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Hermite–Hadamard type inequalities for the product of (α, m)-convex functions

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Abstract

In the paper, the authors establish some Hermite–Hadamard type inequalities for the product of two (α, m)-convex functions.

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1. Introduction

The following definitions are well known in the literature.

Definition 1. *A function $f : I \subseteq \mathbb{R} = (-\infty, \infty) \to \mathbb{R}$ is said to be convex if*

$$f(\lambda x + (1 - \lambda)y) \le \lambda f(x) + (1 - \lambda)f(y) \tag{1.1}$$

holds for all $x, y \in I$ and $\lambda \in [0, 1]$.

Definition 2 ([7])**.** *For $f : [0, b] \to \mathbb{R}$ and $m \in (0, 1]$, if*

$$f(tx + m(1 - t)y) \le tf(x) + m(1 - t)f(y) \tag{1.2}$$

is valid for all $x, y \in [0, b]$ and $t \in [0, 1]$, then we say that $f(x)$ is m-convex on $[0, b]$.

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Definition 3 ([4])**.** *For $f:[0,b]\to\mathbb{R}$ and $(\alpha,m)\in(0,1]\times(0,1]$, if*

$$f(tx+m(1-t)y)\le t^\alpha f(x)+m(1-t^\alpha)f(y) \tag{1.3}$$

is valid for all $x,y\in[0,b]$ and $t\in[0,1]$, then we say that $f(x)$ is (α,m)-convex on $[0,b]$.

In recent decades, many inequalities of the Hermite–Hadamard type for various kinds of convex functions have been established. Some of them may be recited as follows.

Theorem 1.1 ([3])**.** *Let $f:[a,b]\subseteq\mathbb{R}_0=[0,\infty)\to\mathbb{R}$ be m-convex for fixed $m\in(0,1]$. Then*

$$\frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}\,x\le\min\left\{\frac{f(a)+mf(b/m)}{2},\frac{mf(a/m)+f(b)}{2}\right\}. \tag{1.4}$$

Theorem 1.2 ([5])**.** *Let $f,g:[a,b]\subseteq\mathbb{R}\to\mathbb{R}_0$ be convex functions. Then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x\le\frac{1}{3}M(a,b)+\frac{1}{6}N(a,b), \tag{1.5}$$

where $M(a,b)=f(a)g(a)+f(b)g(b)$ and $N(a,b)=f(a)g(b)+f(b)g(a)$.

Theorem 1.3 ([2])**.** *Let $f,g:\mathbb{R}_0\to\mathbb{R}_0$ satisfy $fg\in L([a,b])$, where $0\le a<b<\infty$. If f is m_1-convex and g is m_2-convex on $[a,b]$ for some fixed $m_1,m_2\in(0,1]$, then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x\le\min\{M_1,M_2\}, \tag{1.6}$$

where

$$M_1=\frac{1}{3}\left[f(a)g(a)+m_1m_2f\left(\frac{b}{m_1}\right)g\left(\frac{b}{m_2}\right)\right]+\frac{1}{6}\left[m_2f(a)g\left(\frac{b}{m_2}\right)+m_1f\left(\frac{b}{m_1}\right)g(a)\right]$$

and

$$M_2=\frac{1}{3}\left[f(b)g(b)+m_1m_2f\left(\frac{a}{m_1}\right)g\left(\frac{a}{m_2}\right)\right]+\frac{1}{6}\left[m_1f\left(\frac{a}{m_1}\right)g(b)+m_2f(b)g\left(\frac{a}{m_2}\right)\right].$$

Theorem 1.4 ([2])**.** *Let $f,g:\mathbb{R}_0\to\mathbb{R}_0$ satisfy $fg\in L([a,b])$ with $0\le a<b<\infty$. If f is (α_1,m_1)-convex and g is (α_2,m_2)-convex on $[a,b]$ for $(\alpha_1,m_1),(\alpha_2,m_2)\in(0,1]\times(0,1]$, then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x\le\min\{N_1,N_2\}, \tag{1.7}$$

where

$$\begin{aligned}N_1=&\frac{f(a)g(a)}{\alpha_1+\alpha_2+1}+m_2\left[\frac{1}{\alpha_1+1}-\frac{1}{\alpha_1+\alpha_2+1}\right]f(a)g\left(\frac{b}{m_2}\right)-m_1\left(\frac{1}{\alpha_1+\alpha_2+1}\right.\\&\left.-\frac{1}{\alpha_2+1}\right)g(a)f\left(\frac{b}{m_1}\right)+m_1m_2\left(1-\frac{1}{\alpha_1+1}-\frac{1}{\alpha_2+1}+\frac{1}{\alpha_1+\alpha_2+1}\right)f\left(\frac{b}{m_1}\right)g\left(\frac{b}{m_2}\right)\end{aligned}$$

and

$$\begin{aligned}N_2=&\frac{f(b)g(b)}{\alpha_1+\alpha_2+1}+m_2\left[\frac{1}{\alpha_1+1}-\frac{1}{\alpha_1+\alpha_2+1}\right]f(b)g\left(\frac{a}{m_2}\right)-m_1\left(\frac{1}{\alpha_1+\alpha_2+1}\right.\\&\left.-\frac{1}{\alpha_2+1}\right)g(b)f\left(\frac{a}{m_1}\right)+m_1m_2\left(1-\frac{1}{\alpha_1+1}-\frac{1}{\alpha_2+1}+\frac{1}{\alpha_1+\alpha_2+1}\right)f\left(\frac{a}{m_1}\right)g\left(\frac{a}{m_2}\right).\end{aligned}$$

In recent years, some inequalities of the Hermite–Hadamard type for other kinds of convex functions were created in, for example, [1, 6, 8, 9, 10, 11, 12] and closely related references therein.

The aim of this paper is to present some new inequalities of the Hermite–Hadamard type for the product of two (α,m)-convex functions, which generalizes those results mentioned above.

2. Main results

We are now in a position to establish some new integral inequalities of the Hermite–Hadamard type for the product of two (α, m)-convex functions.

Theorem 2.1. *Let $f, g : \mathbb{R}_0 \to \mathbb{R}_0$ satisfy $f, fg^q \in L([a, b])$, where $0 \le a < b < \infty$ and $q \ge 1$. If f is (α_1, m_1)-convex on $\left[0, \frac{b}{m_1}\right]$ and g^q is (α_2, m_2)-convex on $\left[0, \frac{b}{m_2}\right]$ for $(\alpha_1, m_1), (\alpha_2, m_2) \in (0, 1] \times (0, 1]$, then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x \le \frac{[N(a,b;f,\alpha_1,m_1)]^{1-1/q}\min\bigl\{[M(a,b;f,g^q)]^{1/q},[M(b,a;f,g^q)]^{1/q}\bigr\}}{(\alpha_1+1)[(\alpha_2+1)(\alpha_1+\alpha_2+1)]^{1/q}},$$

where

$$N(a,b;f,\alpha,m) = f(a) + \alpha m f\left(\frac{b}{m}\right) \tag{2.1}$$

and

$$\begin{aligned} M(a,b;f,g) = (\alpha_1+1)(\alpha_2+1)f(a)g(a) + \alpha_2 m_2(\alpha_2+1)f(a)g\left(\frac{b}{m_2}\right) \\ + \alpha_1 m_1(\alpha_1+1)g(a)f\left(\frac{b}{m_1}\right) + \alpha_1\alpha_2(\alpha_1+\alpha_2+2)m_1m_2 f\left(\frac{b}{m_1}\right)g\left(\frac{b}{m_2}\right). \end{aligned} \tag{2.2}$$

Proof. Letting $x = ta + (1-t)b$ for $t \in [0, 1]$ and making use of the Hölder integral inequality yield

$$\begin{aligned} \frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x &= \int_0^1 f(ta+(1-t)b)g(ta+(1-t)b)\,\mathrm{d}\,t \\ &\le \left[\int_0^1 f(ta+(1-t)b)\,\mathrm{d}\,t\right]^{1-1/q}\left[\int_0^1 f(ta+(1-t)b)g^q(ta+(1-t)b)\,\mathrm{d}\,t\right]^{1/q}. \end{aligned}$$

Further employing the conditions that f is (α_1, m_1)-convex on $\left[0, \frac{b}{m_1}\right]$ and g^q is (α_2, m_2)-convex on $\left[0, \frac{b}{m_2}\right]$ leads to

$$\int_0^1 f(ta+(1-t)b)\,\mathrm{d}\,t \le \int_0^1 \left[t^{\alpha_1}f(a) + m_1(1-t^{\alpha_1})f\left(\frac{b}{m_1}\right)\right]\mathrm{d}\,t = \frac{1}{\alpha_1+1}N(a,b;f,\alpha_1,m_1)$$

and

$$\begin{aligned} &\int_0^1 f(ta+(1-t)b)g^q(ta+(1-t)b)\,\mathrm{d}\,t \\ &\le \int_0^1 \left[t^{\alpha_1}f(a) + m_1(1-t^{\alpha_1})f\left(\frac{b}{m_1}\right)\right]\left[t^{\alpha_2}g^q(a) + m_2(1-t^{\alpha_2})g^q\left(\frac{b}{m_2}\right)\right]\mathrm{d}\,t \\ &= \frac{1}{\alpha_1+\alpha_2+1}f(a)g^q(a) + \frac{\alpha_2 m_2}{(\alpha_1+1)(\alpha_1+\alpha_2+1)}f(a)g^q\left(\frac{b}{m_2}\right) \\ &\quad + \frac{\alpha_1 m_1}{(\alpha_2+1)(\alpha_1+\alpha_2+1)}f\left(\frac{b}{m_1}\right)g^q(a) + \frac{\alpha_1\alpha_2(\alpha_1+\alpha_2+2)m_1m_2}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}f\left(\frac{b}{m_1}\right)g^q\left(\frac{b}{m_2}\right) \\ &= \frac{1}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}M(a,b;f,g^q). \end{aligned}$$

The proof of Theorem 2.1 is complete. □

Remark 4. Theorem 2.1 applied to $q = 1$ becomes the inequality (1.7).

Corollary 5. *Under the conditions of Theorem 2.1,*

1. *if* $\alpha_1 = \alpha_2 = \alpha$*, we have*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x \le \frac{[N(a,b;f,\alpha,m_1)]^{1-1/q}\min\big\{[M(a,b;f,g^q)]^{1/q},[M(b,a;f,g^q)]^{1/q}\big\}}{(\alpha+1)^{1+1/q}(2\alpha+1)^{1/q}};$$

2. *if* $m_1 = m_2 = m$*, we have*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x \le \frac{[N(a,b;f,\alpha_1,m)]^{1-1/q}\min\big\{[M(a,b;f,g^q)]^{1/q},[M(b,a;f,g^q)]^{1/q}\big\}}{(\alpha_1+1)[(\alpha_2+1)(\alpha_1+\alpha_2+1)]^{1/q}};$$

3. *if* $\alpha_1 = \alpha_2 = m_1 = m_2 = 1$*, we have*

$$\begin{aligned}\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x &\le \frac{1}{2}\left(\frac{1}{3}\right)^{1/q}[f(a)+f(b)]^{1-1/q}\\ &\quad\times\big[2f(a)g^q(a)+f(a)g^q(b)+f(b)g^q(a)+2f(b)g^q(b)\big]^{1/q}.\end{aligned}$$

Theorem 2.2. *Let* $f,g:\mathbb{R}_0\to\mathbb{R}_0$ *be such that* $f^q, g^{q/(q-1)}\in L([a,b])$*, where* $0\le a<b<\infty$ *and* $q>1$*. If* f^q *is* (α_1,m_1)*-convex on* $\big[0,\frac{b}{m_1}\big]$ *and* $g^{q/(q-1)}$ *is* (α_2,m_2)*-convex on* $\big[0,\frac{b}{m_2}\big]$ *for* $(\alpha_1,m_1),(\alpha_2,m_2)\in(0,1]\times(0,1]$*, then*

$$\begin{aligned}\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x &\le \left[\frac{\min\{N(a,b;f^q,\alpha_1,m_1),N(b,a;f^q,\alpha_1,m_1)\}}{\alpha_1+1}\right]^{1/q}\\ &\quad\times\left[\frac{\min\{N(a,b;g^{q/(q-1)},\alpha_2,m_2),N(b,a;g^{q/(q-1)},\alpha_2,m_2)\}}{\alpha_2+1}\right]^{1-1/q},\end{aligned}\tag{2.3}$$

where $N(a,b;f,\alpha,m)$ *is defined by* (2.1)*.*

Proof. Taking $x=ta+(1-t)b$ for $t\in[0,1]$ and using the Hölder integral inequality generate

$$\begin{aligned}\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x &= \int_0^1 f(ta+(1-t)b)g(ta+(1-t)b)\,\mathrm{d}\,t\\ &\le \left[\int_0^1 f^q(ta+(1-t)b)\,\mathrm{d}\,t\right]^{1/q}\left[\int_0^1 g^{q/(q-1)}(ta+(1-t)b)\,\mathrm{d}\,t\right]^{1-1/q}.\end{aligned}$$

Utilizing properties that f^q is (α_1,m_1)-convex on $\big[0,\frac{b}{m_1}\big]$ and that $g^{q/(q-1)}$ is (α_2,m_2)-convex on $\big[0,\frac{b}{m_2}\big]$ discovers

$$\int_0^1 f^q(ta+(1-t)b)\,\mathrm{d}\,t \le \int_0^1\left[t^{\alpha_1}f^q(a)+m_1(1-t^{\alpha_1})f^q\left(\frac{b}{m_1}\right)\right]\mathrm{d}\,t = \frac{1}{\alpha_1+1}N(a,b;f^q,\alpha_1,m_1).$$

Considering the symmetry of the estimated definite integral with respect to a and b results in

$$\int_0^1 f^q(ta+(1-t)b)\,\mathrm{d}\,t \le \frac{\min\{N(a,b;f^q,\alpha_1,m_1),N(b,a;f^q,\alpha_1,m_1)\}}{\alpha_1+1}.$$

Similarly, we have

$$\int_0^1 g^{q/(q-1)}(ta+(1-t)b)\,\mathrm{d}\,t \le \frac{\min\big\{N(a,b;g^{q/(q-1)},\alpha_2,m_2),N(b,a;g^{q/(q-1)},\alpha_2,m_2\big\}}{\alpha_2+1}.$$

Theorem 2.2 is thus proved. □

Corollary 6. *Under the conditions of Theorem 2.2, if* $\alpha_1 = \alpha_2 = m_1 = m_2 = 1$*, then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x \le \frac{[f^q(a)+f^q(b)]^{1/q}\big[g^{q/(q-1)}(a)+g^{q/(q-1)}(b)\big]^{1-1/q}}{2}. \tag{2.4}$$

Theorem 2.3. *Let* $f,g:\mathbb{R}_0\to\mathbb{R}_0$ *be such that* $f^p g^{q-\ell(q-1)}, f^{(q-p)/(q-1)}g^\ell \in L([a,b])$*, where* $0\le a<b<\infty$*,* $q>1$*,* $q>p>0$*, and* $\frac{q}{q-1}>\ell>0$*. If* f^p *and* $f^{(q-p)/(q-1)}$ *are* (α_1,m_1)*-convex on* $\big[0,\frac{b}{m_1}\big]$ *and if* g^ℓ *and* $g^{q-\ell/(q-1)}$ *are* (α_2,m_2)*-convex on* $\big[0,\frac{b}{m_2}\big]$ *for* $(\alpha_1,m_1),(\alpha_2,m_2)\in(0,1]\times(0,1]$*, then*

$$\begin{aligned}\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x \le{}& \frac{1}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}\big[\min\big\{M\big(a,b;f^p,g^{q-\ell(q-1)}\big),\\ &M\big(b,a;f^p,g^{q-\ell(q-1)}\big)\big\}\big]^{1/q}\big[\min\big\{M\big(a,b;f^{(q-p)/(q-1)},g^\ell\big),M\big(b,a;f^{(q-p)/(q-1)},g^\ell\big)\big\}\big]^{1-1/q},\end{aligned}$$

where $M(a,b;f,g)$ *is defined by* (2.2).

Proof. Letting $x=ta+(1-t)b$ for $t\in[0,1]$ and using the Hölder integral inequality figure out

$$\begin{aligned}\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x &= \int_0^1 f(ta+(1-t)b)g(ta+(1-t)b)\,\mathrm{d}\,t\\ &\le \left[\int_0^1 f^p(ta+(1-t)b)g^{q-\ell(q-1)}(ta+(1-t)b)\,\mathrm{d}\,t\right]^{1/q}\\ &\quad\times\left[\int_0^1 f^{(q-p)/(q-1)}(ta+(1-t)b)g^\ell(ta+(1-t)b)\,\mathrm{d}\,t\right]^{1-1/q}.\end{aligned}$$

Further by virtue of properties that the function f^p is (α_1,m_1)-convex on $\big[0,\frac{b}{m_1}\big]$ and that the function $g^{q-\ell/(q-1)}$ is (α_2,m_2)-convex on $\big[0,\frac{b}{m_2}\big]$, we have

$$\begin{aligned}&\int_0^1 f^p(ta+(1-t)b)g^{q-\ell(q-1)}(ta+(1-t)b)\,\mathrm{d}\,t\\ &\le \int_0^1\left[t^{\alpha_1}f^p(a)+m_1(1-t^{\alpha_1})f^p\left(\frac{b}{m_1}\right)\right]\left[t^{\alpha_2}g^{q-\ell(q-1)}(a)+m_2(1-t^{\alpha_2})g^{q-\ell(q-1)}\left(\frac{b}{m_2}\right)\right]\mathrm{d}\,t\\ &=\frac{1}{\alpha_1+\alpha_2+1}f^p(a)g^{q-\ell(q-1)}(a)+\frac{\alpha_2m_2}{(\alpha_1+1)(\alpha_1+\alpha_2+1)}f^p(a)g^{q-\ell(q-1)}\left(\frac{b}{m_2}\right)\\ &\quad+\frac{\alpha_1m_1}{(\alpha_2+1)(\alpha_1+\alpha_2+1)}f^p\left(\frac{b}{m_1}\right)g^{q-\ell(q-1)}(a)\\ &\quad+\frac{\alpha_1\alpha_2(\alpha_1+\alpha_2+2)m_1m_2}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}f^p\left(\frac{b}{m_1}\right)g^{q-\ell(q-1)}\left(\frac{b}{m_2}\right)\\ &=\frac{1}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}M\big(a,b;f^p,g^{q-\ell(q-1)}\big).\end{aligned}$$

Changing the order of a and b in the above arguments reveals

$$\int_0^1 f^p(ta+(1-t)b)g^{q-\ell(q-1)}(ta+(1-t)b)\,\mathrm{d}\,t \le \frac{\min\big\{M\big(a,b;f^p,g^{q-\ell(q-1)}\big),M\big(b,a;f^p,g^{q-\ell(q-1)}\big)\big\}}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}$$

and

$$\begin{aligned}&\int_0^1 f^{(q-p)/(q-1)}(ta+(1-t)b)g^\ell(ta+(1-t)b)\,\mathrm{d}\,t\\ &\le\frac{\min\big\{M\big(a,b;f^{(q-p)/(q-1)},g^\ell\big),M\big(b,a;f^{(q-p)/(q-1)},g^\ell\big)\big\}}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}.\end{aligned}$$

The proof of Theorem 2.3 is complete. □

Corollary 7. *Under the conditions of Theorem 2.3, if* $p = \ell \le \min\{q, \frac{q}{q-1}\}$, *then we have*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x \le \frac{1}{(\alpha_1+1)(\alpha_2+1)(\alpha_1+\alpha_2+1)}\big[\min\big\{M\big(a,b;f^p,g^{q-p(q-1)}\big), M\big(b,a;f^p,g^{q-p(q-1)}\big)\big\}\big]^{1/q}\big[\min\big\{M\big(a,b;f^{(q-p)/(q-1)},g^p\big), M\big(b,a;f^{(q-p)/(q-1)},g^p\big)\big\}\big]^{1-1/q}.$$

Corollary 8. *Under the conditions of Theorem 2.3, when* $\alpha_1 = \alpha_2 = m_1 = m_2 = 1$, *we have*

$$\frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}\,x \le \frac{1}{6}\big[2f^p(a)g^{q-\ell(q-1)}(a) + f^p(a)g^{q-\ell(q-1)}(b) + f^p(b)g^{q-\ell(q-1)}(a) + 2f^p(b)g^{q-\ell(q-1)}(b)\big]^{1/q}\big[2f^{(q-p)/(q-1)}(a)g^\ell(a) + f^{(q-p)/(q-1)}(a)g^\ell(b) + f^{(q-p)/(q-1)}(b)g^\ell(a) + 2f^{(q-p)/(q-1)}(b)g^\ell(b)\big]^{1-1/q}.$$

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