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Coupled fixed point theorems for compatible mappings in partially ordered G-metric spaces

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Abstract

In this paper, we prove coupled coincidence and coupled common fixed point theorems for compatible mappings in partially ordered G-metric spaces. The results on fixed point theorems are generalizations of some existing results. We also give an example to support our results.

Keywords: partially ordered set, couple coincidence point, coupled fixed point, compatible mappings, G-metric space.
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1. Introduction and Preliminaries

It is well known that fixed point theory is one of the most powerful and fruitful tools in nonlinear analysis, differential equation, and economic theory and has been studied in many various metric spaces. Especially, in 2006, Mustafa and Sims [15] introduced a generalized metric spaces which are called G-metric space. Follow Mustafa and Sims' work, many authors developed and introduced various fixed point theorems in G-metric spaces (see [7, 15, 16, 17, 18, 23]). Some authors have been interested in partially ordered G-metric spaces and prove some fixed point theorem. Simultaneously, fixed point theory has developed rapidly in partially ordered metric spaces [3, 13]. Fixed point theorems have also been considered in partially ordered probabilistic metric spaces [9], in partially ordered cone metric spaces [1, 22], and in partially ordered G-metric spaces [2, 4, 5, 6, 8, 10, 12, 19, 21]. In particular, in [4], Bhaskar and Lakshmikantham introduced notions of a mixed monotone mapping and a coupled fixed point, proved some coupled fixed point theorems for mixed monotone mappings, and discussed the existence and unique of solutions for periodic boundary

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value problems. Afterwards, some coupled fixed point and coupled coincidence point results and their applications have been established.

In this paper, we prove coupled coincidence and coupled common fixed point theorems for compatible mappings in partially ordered G-metric spaces. The results on fixed point theorems are generalizations of some existing results. We give an example to illustrate that our result is better than the results of Aydi at al. [3].

Throughout this paper, let $\mathbf{N}$ denote the set of nonnegative integers, and $\mathbf{R}^+$ be the set of positive real numbers.

Before giving our main results, we recall some basic concepts and results in G-metric spaces.

Definition 1.1. ([15]) Let X be a non-empty set, $G : X \times X \times X \to \mathbf{R}^+$ be a function satisfying the following properties:
(G1) $G(x, y, z) = 0$ if $x = y = z$.
(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$.
(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$.
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$ (symmetry in all three variables).
(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).
Then the function G is called a generalized metric and the pair (X, G) is called a G-metric space.

Definition 1.2. ([15]) Let (X, G) be a G-metric space and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if $\lim_{n,m\to\infty} G(x_n, x_n, x_m) = 0$, and one says the sequence $\{x_n\}$ is G-convergent to x.

Thus, if $x_n \to x$ in G-metric space (X, G) then, for any $\epsilon > 0$, there exists a positive integer N such that $G(x, x_n, x_m) < \epsilon$ for all $n, m > N$.

In [1], the authors have shown that the G-metric induces a Hausdorff topology, and the convergence described in the above definition is relative to this topology. The topology being Hausdorff, a sequence can converge at most to a point. Respectively, the authors achieve the following conclusions.

Definition 1.3. ([15]) Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called G-Cauchy if every $\epsilon > 0$, there exists a positive N such that $G(x_n, x_m, x_l) < \epsilon$ for all $n, m, l > N$, that is, if $G(x_n, x_m, x_l) \to 0$, as $n, m, l \to \infty$.

Lemma 1.4. ([15]) *If (X, G) is a G-metric space, then the following are equivalent.*
(1) $\{x_n\}$ is G-convergent to x.
(2) $G(x_n, x_n, x) \to 0$ as $n \to \infty$.
(3) $G(x_n, x, x) \to 0$ as $n \to \infty$.
(4) $G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Lemma 1.5. ([15]) *If (X, G) is a G-metric space, then the following are equivalent.*
(1) The sequence $\{x_n\}$ is G-Cauchy.
(2) For every $\epsilon > 0$, there exists a positive integer N such that $G(x_n, x_m, x_m) < \epsilon$ for all $n, m > N$.

Lemma 1.6. ([15]) *If (X, G) is a G-metric space, then $G(x, y, y) \leq 2G(y, x, x)$ for all $x, y \in X$.*

Lemma 1.7. ([15]) *If (X, G) is a G-metric space, then $G(x, x, y) \leq G(x, x, z) + G(z, z, y)$ for all $x, y, z \in X$.*

Definition 1.8. ([15]) Let (X, G), (X', G') be two G-metric spaces. Then a function $f : X \to X'$ is G-continuous at a point $x \in X$ if and only if it is G-sequentially continuous at x; that is, whenever $\{x_n\}$ is G-convergent to x, $\{f(x_n)\}$ is G'-convergent to $f(x)$.

Lemma 1.9. ([15]) *Let (X, G) be a G-metric spaces. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables*

Definition 1.10. ([15]) A G-metric space (X, G) is said to be G-complete (or a complete G-metric space) if every G-Cauchy sequence in (X, G) is convergent in X.

Next, we need some notions about partially ordered set.

Definition 1.11. ([4]) Let $(X, \preceq)$ be a partially ordered set and let $F : X \times X \to X$. The mapping F is said to have the mixed monotone property if $F(x, y)$ is monotone non-decreasing in x and is monotone non-increasing in y; that is, for any $x, y \in X$,

$$x_1, x_2 \in X, x_1 \preceq x_2 \Rightarrow F(x_1, y) \preceq F(x_2, y)$$

and

$$y_1, y_2 \in X, y_1 \preceq y_2 \Rightarrow F(x, y_1) \succeq F(x, y_2).$$

Definition 1.12. ([4]) An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if

$$x = F(x, y) \ \text{and} \ \ y = F(y, x).$$

Definition 1.13. ([12]) Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$ and $g : X \to X$ be two mappings. We say that F has the mixed-g-monotone property if $F(x, y)$ is g-monotone nondecreasing in x and it is g-monotone nonincreasing in y, that is, for any $x, y \in X$, we have:

$$x_1, x_2 \in X, \quad g(x_1) \preceq g(x_2) \Rightarrow F(x_1, y) \preceq F(x_2, y)$$

and, respectively,

$$y_1, y_2 \in X, \quad g(y_1) \preceq g(y_2) \Rightarrow F(x, y_1) \succeq F(x, y_2).$$

Definition 1.14. ([12]) An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mapping $F : X \times X \to X$ and $g : X \to X$ if

$$gx = F(x, y) \ \text{and} \ \ gy = F(y, x).$$

Definition 1.15. ([12]) We say that the mapping $F : X \times X \to X$ and $g : X \to X$ are commutative if

$$g(F(x, y)) = F(gx, gy) \ for \ all \ x, y \in X.$$

In [12], Lakshmikantham and Ćirić considered the following class of functions. We denote by Φ the set of functions $\varphi : [0, +\infty) \to [0, +\infty)$ satisfying
(a) $\varphi^{-1}\{0\} = \{0\}$.
(b) $\varphi(t) < t$ for all $t > 0$.
(c) $\lim\limits_{r \to t^+} \varphi(r) < t$ for all $t > 0$.
Hence, it concluded that $\lim\limits_{n \to \infty} \varphi^n(t) = 0$.

Aydi et al. [3] proved the following theorem.

Theorem 1.16. *Let $(X, \preceq)$ be a partially ordered set and suppose there is a G-metric G on X such that $(X, G)$is a complete G-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be such that F is continuous and has the mixed-g-monotone property. Assume there is a function $\varphi \in \Phi$ such that*

$$G(F(x, y), F(u, v), F(w, z)) \leq \varphi(\frac{G(gx, gu, gw) + G(gy, gv, gz)}{2}) \tag{1.1}$$

for all $x, y, z, u, v, w \in X$ with $gw \preceq gu \preceq gx$ and $gy \preceq gv \preceq gz$. Suppose also that $F(X \times X) \subseteq g(X)$ and g is continuous and commutes with F If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0, y_0)$ and $gy_0 \succeq F(y_0, x_0)$, then F and g have a coupled coincidence point, that is, there exists $(x, y) \in X \times X$ such that $g(x) = F(x, y)$ and $g(y) = F(y, x)$.

2. Main results

In this section, we give some fixed point theorems for compatible mappings in G-metric spaces. Our results extend some existing results in [3, 6, 13, 20]. In [11], the authors gave the following definition.

Definition 2.1. The mapping $F : X \times X \to X$ and $g : X \to X$ are said to be compatible if

$$\lim_{n\to\infty} G(gF(x_n, y_n), gF(x_n, y_n), F(gx_n, gy_n)) = 0$$

and

$$\lim_{n\to\infty} G(gF(y_n, x_n), gF(y_n, x_n), F(gy_n, gx_n)) = 0$$

whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x, \quad \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$$

for all $x, y \in X$ are satisfied.

Next, we prove our main results.

Theorem 2.2. *Let $(X, \preceq)$ be a partially ordered set and suppose there is a G-metric G on X such that $(X, G)$is a complete G-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be such that F has the mixed-g-monotone property. Assume there is a function $\varphi \in \Phi$ such that*

$$G(F(x, y), F(u, v), F(w, z)) \le \varphi(\max\{G(gx, gu, gw), G(gy, gv, gz)\}) \tag{2.1}$$

for all $x, y, z, u, v, w \in X$ with $gw \preceq gu \preceq gx$ and $gy \preceq gv \preceq gz$. Suppose $F(X \times X) \subseteq g(X)$ and g is continuous and compatible with F and also suppose either
(a) F is continuous or
(b) X has the following property:

- *(i) if a non-decreasing sequence $\{x_n\}$, then $x_n \preceq x$ for all n,*
- *(i) if a non-increasing sequence $\{y_n\}$, then $y_n \preceq y$ for all n.*

If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0, y_0)$ and $gy_0 \succeq F(y_0, x_0)$, then F and g have a coupled coincidence point, that is, there exists $(x, y) \in X \times X$ such that $g(x) = F(x, y)$ and $g(y) = F(y, x)$.

Proof. Let $x_0, y_0 \in X$ be such that $gx_0 \preceq F(x_0, y_0)$ and $gy_0 \succeq F(y_0, x_0)$. Since $F(X \times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that $gx_1 = F(x_0, y_0)$ and $gy_1 = F(y_0, x_0)$. Again since $F(X \times X) \subseteq g(X)$, we can choose $x_2, y_2 \in X$ such that $gx_2 = F(x_1, y_1)$ and $gy_2 = F(y_1, x_1)$. Since F has the mixed g-monotone property, we have $gx_0 \preceq gx_1 \preceq gx_2$ and $gy_2 \preceq gy_1 \preceq gy_0$. Continuing this process, we can construct two sequences (x_n) and (y_n) in X such that

$$gx_n = F(x_{n-1}, y_{n-1}) \preceq gx_{n+1} = F(x_n, y_n)$$

and

$$gy_{n+1} = F(y_n, x_n) \preceq gy_n = F(y_{n-1}, x_{n-1})$$

If for some n, we have $(gx_{n+1}, gy_{n+1}) = (gx_n, gy_n)$, then $F(x_n, y_n) = gx_n$ and $F(y_n, x_n) = gy_n$, that is, F and g have a coincidence point. So from now on, we assume $(gx_{n+1}, gy_{n+1}) \ne (gx_n, gy_n)$ for all $n \in \mathrm{N}$, that is, we assume that either $gx_{n+1} = F(x_n, y_n) \ne gx_n$ or $gy_{n+1} = F(y_n, x_n) \ne gy_n$. From (2.1), we have

$$\begin{aligned} G(gx_{n+1}, gx_{n+1}, gx_n) &= G(F(x_n, y_n), F(x_n, y_n), F(x_{n-1}, y_{n-1})) \\ &\le \varphi(\max\{G(gx_n, gx_n, gx_{n-1}), G(gy_n, gy_n, gy_{n-1})\}), \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} G(gy_{n+1}, gy_{n+1}, gy_n) &= G(F(y_n, x_n), F(y_n, x_n), F(y_{n-1}, x_{n-1})) \\ &\leq \varphi(\max\{G(gy_n, y_n, y_{n-1}), G(gx_n, gx_n, gx_{n-1})\}). \end{aligned} \tag{2.3}$$

Hence, from (2.2) and (2.3), we can get

$$\max\{G(gx_{n+1}, gx_{n+1}, gx_n), G(gy_{n+1}, gy_{n+1}, gy_n)\} \leq \varphi(\max\{G(gy_n, y_n, y_{n-1}), G(gx_n, gx_n, gx_{n-1})\}).$$

Let $\delta_n = \max\{G(gx_{n+1}, gx_{n+1}, gx_n), G(gy_{n+1}, gy_{n+1}, gy_n)\}$, then

$$\delta_n \leq \varphi(\delta_{n-1}) < \delta_{n-1}. \tag{2.4}$$

Hence, it follows that $\{\delta_n\}$ is monotone decreasing. Therefore, there is some $\delta \geq 0$ such that $\lim_{n\to\infty} \delta_n = \delta^+$. We shall show that $\delta = 0$. Suppose,to the contrary, that $\delta > 0$. In (2.4), let $n \to \infty$, we can get

$$\delta = \lim_{n\to\infty} \delta_n \leq \lim_{n\to\infty} \varphi(\delta_{n-1}) = \lim_{t\to\delta^+} \varphi(t) < \delta, \tag{2.5}$$

which is a contraction. Thus, $\delta = 0$, that is,

$$\lim_{n\to\infty} \delta_n = \lim_{n\to\infty} \max\{G(gx_{n+1}, gx_{n+1}, gx_n), G(gy_{n+1}, gy_{n+1}, gy_n)\} = 0. \tag{2.6}$$

Now we prove that (gx_n) and (gy_n) are G-Cauchy sequences in the G-metric space (X, G). Suppose on the contrary that at least one of (gx_n) and (gy_n) is not a G-Cauchy sequence in (X, G). Then there exists $\epsilon > 0$ and sequences of natural numbers $(m(k))$ and $(l(k))$ such that for every natural number k, $m(k) > l(k) \geq k$ and

$$r_k = \max\{G(gx_{m(k)}, gx_{m(k)}, gx_{l(k)}), G(gy_{m(k)}, gy_{m(k)}, gy_{l(k)})\} \geq \epsilon. \tag{2.7}$$

Now corresponding to $l(k)$ we choose $m(k)$ to be the smallest for which (2.7) holds. So

$$G(gx_{m(k)-1}, gx_{m(k)-1}, gx_{l(k)}) + G(gy_{m(k)-1}, gy_{m(k)-1}, gx_{l(k)}) < \epsilon.$$

Using the rectangle inequality, we get

$$\begin{aligned} \epsilon \leq r_k &= \max\{G(gx_{m(k)}, gx_{m(k)}, gx_{l(k)}), G(gy_{m(k)}, gy_{m(k)}, gy_{l(k)})\} \\ &\leq \max\{G(gx_{m(k)}, gx_{m(k)}, gx_{m(k)-1}), G(gx_{m(k)-1}, gx_{m(k)-1}, gx_{l(k)}), \\ &\qquad G(gy_{m(k)}, gy_{m(k)}, gy_{m(k)-1}), G(gy_{m(k)-1}, gy_{m(k)-1}, gy_{l(k)})\} \\ &< \max\{\delta_{m(k)-1}, \epsilon\}. \end{aligned}$$

Let $k \to \infty$ in the above inequality and using (2.6), we get

$$\lim_{n\to\infty} r_k = \epsilon^+. \tag{2.8}$$

Again, by rectangle inequality, we have

$$\begin{aligned} \epsilon \leq r_k &= \max\{G(gx_{m(k)}, gx_{m(k)}, gx_{l(k)}), G(gy_{m(k)}, gy_{m(k)}, gy_{l(k)})\} \\ &\leq \max\{G(gx_{m(k)}, gx_{m(k)}, gx_{m(k)+1}), G(gx_{m(k)+1}, gx_{m(k)+1}, gx_{l(k)+1}), G(gx_{l(k)+1}, gx_{l(k)+1}, gx_{l(k)}) \\ &\qquad G(gy_{m(k)}, gy_{m(k)}, gy_{m(k)+1}), G(gy_{m(k)+1}, gy_{m(k)+1}, gy_{l(k)+1}), G(gy_{l(k)+1}, gy_{l(k)+1}, gy_{l(k)})\} \\ &\leq \max\{[G(gx_{m(k)}, gx_{m(k)}, gx_{m(k)+1}), G(gy_{m(k)}, gy_{m(k)}, gy_{m(k)+1})]; \\ &\qquad [G(gx_{m(k)+1}, gx_{m(k)+1}, gx_{l(k)+1}), G(gy_{m(k)+1}, gy_{m(k)+1}, gy_{l(k)+1})]; \\ &\qquad [G(gx_{l(k)+1}, gx_{l(k)+1}, gx_{l(k)}), G(gy_{l(k)+1}, gy_{l(k)+1}, gy_{l(k)})]\}. \end{aligned}$$

Using that $G(x, x, y) \leq 2G(x, y, y)$ for any $x, y \in X$, we obtain

$$\begin{aligned} r_k &\leq \max\{[G(gx_{m(k)}, gx_{m(k)}, gx_{m(k)+1}), G(gy_{m(k)}, gy_{m(k)}, gy_{m(k)+1})]; \\ &\qquad [G(gx_{m(k)+1}, gx_{m(k)+1}, gx_{l(k)+1}), G(gy_{m(k)+1}, gy_{m(k)+1}, gy_{l(k)+1})]; \delta_{l(k)}\} \\ &\leq \max\{[2G(gx_{m(k)}, gx_{m(k)+1}, gx_{m(k)+1}), 2G(gy_{m(k)}, gy_{m(k)+1}, gy_{m(k)+1})]; \\ &\qquad [G(gx_{m(k)+1}, gx_{m(k)+1}, gx_{l(k)+1}), G(gy_{m(k)+1}, gy_{m(k)+1}, gy_{l(k)+1})]; \delta_{l(k)}\} \\ &\leq \max\{2\delta_{m(k)}; [G(gx_{m(k)+1}, gx_{m(k)+1}, gx_{l(k)+1}), G(gy_{m(k)+1}, gy_{m(k)+1}, gy_{l(k)+1})]; \delta_{l(k)}\} \end{aligned} \tag{2.9}$$

Now, using inequality (2.1), we have

$$\begin{aligned}G(gx_{m(k)+1}, gx_{m(k)+1}, gx_{l(k)+1}) &= G(F(x_{m(k)}, y_{m(k)}), F(x_{m(k)}, y_{m(k)}), F(x_{l(k)}, y_{l(k)}))\\ &\le \varphi(\max\{G(gx_{m(k)}, gx_{m(k)}, gx_{l(k)}), G(gy_{m(k)}, gy_{m(k)}, gy_{l(k)})\})\\ &= \varphi(r_k).\end{aligned}$$

and

$$\begin{aligned}G(gy_{m(k)+1}, gy_{m(k)+1}, gy_{l(k)+1}) &= G(F(y_{m(k)}, x_{m(k)}), F(y_{m(k)}, x_{m(k)}), F(y_{l(k)}, x_{l(k)}))\\ &\le \varphi(\max\{G(gy_{m(k)}, gy_{m(k)}, gy_{l(k)}, G(gx_{m(k)}, gx_{m(k)}, gx_{l(k)})\})\\ &= \varphi(r_k).\end{aligned}$$

Adding the above inequalities, we get

$$\max\{G(gx_{m(k)+1}, gx_{m(k)+1}, gx_{l(k)+1}), G(gy_{m(k)+1}, gy_{m(k)+1}, gy_{l(k)+1})\} \le \varphi(r_k). \tag{2.10}$$

Hence, from (2.9) and (2.10), it follows that

$$r_k \le \max\left\{2\delta_{m(k)}, \varphi(r_k), \delta_{l(k)}\right\} \tag{2.11}$$

Now, using (2.6), (2.8) and the properties of the function φ, and letting $k \to \infty$ in (2.11), we get

$$\epsilon \le \max\left\{0, \lim_{k\to\infty} \varphi(r_k), 0\right\} = \lim_{k\to\infty} \varphi(r_k) = \lim_{r(k)\to\epsilon^+} \varphi(r_k) < \epsilon, \tag{2.12}$$

which is a contraction. Thus we proved that (gx_n) and (gy_n) are G-Cauchy sequences in the G-metric space (X, G). Now, since (X, G) is G-complete, there are $x, y \in X$ such that (gx_n) and (gy_n) are respectively G-convergent to x and y, that is from Lemma 1.4, we have

$$\lim_{n\to+\infty} F(x_n, y_n) = \lim_{n\to+\infty} g(x_n) = x, \lim_{n\to+\infty} F(y_n, x_n) = \lim_{n\to+\infty} g(y_n) = y. \tag{2.13}$$

and

$$\lim_{n\to+\infty} G(gx_n, gx_n, x) = \lim_{n\to+\infty} G(gx_n, x, x) = 0, \tag{2.14}$$

$$\lim_{n\to+\infty} G(gy_n, gy_n, y) = \lim_{n\to+\infty} G(gy_n, y, y) = 0. \tag{2.15}$$

Since g is continuous and compatible with F, hence we have

$$\lim_{n\to\infty} G(gF(x_n, y_n), gF(x_n, y_n), F(gx_n, gy_n)) = 0 \tag{2.16}$$

and

$$\lim_{n\to\infty} G(gF(y_n, x_n), gF(y_n, x_n), F(gy_n, gx_n)) = 0. \tag{2.17}$$

Now, suppose that assumption (a) holds. From $F(x_n, y_n) = gx_{n+1}$ and $F(y_n, x_n) = gy_{n+1}$, we have

$$G(g(x), g(x), F(gx_n, gy_n)) \le G(g(x), g(x), gF(x_n, y_n)) + G(gF(x_n, y_n), gF(x_n, y_n), F(gx_n, gy_n)). \tag{2.18}$$

In (2.18), let $n \to \infty$ and using (2.16), we can get

$$\lim_{n\to\infty} G(g(x), g(x), F(gx_n, gy_n)) = G(g(x), g(x), F(x, y)) = 0.$$

Hence, $g(x) = F(x, y)$. Similarly, we can show that $g(y) = F(y, x)$. Finally, suppose that (b) holds. Since $\{gx_n\}$ is a non-decreasing sequence and $gx_n \to x$ and as $\{gy_n\}$ is a non-increasing sequence and $gy_n \to y$, we have $g(x_n) \preceq x$ and $g(y_n) \succeq y$ for all n. Then, from (2.1), we have

$$\begin{aligned}G(g(x), g(x), F(x, y)) &\le G(g(x), g(x), g(gx_{n+1})) + G(g(gx_{n+1}), g(gx_{n+1}), F(x, y))\\ &= G(g(x), g(x), g(gx_{n+1})) + G(F(gx_n, gy_n), F(gx_n, gy_n), F(x, y))\\ &\le G(g(x), g(x), g(gx_{n+1})) + \varphi(\max\{G(g(gx_n), g(gx_n), gx), G(g(gy_n), g(gy_n), gy)\}).\end{aligned} \tag{2.19}$$

In (2.19), let $n \to \infty$, we can conclude that $g(x) = F(x, y)$. Similarly, we can show that $g(y) = F(y, x)$. The proof is completed. □

If $\varphi(t) = kt$ in Theorem 2.2, we can get the following corollary.

Corollary 2.3. *Let $(X, \preceq)$ be a partially ordered set and suppose there is a G-metric G on X such that $(X, G)$is a complete G-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be such that F has the mixed-g-monotone property. Assume there is a $k \in [0.1)$ such that*

$$G(F(x, y), F(u, v), F(w, z)) \leq k \max\left\{G(gx, gu, gw), G(gy, gv, gz)\right\} \tag{2.20}$$

for all $x, y, z, u, v, w \in X$ with $gw \preceq gu \preceq gx$ and $gy \preceq gv \preceq gz$. Suppose that $F(X \times X) \subseteq g(X)$ and g is continuous and compatible with F and also suppose either
(a) F is continuous or
(b) X has the following property:

(i) if a non-decreasing sequence $\{x_n\}$, then $x_n \preceq x$ for all n,
(ii) if a non-increasing sequence $\{y_n\}$, then $y_n \preceq y$ for all n.

If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0, y_0)$ and $gy_0 \succeq F(y_0, x_0)$, then F and g have a coupled coincidence point, that is, there exists $(x, y) \in X \times X$ such that $g(x) = F(x, y)$ and $g(y) = F(y, x)$.

Remark 2.4. Corollary 2.3 generalizes the results of Nashine [20].

Let $g = I_x$ in Corollary 2.3, we can get the following corollary.

Corollary 2.5. *Let $(X, \preceq)$ be a partially ordered set and suppose there is a G-metric G on X such that $(X, G)$is a complete G-metric space. Let $F : X \times X \to X$ be such that F has the mixed monotone property. Assume there is a $k \in [0.1)$ such that*

$$G(F(x, y), F(u, v), F(w, z)) \leq k \max\left\{G(x, u, w), G(y, v, z)\right\} \tag{2.21}$$

for all $x, y, z, u, v, w \in X$ with $w \preceq u \preceq x$ and $y \preceq v \preceq z$. Suppose that either
(a) F is continuous or
(b) X has the following property:

(i) if a non-decreasing sequence $\{x_n\}$, then $x_n \preceq x$ for all n,
(ii) if a non-increasing sequence $\{y_n\}$, then $y_n \preceq y$ for all n.

If there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$, then F have a coupled fixed point in X, that is, there exists $(x, y) \in X \times X$ such that $x = F(x, y)$ and $y = F(y, x)$.

Remark 2.6. Corollary 2.5 extends the results of Choudury [6].

Let Ψ denote all functions $\psi : [0, \infty) \to [0, \infty)$ satisfying $\lim_{t \to r} \psi(t) > 0$ for each $r > 0$. Using the definition of Ψ, we can get the following corollary.

Corollary 2.7. *Let $(X, \preceq)$ be a partially ordered set and suppose there is a G-metric G on X such that $(X, G)$is a complete G-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be such that F has the mixed-g-monotone property. Assume there exists $\psi \in \Psi$ such that*

$$\begin{aligned} G(F(x, y), F(u, v), F(w, z)) \leq \max&\left\{G(gx, gu, gw), G(gy, gv, gz)\right\} \\ &-\psi(\max\left\{G(gx, gu, gw), G(gy, gv, gz)\right\}), \end{aligned} \tag{2.22}$$

for all $x, y, z, u, v, w \in X$ with $gw \preceq gu \preceq gx$ and $gy \preceq gv \preceq gz$. Suppose that $F(X \times X) \subseteq g(X)$ and g is continuous and compatible with F and also suppose either
(a) F is continuous or
(b) X has the following property:

(i) if a non-decreasing sequence $\{x_n\}$, then $x_n \preceq x$ for all n,
(ii) if a non-increasing sequence $\{y_n\}$, then $y_n \preceq y$ for all n.

If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0, y_0)$ and $gy_0 \succeq F(y_0, x_0)$, then F and g have a coupled coincidence point, that is, there exists $(x, y) \in X \times X$ such that $g(x) = F(x, y)$ and $g(y) = F(y, x)$.

Proof. Let $\varphi(t) = t - \psi(t)$. Obviously, $\varphi \in \Phi$. Hence, Corollary 2.7 satisfies all conditions of Theorem 2.2. The proof is completed. □

Remark 2.8. Corollary 2.7 extends the results obtained by Luong [13].

Now, we shall prove the uniqueness of the coupled fixed point. Note that, if $(X, \preceq)$ is a partially ordered set, then we endow the product $X \times X$ with the following partial order relation:

$$(x, y), (u, v) \in X \times X, (x, y) \preceq (u, v) \Leftrightarrow x \preceq u, y \succeq v.$$

Theorem 2.9. *In addition to the hypotheses of Theorem 2.2, suppose that for all $(x, y), (x^*, y^*) \in X \times X$, there exists $(u, v) \in X \times X$ such that $(F(u, v), F(v, u))$ is comparable with $(F(x, y)F(y, x))$ and $(F(x^*, y^*), F(y^*, x^*))$. Suppose also that φ is a nondecreasing function. Then F and g have a unique coupled common fixed point, that is, there exists a unique $(x, y) \in X \times X$ such that*

$$x = gx = F(x, y) \text{ and } y = gy = F(y, x). \tag{2.23}$$

Proof. From Theorem 2.2, the set of coupled coincidences is non-empty. We shall show that if (x, y) and (x^*, y^*) are coupled coincidence points, that is, if $g(x) = F(x, y)$, $g(y) = F(y, x)$, $g(x^*) = F(x^*, y^*)$ and $g(y^*) = F(y^*, x^*)$, then $gx = gx^*$ and $gy = gy^*$. By assumption, there exists $(u, v) \in X \times X$ such that $(F(u, v), F(v, u))$ is comparable with $(F(x, y), F(y, x))$ and $(F(x^*, y^*), F(y^*, x^*))$. Without restriction to the generality, we can assume that

$$(F(x, y), F(y, x)) \preceq (F(u, v), F(v, u))$$

and

$$(F(x^*, y^*), F(y^*, x^*)) \preceq (F(u^*, v^*), F(v^*, u^*)).$$

Put $u_0 = u$,$v_0 = v$, and choose $u_1, v_1 \in X$ such that $gu_1 = F(u_0, v_0)$, $gv_1 = F(v_0, u_0)$. Then, similarly as in the proof of Theorem 2.2, we can inductively define sequences (gu_n) and (gv_n) in X by $gu_{n+1} = F(u_n, v_n)$ and $gv_{n+1} = F(v_n, u_n)$. Further, let $x_0 = x, y_0 = y, x_0^* = x^*, y_0^* = y^*$. and, in the same way, define the sequences (gx_n), (gy_n), (gx_n^*) and (gy_n^*). Since

$$(F(x, y), F(y, x)) = (gx_1, gy_1) = (gx, gy) \preceq (F(u, v), F(v, u)) = (gu_1, gv_1),$$

then, $gx \preceq gu_1$ and $gv_1 \preceq gy$. Using that F is a mixed g-monotone mapping, one can show easily that $gx \preceq gu_n$ and $gv_n \preceq gy$ for all $n \geq 1$. Thus, from (2.1), we have

$$\begin{aligned} G(g(u_{n+1}), g(x), g(x)) &= G(F(u_n, v_n), F(x, y), F(x, y)) \\ &\leq \varphi(\max\{G(gu_n, gx, gx), G(gv_n, gy, gy)\}) \end{aligned} \tag{2.24}$$

$$\begin{aligned} G(g(y), g(y), g(v_{n+1})) &= G(F(y, x), F(y, x), F(v_n, u_n)) \\ &\leq \varphi(\max\{G(gy, gy, gv_n), G(gx, gx, gu_n)\}). \end{aligned} \tag{2.25}$$

From (2.24) and (2.25), we can conclude that

$$\max\{G(g(u_{n+1}), g(x), g(x), G(g(y), g(y), g(v_{n+1}))\} \leq \varphi(\max\{G(gu_n, gx, gx), G(gv_n, gy, gy)\})$$

Without restriction to the generality, we can suppose that $(gu_n, gv_n) \neq (gx, gy)$ for all $n \geq 1$. Since φ is non-decreasing, from the previous inequality, we get

$$\max\{G(g(u_{n+1}), g(x), g(x), G(g(y), g(y), g(v_{n+1}))\} \leq \varphi^n(\max\{G(gu_1, gx, gx), G(gv_1, gy, gy)\}). \tag{2.26}$$

In (2.26), let $n \to \infty$, we can get

$$\lim_{n\to\infty} G(g(u_{n+1}), g(x), g(x)) = 0 \ \ and \ \ \lim_{n\to\infty} G(g(y), g(y), g(v_{n+1}) = 0. \tag{2.27}$$

Similarly, one can show that

$$\lim_{n\to\infty} G(g(u_{n+1}), g(x^*), g(x^*)) = 0 \ \ and \ \ \lim_{n\to\infty} G(g(y^*), g(y^*), g(v_{n+1}) = 0. \tag{2.28}$$

Therefore, from (2.27), (2.28) and the uniqueness of the limit, we get

$$gx = gx^* \ \ and \ \ gy = gy^*. \tag{2.29}$$

Since $gx = F(x, y)$ and $gy = F(y, x)$, by compatible of F and g, we have

$$g(gx) = g(F(x, y)) = F(gx, gy) \ \ and \ \ g(g(y)) = g(F(y, x)) = F(gy, gx). \tag{2.30}$$

Put $g(x) = z$ and $g(y) = w$, then by (2.30), we get

$$gz = F(z, w) \ \ and \ \ gw = F(w, z). \tag{2.31}$$

Thus, (z,w) is a coincidence point. Then by (2.29) with $x^* = z$ and $y^* = w$, we have $gx = gz$ and $gy = gw$, that is,

$$g(z) = g(x) = z \ \ and \ \ g(y) = g(w) = w. \tag{2.32}$$

From (2.31) and (2.32), we get $z = gz = F(z, w)$ and $w = gw = F(w, z)$. Then, (z, w) is a coupled fixed point of F and g. To prove the uniqueness, assume that (p, q) is another coupled fixed point. Then by (2.29), we have $p = gp = gz = z$ and $q = gq = gw = w$. The proof is completed. □

Inspired by [14], we give an example to illustrate that Theorem 2.2 is an extension of Theorem 1.16.

Example 2.10. Let $X = [0, 1]$ and $(X, \preceq)$ be a partially ordered set with the natural ordering of real numbers. Let $G(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, z \in X$. Then (X, G) is a complete G-metric space. Let the mapping $g : X \to X$ be defined by

$$g(x) = x^2 \ \ for \ \ all \ \ x \in X,$$

and let the mapping $F : X \times X \to X$ be defined by

$$F(x, y) = \begin{cases} \frac{x^2-y^2}{3} & \text{if } x \succeq y \\ 0 & \text{if } x \prec y \end{cases}$$

for all $x, y \in X$. Then F satisfies the mixed g-monotone property. Let $\varphi(t) : R^+ \to R^+$ be such that $\varphi(t) = \frac{2t}{3}$ for all $t \in R^+$. Suppose that (x_n) and (y_n) are two sequences in X such that

$$\lim_{n\to\infty} F(x_n, y_n) = a, \lim_{n\to\infty} g(x_n) = a, \lim_{n\to\infty} F(y_n, x_n) = b, \lim_{n\to\infty} g(y_n) = b.$$

Then $a = 0$ and $b = 0$. For all $n \geq 1$, we define

$$g(x_n) = x_n^2, \qquad g(y_n) = y_n^2,$$

$$F(x_n, y_n) = \begin{cases} \frac{x_n^2-y_n^2}{3} & \text{if } x_n \succeq y_n, \\ 0 & \text{if } x_n \prec y_n, \end{cases}$$

and

$$F(y_n, x_n) = \begin{cases} \frac{y_n^2-x_n^2}{3} & \text{if } y_n \succeq x_n, \\ 0 & \text{if } y_n \prec x_n. \end{cases}$$

From the above, we see that

$$\lim_{n\to\infty} G(gF(x_n,y_n),gF(x_n,y_n),F(gx_n,gy_n))=0,$$
$$\lim_{n\to\infty} G(gF(y_n,x_n),gF(y_n,x_n),F(gy_n,gx_n))=0.$$

This proves that F and g are compatible. Also, suppose that $x_0=0$ and $y_0=c$ are two points in X such that

$$g(x_0)=g(0)=0=F(0,c)=F(x_0,y_0),$$
$$g(y_0)=g(c)=c^2\succeq\frac{c^2}{3}=F(c,0)=F(y_0,x_0).$$

Now it is left to show that (2.1) of Theorem 2.2 is satisfied with $\varphi(t)=\frac{2t}{3}$ as defined above. Let $x,y,u,v,z,w\in X$ be such that $g(w)\preceq g(u)\preceq g(x)$ and $g(y)\succeq g(v)\succeq g(z)$, that is, $w\preceq u\preceq x$ and $y\preceq v\preceq z$. We have the following possible cases.
Case 1: When $x\succeq y$, $u\succeq v$, and $z\succeq w$. Then we get

$$\begin{aligned}G(F(x,y),F(u,v),F(z,w))&=G(\frac{x^2-y^2}{3},\frac{u^2-v^2}{3},\frac{z^2-w^2}{3})\\&=|\frac{(x^2-y^2)-(u^2-v^2)}{3}|+|\frac{(u^2-v^2)-(z^2-w^2)}{3}|+|\frac{(z^2-w^2)-(x^2-y^2)}{3}|\\&\le\frac{1}{3}\big(|(x^2-y^2)-(u^2-v^2)|+|(u^2-v^2)-(z^2-w^2)|+|(z^2-w^2)-(x^2-y^2)|\big)\\&\le\frac{1}{3}\big(|x^2-u^2|+(y^2-v^2)|+|u^2-z^2|+|v^2-w^2|+|z^2-x^2|+|w^2-y^2)|\big)\\&\le\frac{2}{3}\max\big\{|x^2-u^2|+|u^2-z^2|+|z^2-x^2|,|(y^2-v^2)|+|v^2-w^2|+|w^2-y^2)|\big\}\end{aligned}$$

and

$$\begin{aligned}\varphi(\max\big\{G(gx,gu,gz),G(gy,gv,gw)\big\})&=\varphi(\max\big\{G(x^2,u^2,z^2),G(y^2,v^2,w^2)\big\})\\&=\frac{2}{3}\big(\max\big\{G(x^2,u^2,z^2),G(y^2,v^2,w^2)\big\}\big)\\&=\frac{2}{3}\max\big\{|x^2-u^2|+|u^2-z^2|+|z^2-x^2|,\\&\qquad|y^2-v^2|+|v^2-w^2|+|w^2-y^2|\big\}.\end{aligned}$$

Hence, $G(F(x,y),F(u,v),F(z,w))\le\varphi(\max\big\{G(gx,gu,gz),G(gy,gv,gw)\big\})$, is that, (2.1) holds.
Case 2: When $x\succeq y$, $u\succeq v$, and $z\prec w$. Then we get

$$\begin{aligned}G(F(x,y),F(u,v),F(z,w))&=G(F(x,y),F(u,v),0)=G(\frac{x^2-y^2}{3},\frac{u^2-v^2}{3},0)\\&=|\frac{(x^2-y^2)-(u^2-v^2)}{3}|+|\frac{u^2-v^2}{3}|+|\frac{x^2-y^2}{3}|\\&\le\frac{1}{3}\big(|(x^2-y^2)-(u^2-v^2)|+|u^2-v^2|+|x^2-y^2|\big)\\&\le\frac{1}{3}\big(|x^2-u^2|+(y^2-v^2)|+|u^2-w^2|+|w^2-v^2|+|x^2-z^2|+|z^2-y^2|\big)\\&\le\frac{1}{3}\big(|x^2-u^2|+(y^2-v^2)|+|u^2-z^2|+|w^2-v^2|+|x^2-z^2|+|w^2-y^2|\big)\\&\le\frac{2}{3}\max\big\{|x^2-u^2|+|u^2-z^2|+|z^2-x^2|,|(y^2-v^2)|+|v^2-w^2|+|w^2-y^2|\big\}\end{aligned}$$

and

$$\begin{aligned}\varphi(\max\big\{G(gx,gu,gz),G(gy,gv,gw)\big\})&=\varphi(\max\big\{G(x^2,u^2,z^2),G(y^2,v^2,w^2)\big\})\\&=\frac{2}{3}\big(\max\big\{G(x^2,u^2,z^2),G(y^2,v^2,w^2)\big\}\big)\\&=\frac{2}{3}\max\big\{|x^2-u^2|+|u^2-z^2|+|z^2-x^2|,\\&\qquad|(y^2-v^2)|+|v^2-w^2|+|w^2-y^2|\big\}.\end{aligned}$$

Hence, $G(F(x,y),F(u,v),F(z,w)) \leq \varphi(\max\{G(gx,gu,gz),G(gy,gv,gw)\})$, is that, (2.1) holds.
Case 3: When $x \succeq y$, $u \prec v$, and $z \succeq w$, then we have

$$\begin{aligned}
G(F(x,y),F(u,v),F(z,w)) &= G(F(x,y),0,F(z,w)) = G(\frac{x^2-y^2}{3},0,\frac{z^2-w^2}{3}) \\
&= |\frac{x^2-y^2}{3}|+|\frac{z^2-w^2}{3}|+|\frac{(z^2-w^2)-(x^2-y^2)}{3}| \\
&\leq \frac{1}{3}\big(|x^2-y^2|+|z^2-w^2|+|(z^2-w^2)-(x^2-y^2)|\big) \\
&\leq \frac{1}{3}\big(|x^2-u^2|+|y^2-u^2|+|u^2-z^2|+|w^2-u^2|+|x^2-z^2|+|w^2-y^2|\big) \\
&\leq \frac{1}{3}\big(|x^2-u^2|+|y^2-v^2|+|u^2-z^2|+|w^2-v^2|+|x^2-z^2|+|w^2-y^2)|\big) \\
&\leq \frac{2}{3}\max\{|x^2-u^2|+|u^2-z^2|+|z^2-x^2|,|y^2-v^2|+|v^2-w^2|+|w^2-y^2|\}
\end{aligned}$$

and

$$\begin{aligned}
\varphi(\max\{G(gx,gu,gz),G(gy,gv,gw)\}) &= \varphi(\max\{G(x^2,u^2,z^2),G(y^2,v^2,w^2)\}) \\
&= \frac{2}{3}\big(\max\{G(x^2,u^2,z^2),G(y^2,v^2,w^2)\}\big) \\
&= \frac{2}{3}\max\{|x^2-u^2|+|u^2-z^2|+|z^2-x^2|, \\
&\qquad |(y^2-v^2)|+|v^2-w^2|+|w^2-y^2|\}.
\end{aligned}$$

Hence, $G(F(x,y),F(u,v),F(z,w)) \leq \varphi(\max\{G(gx,gu,gz),G(gy,gv,gw)\})$, is that, (2.1) holds.
Case 4: If $x \prec y$, $u \succeq v$, and $z \succeq w$, then we see that this assumption cannot happen.
Case 5: If $x \prec y$, $u \prec v$, and $z \succeq w$, then we can get

$$\begin{aligned}
G(F(x,y),F(u,v),F(z,w)) &= G(0,0,F(z,w)) = G(0,0,\frac{z^2-w^2}{3}) \\
&= |\frac{z^2-w^2}{3}|+|\frac{z^2-w^2}{3}| \\
&\leq \frac{1}{3}\big(|z^2-u^2|+|u^2-w^2|+|(z^2-x^2)-(x^2-w^2)|\big) \\
&\leq \frac{1}{3}\big(|z^2-u^2|+|v^2-w^2|+|z^2-x^2|+|x^2-w^2|\big) \\
&\leq \frac{2}{3}\max\{|x^2-u^2|+|u^2-z^2|+|z^2-x^2|,|y^2-v^2|+|v^2-w^2|+|w^2-y^2|\}
\end{aligned}$$

and

$$\begin{aligned}
\varphi(\max\{G(gx,gu,gz),G(gy,gv,gw)\}) &= \varphi(\max\{G(x^2,u^2,z^2),G(y^2,v^2,w^2)\}) \\
&= \frac{2}{3}\big(\max\{G(x^2,u^2,z^2),G(y^2,v^2,w^2)\}\big) \\
&= \frac{2}{3}\max\{|x^2-u^2|+|u^2-z^2|+|z^2-x^2|, \\
&\qquad |(y^2-v^2)|+|v^2-w^2|+|w^2-y^2|\}.
\end{aligned}$$

Hence, $G(F(x,y),F(u,v),F(z,w)) \leq \varphi(\max\{G(gx,gu,gz),G(gy,gv,gw)\})$, is that, (2.1) holds.
Case 6: If $x \prec y$, $u \succeq v$, and $z \prec w$, then we see that this assumption cannot happen.
Case 7: If $x \succeq y$, $u \prec v$, and $z \prec w$, then we see that this assumption cannot happen.
Case 8: If $x \prec y$, $u \prec v$, and $z \prec w$, then obviously (2.1) holds.

Thus all the hypotheses of Theorem 2.2 are fulfilled. So, we conclude that F and g have a coupled coincidence point. In this case, $(0,0)$ is a coupled coincidence point of F and g in X.

Next, we illustrate that the example doesn't satisfied with the condition of Theorem 1.16. Since for all $x,y \in X$, we have $g(F(x,y)) \neq F(gx,gy)$. Hence, in this example g does not commute with F. Theorem 1.16 is not application to this example. So our result generalizes and extends Theorem 1.16.

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