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On controllability for nonconvex semilinear differential inclusions

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Abstract

We consider a semilinear differential inclusion and we obtain sufficient conditions for h-local controllability along a reference trajectory.

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1. Introduction

In this paper we are concerned with the following semilinear differential inclusion

$$x' \in Ax + F(t,x), \quad x(0) \in X_0 \tag{1.1}$$

where $F : [0,T] \times X \to \mathcal{P}(X)$ is a set valued map, A is the infinitesimal generator of a C_0-semigroup $\{G(t)\}_{t\geq 0}$ on a separable Banach space X and $X_0 \subset X$. Let S_F be the set of all mild solutions of (1.1) and let $R_F(T)$ be the reachable set of (1.1). For a mild solution $z(.) \in S_F$ and for a locally Lipschitz function $h : X \to X$ we say that the semilinear differential inclusion (1.1) is *h-locally controllable* around $z(.)$ if $h(z(T)) \in \mathrm{int}(h(R_F(T)))$. In particular, if h is the identity mapping the above definitions reduces to the usual concept of local controllability of systems around a solution.

The aim of the present paper is to obtain a sufficient condition for h-local controllability of inclusion (1.1) when X is finite dimensional. This result is derived using a technique developed by Tuan for differential inclusions ([13]). More exactly, we show that inclusion (1.1) is h-locally controlable around the mild solution

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$z(.)$ if a certain linearized inclusion is λ-locally controlable around the null solution for every $\lambda \in \partial h(z(T))$, where $\partial h(.)$ denotes Clarke's generalized Jacobian of the locally Lipschitz function h. The key tools in the proof of our result is a continuous version of Filippov's theorem for mild solutions of semilinear differential inclusions obtained in [2] and a certain generalization of the classical open mapping principle in [14].

Our results may be interpreted as extensions of the results in [13] to semilinear differential inclusions and as extensions of the controllability results in [3] to h-controllability.

We note that existence results and qualitative properties of the mild solutions of problem (1.1) may be found in [2], [3], [4], [5], [6], [8], [9], [10], [12] etc..

The paper is organized as follows: in Section 2 we present some preliminary results to be used in the sequel and in Section 3 we present our main results.

2. Preliminaries

Let denote by I the interval $[0, T]$ and let X be a real separable Banach space with the norm $||.||$ and with the corresponding metric $d(.,.)$. Denote by $\mathcal{L}(I)$ the σ-algebra of all Lebesgue measurable subsets of I, by $\mathcal{P}(X)$ the family of all nonempty subsets of X and by $\mathcal{B}(X)$ the family of all Borel subsets of X. Recall that the Pompeiu-Hausdorff distance of the closed subsets $A, B \subset X$ is defined by

$$d_H(A,B) = \max\{d^*(A,B), d^*(B,A)\}, \quad d^*(A,B) = \sup\{d(a,B); a \in A\},$$

where $d(x,B) = \inf_{y\in B} d(x,y)$.

As usual, we denote by $C(I,X)$ the Banach space of all continuous functions $x(.) : I \to X$ endowed with the norm $||x(.)||_C = \sup_{t\in I} ||x(t)||$ and by $L^1(I,X)$ the Banach space of all (Bochner) integrable functions $x(.) : I \to X$ endowed with the norm $||x(.)||_1 = \int_I ||x(t)||dt$.

We consider $\{G(t)\}_{t\geq 0} \subset L(X,X)$ a strongly continuous semigroup of bounded linear operators from X to X having the infinitesimal generator A and a set valued map $F(.,.)$ defined on $I \times X$ with nonempty closed subsets of X, which define the following differential inclusion:

$$x'(t) \in Ax(t) + F(t,x(t)) \quad a.e.\,(I) \quad x(0) = x_0 \tag{2.1}$$

It is well known that, in general, the Cauchy problem

$$x' = Ax + f(t,x),\ f(t,x) \in F(t,x), \quad x(0) = x_0 \tag{2.2}$$

may not have a classical solution and that a way to overcome this difficulty is to look for continuous solutions of the integral equation

$$x(t) = G(t)x_0 + \int_0^t G(t-u)f(u,x(u))du.$$

This is why the concept of the mild solution is convenient for solving (2.1)

A mapping $x(.) \in C(I,X)$ is called a *mild solution* of (2.1) if there exists a (Bochner) integrable function $f(.) \in L^1(I,X)$ such that

$$f(t) \in F(t,x(t)) \quad a.e.\,(I), \tag{2.3}$$

$$x(t) = G(t)x_0 + \int_0^t G(t-u)f(u)du \quad \forall t \in I, \tag{2.4}$$

i.e., $f(.)$ is a locally (Bochner) integrable selection of the set-valued map $F(.,x(.))$ and $x(.)$ is the mild solution of the initial value problem

$$x'(t) = Ax(t) + f(t), \quad x(0) = x_0. \tag{2.5}$$

We shall call $(x(.), f(.))$ a *trajectory-selection pair* of (2.1) if $f(.)$ verifies (2.3) and $x(.)$ is a mild solution of (2.5).

Hypothesis 2.1. i) *$F(.,.) : I \times X \to \mathcal{P}(X)$ has nonempty closed values and is $\mathcal{L}(I) \otimes \mathcal{B}(X)$ measurable.*

ii) *There exists $L(.) \in L^1(I, \mathbf{R}_+)$ such that, for any $t \in I, F(t,.)$ is $L(t)$-Lipschitz in the sense that*

$$d_H(F(t,x_1), F(t,x_2)) \le L(t)||x_1 - x_2|| \quad \forall\, x_1, x_2 \in X.$$

In the theorem to follow, S is a separable metric space, $X_0 \subset X$, $a(.) : S \to X_0$ and $c(.) : S \to (0, \infty)$ are given continuous mappings.

Hypothesis 2.2. *The continuous mappings $g(.) : S \to L^1(I, X)$, $y(.) : S \to C(I, X)$ are given such that*

$$(y(s))'(t) = Ay(s)(t) + g(s)(t), \quad t \in I, \quad y(s)(0) \in X_0.$$

There exists a continuous function $p(.) : S \to L^1(I, \mathbf{R}_+)$ such that

$$d(g(s)(t), F(t, y(s)(t))) \le p(s)(t) \quad a.e.\,(I),\ \forall\, s \in S.$$

Theorem 2.1. *Assume that Hypotheses 2.1 and 2.2 are satisfied.*

Then there exist $M > 0$ and the continuous functions $x(.) : S \to L^1(I, X)$, $h(.) : S \to C(I, X)$ such that for any $s \in S$ $(x(s)(.), h(s)(.))$ is a trajectory-selection of (1.1) satisfying for any $(t, s) \in I \times S$

$$x(s)(0) = a(s),$$

$$||x(s)(t) - y(s)(t)|| \le M[c(s) + ||a(s) - y(s)(0)|| + \int_0^t p(s)(u)du].$$

The proof of Theorem 2.1 may be found in [2].

In what follows we assume that $X = \mathbf{R}^n$. We recall that if $X = \mathbf{R}^n$ then (2.5) is a Cauchy problem associated to an affine (linear nonhomogenous) differential equation and its solution (2.4) is obtained with the variation of constants method. In this case $G(t) = \exp(tA)$, $A \in L(\mathbf{R}^n, \mathbf{R}^n)$, $t \in I$.

A closed convex cone $C \subset \mathbf{R}^n$ is said to be *regular tangent cone* to the set X at $x \in X$ ([11]) if there exists continuous mappings $q_\lambda : C \cap B \to \mathbf{R}^n$, $\forall \lambda > 0$ satisfying

$$\lim_{\lambda \to 0+} \max_{v \in C \cap B} \frac{||q_\lambda(v)||}{\lambda} = 0,$$

$$x + \lambda v + q_\lambda(v) \in X \quad \forall \lambda > 0, v \in C \cap B.$$

From the multitude of the intrinsic tangent cones in the literature (e.g. [1]) the *contingent*, the *quasitangent* and *Clarke's tangent cones*, defined, respectively, by

$$\begin{aligned} K_xX &= \{v \in \mathbf{R}^n; \quad \exists\, s_m \to 0+,\ x_m \in X : \tfrac{x_m - x}{s_m} \to v\} \\ Q_xX &= \{v \in \mathbf{R}^n; \quad \forall\, s_m \to 0+, \exists x_m \in X : \tfrac{x_m - x}{s_m} \to v\} \\ C_xX &= \{v \in \mathbf{R}^n; \forall\, (x_m, s_m) \to (x, 0+),\ x_m \in X,\ \exists\, y_m \in X : \tfrac{y_m - x_m}{s_m} \to v\} \end{aligned}$$

seem to be among the most often used in the study of different problems involving nonsmooth sets and mappings. We recall that, in contrast with K_xX, Q_xX, the cone C_xX is convex and one has $C_xX \subset Q_xX \subset K_xX$.

The results in the next section will be expressed, in the case when the mapping $g(.) : X \subset \mathbf{R}^n \to \mathbf{R}^m$ is locally Lipschitz at x, in terms of the Clarke generalized Jacobian, defined by ([7])

$$\partial g(x) = \mathrm{co}\{\lim_{i \to \infty} g'(x_i); \quad x_i \to x, \quad x_i \in X \backslash \Omega_g\},$$

where Ω_g is the set of points at which g is not differentiable.

Corresponding to each type of tangent cone, say $\tau_x X$ one may introduce (e.g. [1]) a *set-valued directional derivative* of a multifunction $G(.) : X \subset \mathbf{R}^n \to \mathcal{P}(\mathbf{R}^n)$ (in particular of a single-valued mapping) at a point $(x, y) \in \mathrm{graph}(G)$ as follows

$$\tau_y G(x; v) = \{w \in \mathbf{R}^n;\ (v, w) \in \tau_{(x,y)}\mathrm{graph}(G)\}, \quad \in \tau_x X.$$

We recall that a set-valued map, $A(.) : \mathbf{R}^n \to \mathcal{P}(\mathbf{R}^n)$ is said to be a *convex* (respectively, closed convex) *process* if $\mathrm{graph}(A(.)) \subset \mathbf{R}^n \times \mathbf{R}^n$ is a convex (respectively, closed convex) cone. For the basic properties of convex processes we refer to [1], but we shall use here only the above definition.

Hypothesis 2.3. i) *Hypothesis 2.1 is satisfied and $X_0 \subset \mathbf{R}^n$ is a closed set.*

ii) $(z(.), f(.)) \in C(I, \mathbf{R}^n) \times L^1(I, \mathbf{R}^n)$ *is a trajectory-selection pair of (1.1) and a family $P(t, .) : \mathbf{R}^n \to \mathcal{P}(\mathbf{R}^n)$, $t \in I$ of convex processes satisfying the condition*

$$P(t, u) \subset Q_{f(t)}F(t, .)(z(t); u) \quad \forall\, u \in dom(P(t, .)),\ a.e.\ t \in I \tag{2.6}$$

is assumed to be given and defines the variational inclusion

$$v' \in Av + P(t, v). \tag{2.7}$$

We note that for any set-valued map $F(., .)$, one may find an infinite number of families of convex process $P(t, .)$, $t \in I$, satisfying condition (2.6); in fact any family of closed convex subcones of the quasitangent cones, $\overline{P}(t) \subset Q_{(z(t),f(t))}graph(F(t, .))$, defines the family of closed convex process

$$P(t, u) = \{v \in \mathbf{R}^n;\ (u, v) \in \overline{P}(t)\}, \quad u, v \in \mathbf{R}^n,\ t \in I$$

that satisfy condition (2.6). One is tempted, of course, to take as an "intrinsic" family of such closed convex process, for example Clarke's convex-valued directional derivatives $C_{f(t)}F(t, .)(z(t); .)$.

We recall (e.g. [1]) that, since $F(t, .)$ is assumed to be Lipschitz a.e. on I, the quasitangent directional derivative is given by

$$Q_{f(t)}F(t, .)((z(t); u)) = \{w \in \mathbf{R}^n;\ \lim_{\theta \to 0+} \frac{1}{\theta} d(f(t) + \theta w, F(t, z(t) + \theta u)) = 0\}. \tag{2.8}$$

In what follows B or $B_{\mathbf{R}^n}$ denotes the closed unit ball in $\mathbf{R}^n$ and 0_n denotes the null element in $\mathbf{R}^n$.

Consider $h : \mathbf{R}^n \to \mathbf{R}^m$ an arbitrary given function. Inclusion (1.1) is said to be *h-locally controllable* around $z(.)$ if $h(z(T)) \in \mathrm{int}(h(R_F(T)))$. Inclusion (1.1) is said to be *locally controllable* around the solution $z(.)$ if $z(T) \in \mathrm{int}(R_F(T))$.

Finally a key tool in the proof of our results is the following generalization of the classical open mapping principle due to Warga ([14]).

For $k \in \mathbf{N}$ we define

$$\Sigma_k := \{\gamma = (\gamma_1, ..., \gamma_k); \quad \sum_{i=1}^{k} \gamma_i \le 1, \quad \gamma_i \ge 0,\ i = 1, 2, ..., k\}.$$

Lemma 2.2. *Let $\delta \le 1$, let $g(.) : \mathbf{R}^n \to \mathbf{R}^m$ be a mapping that is C^1 in a neighborhood of 0_n containing $\delta B_{\mathbf{R}^n}$. Assume that there exists $\beta > 0$ such that for every $\theta \in \delta\Sigma_n$, $\beta B_{\mathbf{R}^m} \subset g'(\theta)\Sigma_n$. Then, for any continuous mapping $\psi : \delta\Sigma_n \to \mathbf{R}^m$ that satisfies $\sup_{\theta \in \delta\Sigma_n} ||g(\theta) - \psi(\theta)|| \le \frac{\delta\beta}{32}$ we have $\psi(0_n) + \frac{\delta\beta}{16} B_{\mathbf{R}^m} \subset \psi(\delta\Sigma_n)$.*

3. The main result

In what follows C_0 is a regular tangent cone to X_0 at $z(0)$, denote by S_P the set of all mild solutions of the semilinear differential inclusion

$$v' \in Av + P(t,v), \quad v(0) \in C_0$$

and by $R_P(T) = \{x(T);\ x(.) \in S_P\}$ its reachable set at time T.

Theorem 3.1. *Assume that Hypothesis 2.3 is satisfied and let $h : \mathbf{R}^n \to \mathbf{R}^m$ be a Lipschitz function with Lipschitz constant $l > 0$.*

Then inclusion (1.1) is h-local controllable around the solution $z(.)$ if

$$0_m \in int(\lambda R_P(T)) \quad \forall \lambda \in \partial h(z(T)). \tag{3.1}$$

Proof. By (3.1), since $\lambda R_P(T)$ is a convex cone, it follows that $\lambda R_P(T) = \mathbf{R}^m$ $\forall \lambda \in \partial f(z(T))$. Therefore using the compactness of $\partial f(z(T))$ (e.g. [7]), we have that for every $\beta > 0$ there exist $k \in \mathbf{N}$ and $u_j \in R_P(T)$ $j = 1, 2, ..., k$ such that

$$\beta B_{\mathbf{R}^m} \subset \lambda(u(\Sigma_k)) \quad \forall \lambda \in \partial f(z(T)), \tag{3.2}$$

where

$$u(\Sigma_k) = \{u(\gamma) := \sum_{j=1}^k \gamma_j u_j, \quad \gamma = (\gamma_1, ..., \gamma_k) \in \Sigma_k\}.$$

Using an usual separation theorem we deduce the existence of $\beta_1, \rho_1 > 0$ such that for all $\lambda \in L(\mathbf{R}^n, \mathbf{R}^m)$ with $d(\lambda, \partial f(z(T))) \le \rho_1$ we have

$$\beta_1 B_{\mathbf{R}^m} \subset \lambda(u(\Sigma_k)). \tag{3.3}$$

Since $u_j \in R_P(T)$, $j = 1, ..., k$, there exist $(w_j(.), g_j(.))$, $j = 1, ..., k$ trajectory-selection pairs of (2.7) such that $u_j = w_j(T)$, $j = 1, ..., k$. We note that $\beta > 0$ can be take small enough such that $||w_j(0)|| \le 1$, $j = 1, ..., k$.

Define

$$w(t,s) = \sum_{j=1}^k s_j w_j(t), \quad \overline{g}(t,s) = \sum_{j=1}^k s_j g_j(t), \quad \forall s = (s_1, ..., s_k) \in \mathbf{R}^k.$$

Obviously, $w(.,s) \in S_P$, $\forall s \in \Sigma_k$.

Taking into account the definition of C_0, for every $\varepsilon > 0$ there exists a continuous mapping $o_\varepsilon : \Sigma_k \to \mathbf{R}^n$ such that

$$z(0) + \varepsilon w(0,s) + o_\varepsilon(s) \in X_0, \tag{3.4}$$

$$\lim_{\varepsilon \to 0+} \max_{s \in \Sigma_k} \frac{||o_\varepsilon(s)||}{\varepsilon} = 0. \tag{3.5}$$

Define

$$p_\varepsilon(s)(t) := \frac{1}{\varepsilon} \mathrm{d}(\overline{g}(t,s), F(t, z(t) + \varepsilon w(t,s)) - f(t)),$$

$$q(t) := \sum_{j=1}^k [||g_j(t)|| + L(t)||w_j(t)||], \quad t \in I.$$

Then, for every $s \in \Sigma_k$ one has

$$p_\varepsilon(s)(t) \le ||\overline{g}(t,s)|| + \tfrac{1}{\varepsilon} \mathrm{d}_H(0_n, F(t, z(t) + \varepsilon w(t,s)) - f(t)) \le ||\overline{g}(t,s)|| +$$
$$\tfrac{1}{\varepsilon} \mathrm{d}_H(F(t, z(t)), F(t, z(t) + \varepsilon w(t,s))) \le ||\overline{g}(t,s)|| + L(t)||w(t,s)|| \le q(t).$$

Next, if $s_1, s_2 \in \Sigma_k$ one has

$$|p_\varepsilon(s_1)(t) - p_\varepsilon(s_2)(t)| \leq ||\overline{g}(t,s_1) - \overline{g}(t,s_2)|| + \tfrac{1}{\varepsilon}\mathrm{d}_H(F(t,z(t)+\varepsilon w(t,s_1)),$$
$$F(t,z(t)+\varepsilon w(t,s_2))) \leq ||s_1 - s_2||.\max_{j=\overline{1,k}}[||g_j(t)|| + L(t)||w_j(t)||],$$

thus $p_\varepsilon(.)(t)$ is Lipschitz with a Lipschitz constant not depending on ε.

On the other hand, from (2.8) it follows that

$$\lim_{\varepsilon\to 0} p_\varepsilon(s)(t) = 0 \quad a.e.\,(I), \quad \forall s \in \Sigma_k$$

and hence

$$\lim_{\varepsilon\to 0+}\max_{s\in\Sigma_k} p_\varepsilon(s)(t) = 0 \quad a.e.\ (I). \tag{3.7}$$

Therefore, from (3.6), (3.7) and Lebesgue dominated convergence theorem we obtain

$$\lim_{\varepsilon\to 0+}\int_0^T \max_{s\in\Sigma_k} p_\varepsilon(s)(t)\mathrm{d}t = 0. \tag{3.8}$$

By (3.4), (3.5), (3.8) and the upper semicontinuity of the Clarke generalized Jacobian we can find $\varepsilon_0, e_0 > 0$ such that

$$\max_{s\in\Sigma_k}\frac{||o_{\varepsilon_0}(s)||}{\varepsilon_0} + \int_0^T \max_{s\in\Sigma_k} p_{\varepsilon_0}(s)(t)\mathrm{d}t \leq \frac{\beta_1}{2^8 l^2}, \tag{3.9}$$

$$\varepsilon_0 w(T,s) \leq \frac{e_0}{2} \quad \forall s \in \Sigma_k. \tag{3.10}$$

If we define

$$y(s)(t) := z(t) + \varepsilon_0 w(t,s), \quad g(s)(t) := f(t) + \varepsilon_0\overline{g}(t,s) \quad s \in \mathbf{R}^k,$$
$$a(s) := z(0) + \varepsilon_0 w(0,s) + o_{\varepsilon_0}(s), \quad s \in \mathbf{R}^k,$$

then we apply Theorem 2.1 and we find that there exists the continuous function $x(.) : \Sigma_k \to C(I, \mathbf{R}^n)$ such that for any $s \in \Sigma_k$ the function $x(s)(.)$ is solution of the differential inclusion $x' \in Ax + F(t,x), \quad x(s)(0) = a(s)\ \forall s \in \Sigma_k$ and one has

$$||x(s)(T) - y(s)(T)|| \leq \frac{\varepsilon_0\beta_1}{2^6 l} \quad \forall s \in \Sigma_k. \tag{3.11}$$

We define

$$h_0(x) := \int_{\mathbf{R}^n} h(x - by)\chi(y)\mathrm{d}y, \quad x \in \mathbf{R}^n,$$
$$\phi(s) := h_0(z(T) + \varepsilon_0 w(T,s)),$$

where $\chi(.) : \mathbf{R}^n \to [0,1]$ is a C^∞ function with the support contained in $B_{\mathbf{R}^n}$ that satisfies $\int_{\mathbf{R}^n}\chi(y)\mathrm{d}y = 1$ and $b = \min\{\frac{e_0}{2}, \frac{\varepsilon_0\beta_1}{2^6 l}\}$.

Therefore $h_0(.)$ is of class C^∞ and verifies

$$||h(x) - h_0(x)|| \leq lb, \tag{3.12}$$

$$h_0'(x) = \int_{\mathbf{R}^n} h'(x - by)\chi(y)\mathrm{d}y. \tag{3.13}$$

In particular

$$h_0'(x) \in \overline{\mathrm{co}}\{h'(u); \quad ||u - x|| \leq b, \quad h'(u)\text{ exists}\},$$
$$\phi'(s)\mu = h_0'(z(T) + \varepsilon_0 w(T,\mu)) \quad \forall \mu \in \Sigma_k.$$

Using again the upper semicontinuity of Clarke's generalized Jacobian we obtain

$$\mathrm{d}(h_0'(z(T) + \varepsilon_0 w(T,s)), \partial h(z(T))) \leq \sup\{\mathrm{d}(h_0'(u), \partial h(z(T)));\ ||u - z(t)||$$
$$\leq ||u - (z(T) + \varepsilon_0 w(T,s))|| + ||\varepsilon_0 w(t,s)|| \leq e_0, \quad h'(u)\ \text{exists}\} < \rho_1.$$

The last inequality with (3.3) gives

$$\varepsilon_0\beta_1 B_{\mathbf{R}^m} \subset \phi'(s)\Sigma_k \quad \forall s \in \Sigma_k.$$

Finally, for $s \in \Sigma_k$, we put $\psi(s) = h(x(s)(T))$.
Obviously, $\psi(.)$ is continuous and from (3.11), (3.12), (3.13) one has

$$||\psi(s) - \phi(s)|| = ||h(x(s)(T)) - h_0(y(s)(T))|| \leq ||h(x(s)(T)) - h(y(s)(T))|| +$$
$$||h(y(s)(T)) - h_0(y(s)(T))|| \leq l||x(s)(T) - y(s)(T)|| + lb \leq \tfrac{\varepsilon_0\beta_1}{64} + \tfrac{\varepsilon_0\beta_1}{64} = \tfrac{\varepsilon_0\beta_1}{32}.$$

We apply Lemma 2.2 and we find that

$$h(x(0_k)(T)) + \frac{\varepsilon_0\beta_1}{16} B_{\mathbf{R}^m} \subset \psi(\Sigma_k) \subset h(R_F(T)).$$

On the other hand, $||h(z(T)) - h(x(0_k)(T))|| \leq \frac{\varepsilon_0\beta_1}{64}$, so we have $h(z(T)) \in \mathrm{int}(R_F(T))$ and the proof is complete. □

Remark 3.2. If in Theorem 3.1, $A \equiv 0$, then the semilinear differential inclusion (1.1) reduces to the classical differential inclusion

$$x' \in F(t,x), \quad x(0) \in X_0. \tag{3.14}$$

A similar result to the one in Theorem 3.1 for problem (3.14) may be found in [13]. On the other hand, if $m = n$ and $h(x) \equiv x$, Theorem 3.1 yields Theorem 3.4 in [3].

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