p(Fx, Fy) \leq kmax\left\{p(x, y), p(x, Fx), p(y, Fy), \frac{1}{2}[p(x, Fy) + p(y, Fx)]\right\},$$

for all $x \in A$ and $y \in B$ where $0 \leq k < 1$. Then F has a unique fixed point in $A \cap B$.

Proof. It follows from the Theorem 2.1 by taking $\phi(t) = kt$. □

3. Conclusion

In this work a fixed point theorem for generalized contraction defined on a cyclic map in partial metric space is established.

References

[1] T. Abdeljawed, J. O. Alzabut, A. Mukheimer, Y. Zaidan, *Banach contraction principle for cyclical mappings on partial metric spaces*, Fixed Point Theory Appl., **2012** (2012), 7 pages. 1, 1.1, 1.5, 1.6

[2] T. Abdeljawed, E. Karapiner, K. Tas, *Existence and uniqueness of a common fixed point on partial metric spaces*, Appl. Math. Lett., **24** (2011), 1900–1904. 1

[3] T. Abdeljawed, E. Karapiner, K. Tas, *A generalized contraction principle with control functions on partial metric spaces*, Comput. Math. Appl., **63** (2012), 716–719. 1, 1.1, 1.2, 1.3, 1.5, 1.6

[4] R. P. Agarwal, M. Alghamdi, N. Shahzad, *Fixed point theory for cyclic generalized contractions in partial metric spaces*, Fixed Point Theory Appl., **2012** (2012), 11 pages. 1

[5] I. Altun, A. Erduran, *Fixed point theorem for monotone mappings on partial metric spaces*, Fixed Point Theory Appl., **2011** (2011), 10 pages.

[6] H. Aydi, E. Karapinar, *A Meir-Keeler common type fixed point theorem on partial metric spaces*, Fixed Point Theory Appl., **2012** (2012), 10 pages.

[7] S. Banach, *Sur les oprations dansles ensembles abstraits et leur application aux equations integrals*, Fund. Math., **3** (1922), 133–181. 1

[8] S. Chandok, M. Postolache, *Fixed point theorem for weakly Chatterjea type cyclic contractions*, Fixed Point Theory Appl., **2013** (2013), 9 pages.

[9] K. P. Chi, E. Karapiner, T. D. Thanh, *A generalized contraction principle in partial metric spaces*, Math. Comput. Modelling, **55** (2012), 1673–1681.

[10] E. Karapinar, *Generalizations of Caristi Kirk's theorems on partial metric spaces*, Fixed Point Theory Appl., **2011** (2011), 7 pages.

[11] E. Karapinar, I. M. Erhan, A. Y. Ulus, *Fixed point theorem for cyclic maps on partial metric spaces*, Appl. Math. Inf. Sci., **6** (2012), 239–244. 1

[12] E. Karapiner, H. K. Nashine, *Fixed point theorems for Kannan type cyclic weakly contractions*, J. Nonlinear Anal. Optim., **4** (2013), 29–35. 1, 1.7

[13] E. Karapinar, K. Sadarangani, *Fixed point theory for cyclic $\phi - \psi$contractions*, Fixed Point Theory Appl., **2011** (2011), 8 pages. 1

[14] E. Karapinar, N. Shobkolaei, S. Sedghi, S. M. Vaezpour, *A common fixed point theorem for cyclic operators on partial metric spaces*, Filomat, **26** (2012), 407–414.

[15] E. Karapinar, U. Yuskel, *Some common fixed point theorems in partial metric spaces*, J. Appl. Math., **2011** (2011), 16 pages.

[16] W. A. Kirk, P. S. Srinivasan, P. Veeramani, *Fixed points for mappings satisfying cyclical contractive conditions*, Fixed Point Theory, **4** (2003), 79–89.

[17] S. G. Matthews, *Partial metric topology*, Ann. New York Acad., **728** (1994), 183–197.

[18] S. G. Matthews, *Partial metric spaces*, In Research Report 212, Dept. of Computer Science, University of Warwick, (1992). 1

[19] J. Matkowski, *Integrable solutions of functional equations*, Dissertationes Math., **127** (1975), 1–68.

[20] J. Matkowski, *Fixed point theorems for mappings with a contractive iterate at a point*, Proc. Amer. Math. Soc., **62** (1977), 344–348. 1, 1.2, 1.3

[21] B. Michael, R. Kopperman, S. G. Matthews, H. Pajoohesh, *Partial metric spaces*, Amer. Math. Monthly, **116** (2009), 708–718.

[22] S. Oltra, O. Valero, *Banach's fixed point theorem for partial metric spaces*, Rend. Istit. Math. Univ. Trieste, **36** (2004), 17–26. 1.5, 1.6

[23] M. A. Petric, *Some results concerning cyclical contractive mappings*, General Math., **18** (2010), 213–226. 1.8

[24] O. Valero, *On Banach fixed point theorems for partial metric spaces*, Appl. Gen. Topol., **6** (2005), 229–240. 1.8