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Some fixed point results for nonlinear mappings in convex metric spaces

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Abstract

In this paper, we consider an iteration process to approximate a common random fixed point of a finite family of asymptotically quasi-nonexpansive random mappings in convex metric spaces. Our results extend and improve several known results in recent literature.

Keywords: Asymptotically quasi-nonexpansive random mappings, random iteration process, common random fixed point, convex metric spaces.
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1. Introduction and Preliminaries

Random fixed point theorems are stochastic generalizations of classical fixed point theorems, which are usually used to obtain the solutions of nonlinear random systems [3]. Some random fixed point theorems for random mappings on separable metric spaces were first proved by Spacek [18] and Hans [7]. Itoh [8] introduced multivalued random contractive mappings on separable metric spaces and considered some random fixed point theorems for the mappings. Choudhury [5] gave a random Ishikawa iteration process to converge to fixed points of the given random mappings. After that, many authors [1, 2, 5, 11, 12, 13, 14, 17, 16] have worked on random iterative algorithms for contractive and asymptotically nonexpansive random mappings in separable normed spaces, Banach spaces and uniformly convex Banach spaces.

In 1970, Takahashi [19] introduced a notion of convex metric space which is a more general space, and each linear normed space is a special example of a convex metric space. Recently [4, 10, 21, 22] have discussed different iteration processes to obtain fixed point of asymptotically quasi-nonexpansive mappings in convex metric spaces.

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Inspried and motived by the above facts, we will construct an iteration process which converges strongly to a common random fixed point of a finite family of asymptotically quasi-nonexpansive random mappings in convex metric spaces. The results extend and improve the corresponding results in [1, 2, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 17, 16, 20, 21, 22].

Let (Ω, Σ) be a mesurable space with Σ being a σ-algebra of subsets of Ω, and let K be a nonempty subset of a metric space (X, d).

Definition 1.1 ([1]). (i) A mapping $\xi : \Omega \to X$ is measurable if $\xi^{-1}(U) \in \Sigma$ for each open subset U of X;

(ii) The mapping $T : \Omega \times K \to X$ is a random mapping if and only if for each fixed $x \in K$, the mapping $T(\cdot, x) : \Omega \to X$ is measurable, and it is continuous if for each $\omega \in \Omega$, the mapping $T(\omega, \cdot) : K \to X$ is continuous;

(iii) A measurable mapping $\xi : \Omega \to K$ is a random fixed point of the random mapping $T : \Omega \times K \to X$ if and only if $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

We denote by $\mathbb{N}$ the set of natural numbers, $F(T)$ the set of all random fixed points of a random map T and $T^n(\omega, x)$ the nth iteration $T(\omega, T(\omega, T(\omega, \cdots T(\omega, x) \cdots)))$ of T for each $\omega \in \Omega$. The letter I denotes the random mapping $T : \Omega \times K \to K$ defined by $I(\omega, x) = x$ and $T^0 = I$ for each $\omega \in \Omega$.

Next, we introduce some random mappings in metric spaces.

Definition 1.2. Let K be a nonempty subset of a separable metric space (X, d) and $T : \Omega \times K \to K$ be a random mapping. The mapping T is said to be

(i) a nonexpansive random mapping if

$$d(T(\omega, x), T(\omega, y)) \leq d(x, y)$$

for each $\omega \in \Omega$ and $x, y \in K$;

(ii) an asymptotically nonexpansive random mapping if there exists a sequence of measurable mappings $\{r_n(\omega)\} : \Omega \to [0, \infty)$ with $\lim_{n\to\infty} r_n(\omega) = 0$ such that

$$d(T^n(\omega, x), T^n(\omega, y)) \leq (1 + r_n(\omega))d(x, y)$$

for each $\omega \in \Omega$, $n \in \mathbb{N}$ and $x, y \in K$;

(iii) an asymptotically quasi-nonexpansive random mapping if there exists a sequence of measurable mappings $\{r_n(\omega)\} : \Omega \to [0, \infty)$ with $\lim_{n\to\infty} r_n(\omega) = 0$ such that

$$d(T^n(\omega, \eta(\omega)), \xi(\omega)) \leq (1 + r_n(\omega))d(\eta(\omega), \xi(\omega))$$

for each $\omega \in \Omega$ and $n \in \mathbb{N}$, where $\xi \in F(T) \neq \emptyset$ and $\eta : \Omega \to K$ is any measurable mapping.

(iv) an semicompact random mapping if for any sequence of measurable mappings $\{\xi_n(\omega)\} : \Omega \to K$, with $\lim_{n\to\infty} d(T(\omega, \xi_n(\omega)), \xi_n(\omega)) = 0$ for each $\omega \in \Omega$ and $n \in \mathbb{N}$, there exists a subsequence $\{\xi_{n_j}\}$ of $\{\xi_n\}$ which converges pointwise to ξ, where $\xi : \Omega \to K$ is a measurable mapping.

Remark 1.3. It is easy to see that if T is an asymptotically nonexpansive random mapping and $F(T) \neq \emptyset$, then T is an asymptotically quasi-nonexpansive random mapping.

Definition 1.4 ([19]). A convex structure in a metric space (X, d) is a mapping $W : X \times X \times [0, 1] \to X$ satisfying, for each $x, y, u \in X$ and each $\lambda \in [0, 1]$

$$d(u, W(x, y; \lambda)) \leq \lambda d(u, x) + (1 - \lambda)d(u, y).$$

A metric space together with a convex structure is called a convex metric space.

A nonempty subset K of X is said to be convex if $W(x, y; \lambda) \in K$ for all $(x, y; \lambda) \in K \times K \times [0, 1]$. The mapping $W : K \times K \times [0, 1] \to K$ is said to be a measurable convex structure if for any two measurable mappings $\xi, \eta : \Omega \to K$ and each fixed $\lambda \in [0, 1]$, the mapping $W(\xi(\cdot), \eta(\cdot); \lambda) : \Omega \to K$ is measurable.

In Banach spaces, Khan et al. [9] introduced the following iteration process for common fixed points of asymptotically quasi-nonexpansive mappings $\{T_i : i \in J = \{1, 2, \cdots, k\}\}$: any initial point $x_1 \in K$,

$$\begin{cases} x_{n+1} = (1-\alpha_{kn})x_n + \alpha_{kn}T_k^n y_{(k-1)n}, \\ y_{(k-1)n} = (1-\alpha_{(k-1)n})x_n + \alpha_{(k-1)n}T_{(k-1)}^n y_{(k-2)n}, \\ y_{(k-2)n} = (1-\alpha_{(k-2)n})x_n + \alpha_{(k-2)n}T_{(k-2)}^n y_{(k-3)n}, \\ \quad\vdots \\ y_{1n} = (1-\alpha_{1n})x_n + \alpha_{1n}T_1^n y_{0n}, \end{cases} \tag{1.1}$$

where $y_{0n} = x_n$ and $\{\alpha_{in}\}$ are real sequences in $[0, 1]$ for all $n \in \mathbb{N}$. And then, Khan and Ahmed [10] considered the iteration process (1.1) in convex metric spaces as follows:

$$\begin{cases} x_{n+1} = W(T_k^n y_{(k-1)n}, x_n; \alpha_{kn}), \\ y_{(k-1)n} = W(T_{k-1}^n y_{(k-2)n}, x_n; \alpha_{(k-1)n}), \\ y_{(k-2)n} = W(T_{k-2}^n y_{(k-3)n}, x_n; \alpha_{(k-2)n}), \\ \quad\vdots \\ y_{1n} = W(T_1^n y_{0n}, x_n; \alpha_{1n}), \end{cases} \tag{1.2}$$

where $y_{0n} = x_n$ and $\{\alpha_{in}\}$ are real sequences in $[0, 1]$ for all $n \in \mathbb{N}$.

From (1.1) and (1.2), we investigate the following random iteration process in convex metric space.

Definition 1.5. Let $\{T_i : i \in J\}$ be a finite familiy of asymptotically quasi-nonexpansive random mappings from $\Omega \times K$ to K , where K is a nonempty closed convex subset of a separable convex metric space (X, d). Let $\xi_1 : \Omega \to K$ be a measurable mapping, for each $\omega \in \Omega$, the sequence $\{\xi_n(\omega)\}$ is defined as follows:

$$\begin{cases} \xi_{n+1}(\omega) = W(T_k^n(\omega, \eta_{(k-1)n}(\omega)), \xi_n(\omega); \alpha_{kn}), \\ \eta_{(k-1)n}(\omega) = W(T_{k-1}^n(\omega, \eta_{(k-2)n}(\omega)), \xi_n(\omega); \alpha_{(k-1)n}), \\ \eta_{(k-2)n}(\omega) = W(T_{k-2}^n(\omega, \eta_{(k-3)n}(\omega)), \xi_n(\omega); \alpha_{(k-2)n}), \\ \quad\vdots \\ \eta_{1n}(\omega) = W(T_1^n(\omega, \eta_{0n}(\omega)), \xi_n(\omega); \alpha_{1n}), \end{cases} \tag{1.3}$$

where $\eta_{0n}(\omega) = \xi_n(\omega)$ and $\{\alpha_{in}\}$ are real sequences in $[0, 1]$ for all $n \in \mathbb{N}$.

We need the following two results for proving our main results.

Lemma 1.6 ([20]). *Let X be a separable metric space and Y be a metric space. If $f : \Omega \times X \to Y$ is measurable in $\omega \in \Omega$ and continuous in $x \in X$, and if $x : \Omega \to X$ is measurable, then $f(\cdot, x(\cdot)) : \Omega \to Y$ is measurable.*

Lemma 1.7 ([15]). *Let $\{\beta_n\}$ and $\{\gamma_n\}$ be sequences of nonnegative real numbers satisfying the following conditions:*

$$\beta_{n+1} \le (1+\gamma_n)\beta_n, \qquad \sum_{n=1}^{\infty} \gamma_n < \infty$$

We have

(i) $\lim_{n\to\infty} \beta_n$ *exists;*

(ii) if $\liminf_{n\to\infty} \beta_n = 0$*, then* $\lim_{n\to\infty} \beta_n = 0$.

2. Main results

In this section, we give some conditions for the convergence of the random iteration process (1.3) to a common random fixed point of a finite family asymptotically quasi-nonexpansive random mappings $\{T_i, i \in J\}$. We first prove the following lemma.

Lemma 2.1. *Let K be a nonempty closed convex subset of a separable complete convex metric space (X, d). Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of asymptotically quasi-nonexpansive random mappings with $r_{in}(\omega) : \Omega \to [0, \infty)$ for each $\omega \in \Omega$. Suppose that the sequence $\{\xi_n(\omega)\}$ is defined as (1.3) and $\sum_{n=1}^{\infty} \alpha_{kn} < \infty$. If $F = \bigcap_{i=1}^{k} F(T_i) \neq \emptyset$, then*

(i) *there exists a constant $M_0 > 0$ such that*

$$d(\xi_{n+1}(\omega), \xi(\omega)) \leq (1 + \alpha_{kn} M_0) d(\xi_n(\omega), \xi(\omega))$$

for all $\xi(\omega) \in F$ and $n \in \mathbb{N}$;

(ii) *there exists a constant $M_1 > 0$ such that*

$$d(\xi_{n+m}(\omega), \xi(\omega)) \leq M_1 d(\xi_n(\omega), \xi(\omega))$$

for all $\xi(\omega) \in F$ and $n, m \in \mathbb{N}$.

Proof. (i) Since $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of asymptotically quasi-nonexpansive random mappings with $r_{in} : \Omega \to [0, \infty)$ for each $\omega \in \Omega$, there exists a measurable mapping $r_n(\omega) = max\{r_{1n}(\omega), r_{2n}(\omega), \cdots, r_{kn}(\omega)\}$ for each $\omega \in \Omega$ with $\lim_{n\to\infty} r_n(\omega) = 0$, such that

$$d(T_i^n(\omega, \eta(\omega)), \xi(\omega)) \leq (1 + r_n(\omega)) d(\eta(\omega), \xi(\omega))$$

where $i \in J$ and $\eta : \Omega \to K$ is any measurable mapping. By (1.3), we have

$$\begin{aligned} d(\eta_{1n}(\omega), \xi(\omega)) =& d(W(T_1^n(\omega, \eta_{0n}(\omega)), \xi_n(\omega); \alpha_{1n}), \xi(\omega)) \\ \leq& \alpha_{1n} d(T_1^n(\omega, \eta_{0n}(\omega)), \xi(\omega)) + (1 - \alpha_{1n}) d(\xi_n(\omega), \xi(\omega)) \\ \leq& \alpha_{1n}(1 + r_n(\omega)) d(\xi_n(\omega), \xi(\omega)) + (1 - \alpha_{1n}) d(\xi_n(\omega), \xi(\omega)) \\ \leq& (1 + \alpha_{1n}(1 + r_n(\omega))) d(\xi_n(\omega), \xi(\omega)). \end{aligned}$$

Since $r_n(\omega) : \Omega \to [0, \infty)$ and $\lim_{n\to\infty} r_n(\omega) = 0$, there exists a constant $L > 0$ such that

$$L = \sup_{n\geq 1}\{1 + r_n(\omega)\} < \infty.$$

Therefore,

$$d(\eta_{1n}(\omega), \xi(\omega)) \leq (1 + L) d(\xi_n(\omega), \xi(\omega)).$$

Assume that

$$d(\eta_{in}(\omega), \xi(\omega)) \leq (1 + L)^i d(\xi_n(\omega), \xi(\omega))$$

holds for some $1 \leq i \leq k - 1$. Then

$$\begin{aligned} d(\eta_{(i+1)n}(\omega), \xi(\omega)) =& d(W(T_{i+1}^n(\omega, \eta_{in}(\omega)), \xi_n(\omega); \alpha_{(i+1)n}), \xi(\omega)) \\ \leq& \alpha_{(i+1)n} d(T_{i+1}^n(\omega, \eta_{in}(\omega)), \xi(\omega)) + (1 - \alpha_{(i+1)n}) d(\xi_n(\omega), \xi(\omega)) \\ \leq& \alpha_{(i+1)n}(1 + r_n(\omega)) d(\eta_{in}(\omega), \xi(\omega)) + (1 - \alpha_{(i+1)n}) d(\xi_n(\omega), \xi(\omega)) \\ \leq& (1 - \alpha_{(i+1)n} + \alpha_{(i+1)n} L(1 + L)^i) d(\xi_n(\omega), \xi(\omega)) \\ \leq& (1 + L(1 + L)^i) d(\xi_n(\omega), \xi(\omega)) \\ \leq& (1 + L)^{i+1} d(\xi_n(\omega), \xi(\omega)) \end{aligned}$$

So, by induction, we obtain

$$d(\eta_{in}(\omega), \xi(\omega)) \leq (1+L)^i d(\xi_n(\omega), \xi(\omega))$$

for all $1 \leq i \leq k$. Now, by (1.3) and the above inequality, we get

$$\begin{aligned} d(\xi_{n+1}(\omega), \xi(\omega)) =& d(W(T_k^n(\omega, \eta_{(k-1)n}(\omega)), \xi_n(\omega); \alpha_{kn}), \xi(\omega)) \\ \leq& \alpha_{kn} d(T_k^n(\omega, \eta_{(k-1)n}(\omega)), \xi(\omega)) + (1-\alpha_{kn}) d(\xi_n(\omega), \xi(\omega)) \\ \leq& \alpha_{kn}(1+r_n(\omega)) d(\eta_{(k-1)n}(\omega), \xi(\omega)) + (1-\alpha_{kn}) d(\xi_n(\omega), \xi(\omega)) \\ \leq& (1-\alpha_{kn}+\alpha_{kn} L(1+L)^k) d(\xi_n(\omega), \xi(\omega)) \\ \leq& (1+\alpha_{kn} M_0) d(\xi_n(\omega), \xi(\omega)) \end{aligned}$$

where $M_0 = (1+L)^k > 0$.

(ii)Notice that $1+x \leq e^x$ for all $x \geq 0$. Using this and $\sum_{n=1}^{\infty} \alpha_{kn} < \infty$, we have

$$\begin{aligned} d(\xi_{n+m}(\omega), \xi(\omega)) \leq& (1+\alpha_{k(n+m-1)} M_0) d(\xi_{n+m-1}(\omega), \xi(\omega)) \\ \leq& e^{\alpha_{k(n+m-1)} M_0} (1+\alpha_{k(n+m-2)} M_0) d(\xi_{n+m-2}(\omega), \xi(\omega)) \\ \leq& e^{[\alpha_{k(n+m-1)}+\alpha_{k(n+m-2)}] M_0} d(\xi_{n+m-2}(\omega), \xi(\omega)) \\ & \cdots\cdots \\ \leq& e^{M_0 \Sigma_{j=1}^{\infty} \alpha_{kj}} d(\xi_n(\omega), \xi(\omega)) \\ \leq& M_1 d(\xi_n(\omega), \xi(\omega)), \end{aligned}$$

where $M_1 = e^{M_0 \Sigma_{j=1}^{\infty} \alpha_{kj}} > 0$. □

Theorem 2.2. *Let K be a nonempty closed convex subset of a separable complete convex metric space (X, d) with a measurable convex structure W. Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of continuous asymptotically quasi-nonexpansive random mappings with $r_{in}(\omega) : \Omega \to [0, \infty)$ for each $\omega \in \Omega$. Suppose that the sequence $\{\xi_n(\omega)\}$ is defined as (1.3) and $\sum_{n=1}^{\infty} \alpha_{kn} < \infty$. If $F = \bigcap_{i=1}^{k} F(T_i) \neq \emptyset$, then $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$ if and only if $\liminf_{n\to\infty} d(\xi_n(\omega), F) = 0$, where $d(\xi_n(\omega), F) = \inf\{d(\xi_n(\omega), \eta(\omega)) : \forall \eta(\omega) \in F\}$ for each $\omega \in \Omega$.*

Proof. The necessity is obvious. Thus, we only need prove the sufficiency. From Lemma 2.1 (i), we have

$$d(\xi_{n+1}(\omega), F) \leq (1+\alpha_{kn} M_0) d(\xi_n(\omega), F).$$

By Lemma 1.7 and $\sum_{n=1}^{\infty} \alpha_{kn} < \infty$, we know that

$$\lim_{n\to\infty} d(\xi_n(\omega), F)$$

exists. Since $\liminf_{n\to\infty} d(\xi_n(\omega), F) = 0$, we obtain

$$\lim_{n\to\infty} d(\xi_n(\omega), F) = 0$$

for each $\omega \in \Omega$.

Next, We show that $\{\xi_n(\omega)\}$ is a Cauchy sequence. Indeed, for any $\varepsilon > 0$, there exists a constant N_0 such that for all $n \geq N_0$, we have

$$d(\xi_n(\omega), F) \leq \frac{\varepsilon}{2M_1}.$$

In particular, there exist a $p_1(\omega) \in F$ and a constant $N_1 > N_0$ such that

$$d(\xi_{N_1}(\omega), p_1(\omega)) \leq \frac{\varepsilon}{2M_1}.$$

It follows from Lemma 2.1 (ii) that for $n > N_1$, we have

$$\begin{aligned} d(\xi_{n+m}(\omega), \xi_n(\omega)) \leq& d(\xi_{n+m}(\omega), p_1(\omega)) + d(p_1(\omega), \xi_n(\omega)) \\ \leq& M_1 d(\xi_{N_1}(\omega), p_1(\omega)) + M_1 d(\xi_{N_1}(\omega), p_1(\omega)) \\ \leq& 2M_1 \frac{\varepsilon}{2M_1} = \varepsilon. \end{aligned}$$

This implies that $\{\xi_n\}$ is a Cauchy sequence in closed convex subset of a complete convex metric space. Therefore, $\{\xi_n(\omega)\}$ converges to a point in K.

Suppose $\lim_{n\to\infty} \xi_n(\omega) = p(\omega)$ for each $\omega \in \Omega$. Since T_i are continuous, by Lemma 1.6, we know that for any measurable mapping $f : \Omega \to K$, $T_i^n(\omega, f(\omega)) : \Omega \to K$ are measurable mappings. Thus, $\{\xi_n(\omega)\}$ is a sequence of measurable mappings. Hence, $p(\omega) : \Omega \to K$ is also measurable. Notice that

$$d(p(\omega), F) \leq d(\xi_n(\omega), p(\omega)) + d(\xi_n(\omega), F),$$

together with $\lim_{n\to\infty} d(\xi_n(\omega), F) = 0$ and $\lim_{n\to\infty} d(\xi_n(\omega), p(\omega)) = 0$, we can conclude that $d(p(\omega), F) = 0$. Therefore, $p(\omega) \in F$.

□

Remark 2.3. (i) *Theorem 2.2 extends the corresponding results in [1, 2, 5, 6, 8, 11, 12, 13, 14, 17, 16] to the convex metric space, which is a more general space;*

(ii) *Theorem 2.2 extends the corresponding results in [4, 9, 10, 20, 21, 22] to a finite family of asymptotically quasi-nonexpansive random mappings, which are stochastic generalizations of asymptotically quasi-nonexpansive mappings;*

(iii) *In Theorem 2.2, we remove the condition: "* $\sum_{n=1}^{\infty} r_{in} < \infty, i \in J$*", which is required in many other papers (see, e.g., [1, 2, 4, 9, 10, 16, 20, 22]). And the condition "*$\sum_{n=1}^{\infty} \alpha_{in} < \infty, i \in J$*" is replaced with "*$\sum_{n=1}^{\infty} \alpha_{kn} < \infty$*".*

By Remark 1.3, we can get the following result:

Corollary 2.4. *Let K be a nonempty closed convex subset of a separable complete convex metric space (X, d) with a measurable convex structure W. Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of asymptotically nonexpansive random mappings with $r_{in}(\omega) : \Omega \to [0, \infty)$ for each $\omega \in \Omega$. Suppose that the sequence $\{\xi_n(\omega)\}$ is defined as* (1.3) *and $\sum_{n=1}^{\infty} \alpha_{kn} < \infty$. If $F = \bigcap_{i=1}^{k} F(T_i) \neq \emptyset$, then $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$ if and only if $\liminf_{n\to\infty} d(\xi_n(\omega), F) = 0$, where $d(\xi_n(\omega), F) = \inf\{d(\xi_n(\omega), \eta(\omega)) : \forall \eta(\omega) \in F\}$ for each $\omega \in \Omega$.*

Theorem 2.5. *Let K be a nonempty closed convex subset of a separable complete convex metric space (X, d) with a measurable convex structure W. Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of continuous asymptotically quasi-nonexpansive random mappings with $r_{in}(\omega) : \Omega \to [0, \infty)$ for each $\omega \in \Omega$. Suppose that the sequence $\{\xi_n(\omega)\}$ is defined as* (1.3) *,$\sum_{n=1}^{\infty} \alpha_{kn} < \infty$ and $F = \bigcap_{i=1}^{k} F(T_i) \neq \emptyset$. If for some given $1 \leq l \leq k$ and each $\omega \in \Omega$,*

(i) $\lim_{n\to\infty} d(T_l(\omega, \xi_n(\omega)), \xi_n(\omega)) = 0,$

(ii) *there exists a constant $M_2 > 0$ such that*

$$d(T_l(\omega, \xi_n(\omega)), \xi_n(\omega)) \geq M_2 d(\xi_n(\omega), F).$$

Then $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$.

Proof. From the conditions (i) and (ii), we have

$$\lim_{n\to\infty} d(\xi_n(\omega), F) = 0.$$

Therefore, from the proof of Theorem 2.2, we know that $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$ □

Theorem 2.6. *Let K be a nonempty closed convex subset of a separable complete convex metric space (X, d) with a measurable convex structure W. Let $\{T_i : i \in J\} : \Omega \times K \to K$ be a finite family of continuous asymptotically quasi-nonexpansive random mappings with $r_{in}(\omega) : \Omega \to [0, \infty)$ for each $\omega \in \Omega$. Suppose that the sequence $\{\xi_n(\omega)\}$ is defined as (1.3), $\sum_{n=1}^{\infty} \alpha_{kn} < \infty$ and $F = \bigcap_{i=1}^{k} F(T_i) \neq \emptyset$. If*

(i) *for all $1 \le i \le k$ and each $\omega \in \Omega$, $\lim\limits_{n\to\infty} d(T_i(\omega, \xi_n(\omega)), \xi_n(\omega)) = 0$;*

(ii) *for some $1 \le l' \le k$, $T_{l'}$ is semicompact.*

Then $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$.

Proof. Since $T_{l'}$ is semicompact and $\lim_{n\to\infty} d(T_{l'}(\omega, \xi_n(\omega)), \xi_n(\omega)) = 0$, there exists a subsequence $\{\xi_{n_j}(\omega)\} \subset \{\xi_n(\omega)\}$ such that $\lim_{j\to\infty} \xi_{n_j}(\omega) = \xi'(\omega)$ for each $\omega \in \Omega$. Since T_i are continuous, it follows that $\{\xi_n\}$ is a sequence of measurable mappings. Therefore, $\xi'(\omega) : \Omega \to K$ is also measurable. Hence, it follows from

$$d(T_i(\omega, \xi'(\omega)), \xi'(\omega)) = \lim_{n\to\infty} d(T_i(\omega, \xi_{n_j}(\omega)), \xi_{n_j}(\omega)) = 0$$

that $\xi'(\omega) \in F$. By Lemma 2.1 (i), we have

$$d(\xi_{n+1}(\omega), \xi'(\omega)) \le (1 + \alpha_{kn} M_0) d(\xi_n(\omega), \xi'(\omega)).$$

According to Lemma 1.7 and $\sum_{n=1}^{\infty} \alpha_{kn} < \infty$, there exists a constant $\delta \ge 0$ such that

$$\lim_{n\to\infty} d(\xi_n(\omega), \xi'(\omega)) = \delta.$$

Since $\lim\limits_{j\to\infty} \xi_{n_j}(\omega) = \xi'(\omega)$, we have $\delta = 0$. Therefore, $\{\xi_n(\omega)\}$ converges to a common fixed point of $\{T_i : i \in J\}$. □

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