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A note on common fixed point theorems for isotone increasing mappings in ordered b-metric spaces

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Abstract

In this article we prove the existence of common fixed points for isotone increasing mappings in ordered b-metric spaces. Our results unite and improve the recent remarkable results, established by Roshan et al. [J. R. Roshan, V. Parvaneh, Z. Kadelburg, J. Nonlinear Sci. Appl. 7 (2014), 229–245], with much more general conditions and shorter proofs. An example is given to show the superiority of our genuine generalization.

Keywords: Common fixed point, b-metric space, g-weakly isotone increasing, well ordered.
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1. Introduction and Preliminaries

Fixed point theory has fascinated thousands of researchers during the last several decades. This is because it has extensive application prospects, such as physics, engineering, economics, applied science. Many practical problems in these aspects, can be reformulated directly as a problem of finding fixed points of nonlinear mappings. The celebrated Banach fixed point theorem [4] is a fundamental result in fixed point theory. It guarantees the existence and uniqueness of fixed points of certain contractive self-maps in complete metric spaces. Also, it provides a constructive method to approximate the fixed points. In recent years, this famous theorem has be extended and generalized in a variety of forms. One of the most important generalizations is generalization of spaces. Whereas, one influential generalization is b-metric spaces [3] or metric type spaces (see [6, 8, 9, 18]).

In order to start this paper, we first need to briefly recall some basic terms and notions as follows.

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Definition 1.1 ([3]). Let X be a (nonempty) set and $s \geq 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is called a b-metric on X if, for all $x, y, z \in X$, it satisfies

($b1$) $d(x, y) = 0$ if and only if $x = y$;

($b2$) $d(x, y) = d(y, x)$;

($b3$) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

In this case, the pair (X, d) is called a b-metric space or metric type space.

Further, for more notions such as b-convergence, b-completeness, b-Cauchy sequence in the setting of b-metric spaces, the reader refers to [1, 2, 5, 6, 7, 8, 9, 10, 13, 14, 16].

Definition 1.2 ([15]). A triple $(X, \preceq, d)$ is called a partially ordered b-metric space if $(X, \preceq)$ is a partially ordered set and d is a b-metric on X.

Let $(X, \preceq)$ be a partially ordered set and let f, g be two self-maps on X. We shall utilize the following terminology from [16].

(1) elements $x, y \in X$ are called comparable if $x \preceq y$ or $y \preceq x$ holds;

(2) a subset K of X is said to be well ordered if every two elements of K are comparable;

(3) f is called nondecreasing w.r.t. $\preceq$ if $x \preceq y$ implies $fx \preceq fy$;

(4) the pair (f, g) is said to be weakly increasing if $fx \preceq gfx$ and $gx \preceq fgx$ for all $x \in X$;

(5) f is said to be g-weakly isotone increasing if $fx \preceq gfx \preceq fgfx$ for all $x \in X$.

Note that two weakly increasing mappings need not be nondecreasing (see for example [2] from [16]). If $f, g : X \to X$ are weakly increasing, then f is a g-weakly isotone increasing. Also, if $f = g$ in (5), we say that f is weakly isotone increasing. In this case for each $x \in X$, we have $fx \preceq ffx$.

Definition 1.3 ([7]). An ordered b-metric space $(X, \preceq, d)$ is called regular if one of the following conditions holds:

(r1) if for any nondecreasing sequence $\{x_n\}$ in X such that $x_n \to x$, as $n \to \infty$, one has $x_n \preceq x$ for all $n \in \mathbb{N}$;

(r2) if for any nonincreasing sequence $\{y_n\}$ in X such that $y_n \to y$, as $n \to \infty$, one has $y_n \succeq y$ for all $n \in \mathbb{N}$.

Otherwise, fixed point results in partially ordered metric spaces were firstly obtained by Ran and Reurings [15] and then by Nieto and López [11, 12]. Afterwards, many authors obtained numerous interesting results in ordered metric spaces as well as in ordered b-metric spaces (see [1, 7, 11, 12, 13, 15, 16, 17]).

Recently, in [16] authors introduced and proved the following:

Let $(X, \preceq, d)$ be an ordered b-metric space with $s > 1$, and $f, g : X \to X$ be two mappings. For all $x, y \in X$, set

$$M_s(x, y) = \max\left\{\psi(d(x, y)), \psi(d(x, fx)), \psi(d(y, gy)), \psi\left(\frac{d(x, gy) + d(y, fx)}{2s}\right)\right\},$$

where $\psi : [0, \infty) \to [0, \infty)$ is a continuous function with $\psi(t) < t$ for $t > 0$ and $\psi(0) = 0$.

Theorem 1.4 ([16]). *Let $(X, \preceq, d)$ be a complete partially ordered b-metric space with $s > 1$. Let $f, g : X \to X$ be two mappings such that f is g-weakly isotone increasing. Suppose that for every two comparable elements $x, y \in X$, we have*

$$s^4 d(fx, gy) \leq M_s(x, y). \tag{1.1}$$

Then, the pair (f, g) has a common fixed point z in X if one of f or g is continuous (resp. $(X, \preceq, d)$ is regular). Moreover, the set of common fixed points of f and g is well ordered if and only if f and g have one and only one common fixed point.

Taking $f = g$ in Theorem 1.4, ones obtained the following fixed point results:

Corollary 1.5 ([16]). *Let $(X, \preceq, d)$ be a complete partially ordered b-metric space with $s > 1$. Let $f : X \to X$ be a mapping such that f is weakly isotone increasing. Suppose that for every two comparable elements $x, y \in X$ we have*

$$s^4 d(fx, fy) \leq M_s(x, y), \tag{1.2}$$

where

$$M_s(x, y) = \max\left\{\psi(d(x, y)), \psi(d(x, fx)), \psi(d(y, fy)), \psi\left(\frac{d(x, fy) + d(y, fx)}{2s}\right)\right\}.$$

Then f has a fixed point z in X if either:
(a) f is continuous, or
(b) $(X, \preceq, d)$ is regular.
Moreover, the set of fixed points of f is well ordered if and only if f has one and only one fixed point.

Further, in [16] authors presented the following result for so-called quasicontractions:

Theorem 1.6 ([16]). *Let $(X, \preceq, d)$ be a complete partially ordered b-metric space with $s > 1$. Let $f, g : X \to X$ be two mappings such that f is g-weakly isotone increasing. Suppose that for every two comparable elements $x, y \in X$, we have*

$$s^4 d(fx, gy) \leq N(x, y), \tag{1.3}$$

where

$$N(x, y) = \max\{\psi(d(x, y)), \psi(d(x, fx)), \psi(d(y, gy)), \psi(d(x, gy)), \psi(d(y, fx))\},$$

and $\psi : [0, \infty) \to [0, \infty)$ is a continuous function with $\psi(t) < \frac{t}{2s}$ for each $t > 0$ and $\psi(0) = 0$. Then, the pair (f, g) has a common fixed point z in X if one of f or g is continuous (resp. $(X, \preceq, d)$ is regular). Moreover, the set of common fixed points of f and g is well ordered if and only if f and g have one and only one common fixed point.

It needs mentioning that the following two crucial lemmas are used often in proving of all main results in [16].

Lemma 1.7 ([16]). *Let (X, d) be a b-metric space with $s \geq 1$, and suppose that $\{x_n\}$ and $\{y_n\}$ are b-convergent to x, y, respectively. Then we have*

$$\frac{1}{s^2} d(x, y) \leq \liminf_{n\to\infty} d(x_n, y_n) \leq \limsup_{n\to\infty} d(x_n, y_n) \leq s^2 d(x, y).$$

In particular, if $x = y$, then we have $\lim_{n\to\infty} d(x_n, y_n) = 0$. Moreover, for each $z \in X$, we have

$$\frac{1}{s} d(x, z) \leq \liminf_{n\to\infty} d(x_n, z) \leq \limsup_{n\to\infty} d(x_n, z) \leq s d(x, z).$$

Lemma 1.8 ([16]). *Let (X, d) be a b-metric space with $s \geq 1$ and let $\{x_n\}$ be a sequence in X such that*

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0.$$

If $\{x_n\}$ is not a b-Cauchy sequence, then there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that for the following four sequences

$$d\left(x_{m(k)}, x_{n(k)}\right), d\left(x_{m(k)}, x_{n(k)+1}\right), d\left(x_{m(k)+1}, x_{n(k)}\right), d\left(x_{m(k)+1}, x_{n(k)+1}\right),$$

it holds:

$$\varepsilon \leq \liminf_{k\to\infty} d\left(x_{m(k)}, x_{n(k)}\right) \leq \limsup_{k\to\infty} d\left(x_{m(k)}, x_{n(k)}\right) \leq s\varepsilon,$$

$$\frac{\varepsilon}{s} \le \liminf_{k\to\infty} d\left(x_{m(k)}, x_{n(k)+1}\right) \le \limsup_{k\to\infty} d\left(x_{m(k)}, x_{n(k)+1}\right) \le s^2\varepsilon,$$
$$\frac{\varepsilon}{s} \le \liminf_{k\to\infty} d\left(x_{m(k)+1}, x_{n(k)}\right) \le \limsup_{k\to\infty} d\left(x_{m(k)+1}, x_{n(k)}\right) \le s^2\varepsilon,$$
$$\frac{\varepsilon}{s^2} \le \liminf_{k\to\infty} d\left(x_{m(k)+1}, x_{n(k)+1}\right) \le \limsup_{k\to\infty} d\left(x_{m(k)+1}, x_{n(k)+1}\right) \le s^3\varepsilon.$$

2. Main results

In this section, we will prove that the conclusions about common fixed points in the previous results are valid under much more general assumption than (1.1), (1.2) and (1.3). Also, we will dismiss Lemma 1.7 and Lemma 1.8 with shorter proofs as compared to the proofs of all main results of [16].

First of all, we introduce the following denotations.

Let $(X, \preceq, d)$ be an ordered b-metric space with $s > 1$, and $f, g : X \to X$ be two mappings. For all $x, y \in X$, set

$$M_s^{f,g}(x, y) = \max\left\{d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(y, fx)}{2s}\right\}.$$

Theorem 2.1. *Let $(X, \preceq, d)$ be a complete partially ordered b-metric space with $s > 1$, and $f, g : X \to X$ be two mappings such that f is g-weakly isotone increasing. Suppose that for every two comparable elements $x, y \in X$, we have*

$$s^{\varepsilon} d(fx, gy) \le M_s^{f,g}(x, y), \tag{2.1}$$

where $\varepsilon > 1$ is a constant. Then the pair (f, g) has a common fixed point z in X if one of f or g is continuous (resp. $(X, \preceq, d)$ is regular). Moreover, the set of common fixed points of f and g is well ordered if and only if f and g have one and only one common fixed point.

Proof. Let $x_0 \in X$ be arbitrary and form a sequence $\{x_n\}$ such that $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$ for all $n \ge 0$. As f is g-weakly isotone increasing, we have

$$x_1 = fx_0 \preceq gfx_0 = gx_1 = x_2 \preceq fgfx_0 = fx_2 = x_3.$$

Continuing this process, we obtain $x_n \preceq x_{n+1}$, for all $n \ge 1$.

In the sequel, we shall complete the proof in the following three steps.

Step I. We prove that

$$d(x_{n+1}, x_{n+2}) \le \lambda d(x_n, x_{n+1}) \tag{2.2}$$

for all $n \ge 1$, where $\lambda \in [0, \frac{1}{s})$.

On the one hand, since x_{2n} and x_{2n+1} are comparable, then by (2.1), we get that

$$\begin{aligned} s^{\varepsilon} d(x_{2n+1}, x_{2n+2}) &= s^{\varepsilon} d(fx_{2n}, gx_{2n+1}) \le M_s^{f,g}(x_{2n}, x_{2n+1}) \\ &= \max\left\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}), \frac{d(x_{2n}, x_{2n+2}) + 0}{2s}\right\} \\ &\le \max\left\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}), \frac{d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})}{2}\right\} \\ &\le \max\left\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\right\}. \end{aligned} \tag{2.3}$$

If $d(x_{2n}, x_{2n+1}) \le d(x_{2n+1}, x_{2n+2})$, then (2.3) follows that

$$s^{\varepsilon} d(x_{2n+1}, x_{2n+2}) \le d(x_{2n+1}, x_{2n+2}). \tag{2.4}$$

In view of $s^{\varepsilon} > 1$, (2.4) leads to $d(x_{2n+1}, x_{2n+2}) = 0$, hence $d(x_{2n}, x_{2n+1}) = 0$. This implies that (2.2) holds trivially. If $d(x_{2n+1}, x_{2n+2}) \leq d(x_{2n}, x_{2n+1})$, then

$$s^{\varepsilon} d(x_{2n+1}, x_{2n+2}) \leq d(x_{2n}, x_{2n+1}). \tag{2.5}$$

On the other hand, since x_{2n} and x_{2n-1} are comparable, then by (2.1), it may be verified that

$$\begin{aligned} s^{\varepsilon} d(x_{2n}, x_{2n+1}) &= s^{\varepsilon} d(x_{2n+1}, x_{2n}) \\ &= s^{\varepsilon} d(fx_{2n}, gx_{2n-1}) \leq M_s^{f,g}(x_{2n}, x_{2n-1}) \\ &= \max\left\{ d(x_{2n}, x_{2n-1}), d(x_{2n}, x_{2n+1}), \frac{0 + d(x_{2n-1}, x_{2n+1})}{2s} \right\} \\ &\leq \max\left\{ d(x_{2n}, x_{2n-1}), d(x_{2n}, x_{2n+1}), \frac{d(x_{2n-1}, x_{2n}) + d(x_{2n}, x_{2n+1})}{2} \right\} \\ &\leq \max\{ d(x_{2n-1}, x_{2n}), d(x_{2n}, x_{2n+1}) \}. \end{aligned} \tag{2.6}$$

If $d(x_{2n-1}, x_{2n}) \leq d(x_{2n}, x_{2n+1})$, then (2.6) establishes that

$$s^{\varepsilon} d(x_{2n}, x_{2n+1}) \leq d(x_{2n}, x_{2n+1}). \tag{2.7}$$

By virtue of $s^{\varepsilon} > 1$, (2.7) implies $d(x_{2n}, x_{2n+1}) = 0$, thus $d(x_{2n-1}, x_{2n}) = 0$. This means that (2.2) holds trivially. If $d(x_{2n}, x_{2n+1}) \leq d(x_{2n-1}, x_{2n})$, then

$$s^{\varepsilon} d(x_{2n}, x_{2n+1}) \leq d(x_{2n-1}, x_{2n}). \tag{2.8}$$

Combining (2.5) and (2.8), we claim that (2.2) holds, where $\lambda = \frac{1}{s^{\varepsilon}} \in [0, \frac{1}{s})$.

Step II. We show that f has a fixed point z.

Actually, by Step I and [8, Lemma 3.1], we conclude that $\{x_n\}$ is a b-Cauchy sequence. Now that (X, d) is b-complete, then $\{x_n\}$ b-converges to some $z \in X$. In order to end $fz = z$, we divide it into two cases.

Case 1. Let f or g be continuous. Without loss of generality, we assume that f is continuous. Then, using the triangle inequality, we arrive at

$$\frac{1}{s} d(z, fz) \leq d(z, fx_{2n}) + d(fx_{2n}, fz). \tag{2.9}$$

Letting $n \to \infty$ in (2.9) and applying the fact that $x_{2n+1} = fx_{2n}$ b-converges to z, we get

$$\frac{1}{s} d(z, fz) \leq \lim_{n\to\infty} d(z, fx_{2n}) + \lim_{n\to\infty} d(fx_{2n}, fz) = 0,$$

which implies that $fz = z$.

Case 2. Let $(X, \preceq, d)$ be a regular ordered b-metric space. Using this hypothesis and $x_n \preceq x_{n+1}$ $(n \geq 1)$ together with $x_n \to z$ $(n \to \infty)$, we infer $x_n \preceq z$ for all $n \geq 1$. Next we show that $fz = z$. In fact, for one thing, we have

$$\frac{1}{s} d(fz, z) \leq d(fz, gx_{2n+1}) + d(x_{2n+2}, z). \tag{2.10}$$

For another thing, we deduce from (2.1) that

$$\begin{aligned} &d(fz, gx_{2n+1}) \\ &\leq \frac{1}{s^{\varepsilon}} \max\left\{ d(z, x_{2n+1}), d(z, fz), d(x_{2n+1}, x_{2n+2}), \frac{d(z, x_{2n+2}) + d(x_{2n+1}, fz)}{2s} \right\} \\ &\leq \frac{1}{s^{\varepsilon}} \max\left\{ d(z, x_{2n+1}), d(z, fz), d(x_{2n+1}, x_{2n+2}), \frac{d(z, x_{2n+2})}{2s} + \frac{d(x_{2n+1}, z) + d(z, fz)}{2} \right\}. \end{aligned} \tag{2.11}$$

Taking the limits as $n \to \infty$ from (2.10) and (2.11), we demonstrate that

$$\frac{1}{s} d(fz, z) \leq \frac{1}{s^{\varepsilon}} \max\left\{0, d(z, fz), 0, 0 + \frac{0 + d(z, fz)}{2}\right\} + 0 = \frac{1}{s^{\varepsilon}} d(z, fz). \tag{2.12}$$

Clearly, (2.12) is a contradiction too if $d(z, fz) > 0$. In other words, $fz = z$.

Step III. We claim $z \in X$ is a fixed point of f if and only if z is a fixed point of g.

As a matter of fact, let $z \in X$ be a fixed point of f, that is, $fz = z$. Then we shall prove $gz = z$, i.e., $d(z, gz) = 0$. Indeed, we suppose for absurd that $d(z, gz) > 0$. Note that z and z are comparable, then by (2.1), it ensures us that

$$s^{\varepsilon} d(z, gz) = s^{\varepsilon} d(fz, gz) \leq M_s^{f,g}(z, z), \tag{2.13}$$

where

$$\begin{aligned} M_s^{f,g}(z, z) &= \max\left\{d(z, z), d(z, fz), d(z, gz), \frac{d(z, gz) + d(z, fz)}{2s}\right\} \\ &= \max\left\{d(z, gz), \frac{d(z, gz)}{2s}\right\} = d(z, gz). \end{aligned} \tag{2.14}$$

Accordingly, (2.13) and (2.14) imply that $s^{\varepsilon} d(z, gz) \leq d(z, gz)$. This is a contradiction. So $d(z, gz) = 0$.

Conversely, let $z \in X$ be a fixed point of g. Making full use of the same method, we are not hard to verify that $z \in X$ is also a fixed point of f.

Finally, by steps II and III, we claim that z is a common fixed point of f and g. □

Corollary 2.2. *Let $(X, \preceq, d)$ be a complete partially ordered b-metric space with $s > 1$. Let $f : X \to X$ be a mapping such that f is weakly isotone increasing. Suppose that for every two comparable elements $x, y \in X$ we have*

$$s^{\varepsilon} d(fx, fy) \leq M_s^{f}(x, y), \tag{2.15}$$

where $\varepsilon > 1$ is a constant and

$$M_s^{f}(x, y) = \max\left\{d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2s}\right\}.$$

Then f has a fixed point z in X if either:

(a) f is continuous, or

(b) $(X, \preceq, d)$ is regular.

Moreover, the set of fixed points of f is well ordered if and only if f has one and only one fixed point.

Proof. Taking $f = g$ in Theorem 2.1, we obtain the desired result. □

We present a result for so-called quasicontraction.

Theorem 2.3. *Let $(X, \preceq, d)$ be a a complete partially ordered b-metric space with $s > 1$, and $f, g : X \to X$ be two mappings such that f is a g-weakly isotone increasing. Suppose that for every two comparable elements $x, y \in X$, we have*

$$s^{\varepsilon} d(fx, gy) \leq N_s^{f,g}(x, y),$$

where $\varepsilon > 1$ is a constant and

$$N_s^{f,g}(x, y) = \max\left\{d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy)}{s}, \frac{d(y, fx)}{s}\right\}.$$

Then, the pair (f, g) has a common fixed point z in X if one of f or g is continuous (resp. $(X, \preceq, d)$ is regular). Moreover, the set of common fixed points of f and g is well ordered if and only if f and g have one and only one common fixed point.

Proof. The proof of this theorem including both cases (f or g is continuous and $(X, \preceq, d)$ is regular) is very similar to the proof of previous results. Thus we omit it. □

Remark 2.4. It is clear that Theorem 2.1, Corollary 2.2 and Theorem 2.3 improve and generalize all results of [16] (also see Theorem 1.4, Corollary 1.5, Theorem 1.6) in several directions. Indeed, comparing with the conditions of the main results of [16], we delete the assumption of the function ψ. Moreover, because of $\varepsilon > 1$, our condition is much more general than the counterpart of [16]. Further, our proofs are very compact since they have nothing to do with Lemma 1.7 and Lemma 1.8 but the proofs of [16] are strongly dependent on these two lemmas. As a consequence, our conclusions are more useful and meaningful in applications. The following example shows the superiority of our assertions.

Example 2.5. (see [16, Example 2.7]) Let $X = \{0, 1, 2\}$ and $d : X \times X \to [0, \infty)$ be defined by $d(x, x) = 0$ for all $x \in X, d(0, 1) = d(1, 0) = d(1, 2) = d(2, 1) = 1, d(0, 2) = d(2, 0) = \frac{9}{4}$. Then, (X, d) is a b-metric space with $s = \frac{9}{8}$, which is not a metric space. Define an order on X by $\preceq := \{(0, 0), (1, 1), (2, 2), (2, 0), (2, 1)\}$ and obtain a complete ordered b-metric space. Consider the mapping $f : X \to X$ given by $f0 = f2 = 0, f1 = 1$. The mapping f is obviously weakly isotone increasing and continuous. The contractive condition (2.15) needs to be checked only for $x = 2, y = 1$. For this case we get that

$$s^{\varepsilon} d(f2, f1) = \left(\frac{9}{8}\right)^{\varepsilon} d(0, 1) = \left(\frac{9}{8}\right)^{\varepsilon}$$

and

$$M^{f}_{\frac{9}{8}}(2, 1) = \max\left\{d(2, 1), d(2, f2), d(1, f1), \frac{4}{9}\left[d(2, f1) + d(1, f2)\right]\right\} = \frac{9}{4}.$$

Now all the conditions of Corollary 2.2 are satisfied, if for example $\varepsilon = 3$. The mapping has a unique fixed point $z = 0$. However, if $\varepsilon = 4$ and ψ is arbitrary (for instance $\psi(t) = kt, k \leq \frac{729}{1024}$), then Corollary 2.4 from [16] (also see Corollary 1.5) is false.

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