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Approximating common fixed points for a pair of generalized nonlinear mappings in convex metric space

Chao Wang*, Taizhong Zhang

School of Mathematics and Statistics, Nanjing University of Information Science and Technology, Nanjing 210044, P. R. China

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Abstract

In this paper, a pair of generalized nonlinear mappings are introduced. Sufficient conditions for the existence of common fixed points for a pair of generalized nonlinear mappings in convex metric spaces are obtained and Krasnoselskii type iterations are used to approximate common fixed points. Our results generalize and extend various known results.

Keywords: Nonlinear mappings, common fixed point, convex metric spaces, existence conditions, Krasnoselskii type iterations.
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1. Introduction and Preliminaries

In the past few decades, fixed point theorems for contractive type mappings have been extensively studied by many authors in the framework of Banach spaces and metric spaces [2, 4, 5, 8, 14]. If the domain is a closed convex subset, some common fixed point theorems for generalized contractive mappings in uniformly convex Banach spaces are proved in [1]. In 1984, Wang *et al.* [13] gave some fixed point theorems for expansive type mappings which correspond to some contractive type mappings. Later, Daffer and Kaneko [3] defined a pair of expansive type mappings and prove some common fixed point theorems for two mappings in metric spaces. In metric spaces, Takahashi [9] introduced a convex structure:

*Corresponding author
Email addresses: wangchaosx@126.com (Chao Wang), ztz@nuist.edu.cn (Taizhong Zhang)

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A convex structure in a metric space (X, d) is a mapping $W : X \times X \times [0, 1] \to X$ satisfying, for each $x, y, u \in X$ and $\lambda \in [0, 1]$,

$$d(u, W(x, y; \lambda)) \le \lambda d(u, x) + (1 - \lambda)d(u, y).$$

A metric space (X, d) together with a convex structure W is called a convex metric space (X, d, W). Moreover, a nonempty subset K of X is said to be convex if $W(x, y; \lambda) \in K$ for all $(x, y; \lambda) \in K \times K \times [0, 1]$.

In fact, every normed space and its convex subset are special examples of convex metric spaces. Many authors ([6, 7, 10, 11, 12]) have considered some existence and convergence theorems for fixed points of contractive type mappings in convex metric spaces.

Inspired and motived by the above results, we first define a pair of generalized nonlinear mappings (which include many contractive type and expansive type mappings), if the domain is a closed convex subset of a convex metric space, we study some sufficient conditions for existence of common fixed points for a pair of generalized nonlinear mappings and use Krasnoselskii type iterations to approximate common fixed points. Our results generalize and improve the corresponding results in [1, 2, 3, 4, 5, 6, 7, 8, 10, 13, 14].

Definition 1.1. Let K be a nonempty subset of a metric space (X, d). Two mappings $T, S : K \to K$ are said to be a pair of generalized nonlinear mappings if there exist k, a, b, c such that

$$kd(Tx, Sy) \le ad(x, y) + b[d(x, Tx) + d(y, Sy)] + c[d(x, Sy) + d(y, Tx)] \tag{1.1}$$

for all $x, y \in K$.

It is easy to see that contractive and expansive type mappings considered in [1, 2, 3, 4, 5, 6, 8, 10, 13, 14] can be obtain from a pair of generalized nonlinear mappings (1.1) by suitably choosing the mappings T, S and the coefficients (k, a, b, c). For example, let $T = S$ and $k = 1,\ a \ge 0,\ b \ge 0,\ c \ge 0$, a pair of generalized nonlinear mappings (1.1) change into the generalized contractive mapping in [2, 4, 8, 10].

We need the following notations for a pair of generalized nonlinear mappings (1.1):

$$\Gamma_1 = \bigcup_{\lambda\in[0,1)} A_\lambda, \quad \Gamma_2 = \bigcup_{\lambda\in[0,1)} B_\lambda, \quad \Gamma_3 = \bigcup_{\lambda\in[0,1)} C_\lambda, \quad \Gamma_4 = \bigcup_{\lambda\in[0,1)} D_\lambda,$$

where

$$\begin{aligned}
A_\lambda &= \{(k, a, b, c)|\ b + |c| \le a + 2b + c + |c| + (|k| - a)\lambda < k\},\\
B_\lambda &= \{(k, a, b, c)|\ b + c \le a + 2b + c + |c| + (|k| + c - |c| - a)\lambda < k\},\\
C_\lambda &= \{(k, a, b, c)|\ b + |c| \le a + 2b + c + |c| - (k + a)\lambda < -|k|\},\\
D_\lambda &= \{(k, a, b, c)|\ b + c \le a + 2b + c + |c| - (k + a + |c| - c)\lambda < -|k|\}.
\end{aligned}$$

Note that if $c \ge 0$, then $\Gamma_1 = \Gamma_2$ and $\Gamma_3 = \Gamma_4$. And if $k \le 0$, then $\Gamma_1 = \Gamma_3$ and $\Gamma_2 = \Gamma_4$.

2. Main results

For the proof of our main results, we require the following lemma in [9].

Lemma 2.1. *Let (x, d, W) be a convex metric space, then the following statements hold:*

(i) $d(x, y) = d(x, W(x, y; \lambda)) + d(y, W(x, y; \lambda))$,

(ii) $d(x, W(x, y; \lambda)) = (1 - \lambda)d(x, y), \quad d(y, W(x, y; \lambda)) = \lambda d(x, y),$ *for all $x, y \in X$ and $\lambda \in [0, 1]$.*

Now, we give our main results.

Theorem 2.2. *Let (X, d, W) be a complete convex metric space, and K be a nonempty, closed and convex subset of X. Suppose $T, S : K \to K$ is a pair of generalized nonlinear mappings (1.1) such that $(k, a, b, c) \in \Gamma_\rho$, for any $\rho \in \{1, 2, 3, 4\}$. Then, T and S have at least one common fixed point in K. Furthermore, if $a + 2c < k$, then T and S have a unique common fixed point in K.*

Proof. For any given point $\lambda \in [0,1)$, let $x_0 \in K$, define the sequence $\{x_n\}_\lambda$ by

$$\begin{cases} x_{2n+1} = W(x_{2n}, Tx_{2n}; \lambda), \\ x_{2n+2} = W(x_{2n+1}, Sx_{2n+1}; \lambda), \end{cases} \tag{2.1}$$

for all $n \geq 0$.

Claim I. There exists $\theta \in [0,1)$ such that for all $n \geq 0$,

$$d(x_{2n+1}, x_{2n+2}) \leq \theta d(x_{2n}, x_{2n+1}).$$

From Lemma 2.1 and (2.1), we get

$$d(x_{2n+1}, x_{2n+2}) = d(x_{2n+1}, W(x_{2n+1}, Sx_{2n+1}; \lambda)) = (1-\lambda) d(x_{2n+1}, Sx_{2n+1}), \tag{2.2}$$

$$d(x_{2n}, x_{2n+1}) = d(x_{2n}, W(x_{2n}, Tx_{2n}; \lambda)) = (1-\lambda) d(x_{2n}, Tx_{2n}), \tag{2.3}$$

for all $n \geq 0$. Now, substituting x with x_{2n} and y with x_{2n+1} in (1.1), we obtain

$$\begin{aligned} kd(Tx_{2n}, Sx_{2n+1}) \leq{}& ad(x_{2n}, x_{2n+1}) + b[d(x_{2n}, Tx_{2n}) + d(x_{2n+1}, Sx_{2n+1})] \\ &+ c[d(x_{2n}, Sx_{2n+1}) + d(x_{2n+1}, Tx_{2n})]. \end{aligned} \tag{2.4}$$

By (2.1), Lemma 2.1 and (2.3), we get

$$d(x_{2n+1}, Tx_{2n}) = d(W(x_{2n}, Tx_{2n}; \lambda), Tx_{2n}) = \lambda d(x_{2n}, Tx_{2n}) = \frac{\lambda}{1-\lambda} d(x_{2n}, x_{2n+1}). \tag{2.5}$$

By the triangle inequality for $d(Tx_{2n}, Sx_{2n+1})$, we have

$$\begin{cases} d(Tx_{2n}, Sx_{2n+1}) \leq d(Tx_{2n}, x_{2n+1}) + d(x_{2n+1}, Sx_{2n+1}), \\ d(Tx_{2n}, Sx_{2n+1}) \geq -d(Tx_{2n}, x_{2n+1}) + d(x_{2n+1}, Sx_{2n+1}), \\ d(Tx_{2n}, Sx_{2n+1}) \geq d(Tx_{2n}, x_{2n+1}) - d(x_{2n+1}, Sx_{2n+1}). \end{cases}$$

Therefore, it follows from (2.2) and (2.5) that

$$kd(Tx_{2n}, Sx_{2n+1}) \geq \frac{-|k|\lambda}{1-\lambda} d(x_{2n}, x_{2n+1}) + \frac{k}{1-\lambda} d(x_{2n+1}, x_{2n+2}), \tag{2.6}$$

$$kd(Tx_{2n}, Sx_{2n+1}) \geq \frac{k\lambda}{1-\lambda} d(x_{2n}, x_{2n+1}) - \frac{|k|}{1-\lambda} d(x_{2n+1}, x_{2n+2}). \tag{2.7}$$

Similarly, by the triangle inequality for $d(x_{2n}, Sx_{2n+1})$, we have

$$\begin{cases} d(x_{2n}, Sx_{2n+1}) \leq d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, Sx_{2n+1}), \\ d(x_{2n}, Sx_{2n+1}) \geq d(x_{2n}, x_{2n+1}) - d(x_{2n+1}, Sx_{2n+1}), \\ d(x_{2n}, Sx_{2n+1}) \geq -d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, Sx_{2n+1}). \end{cases}$$

Therefore, it follows from (2.2) that

$$cd(x_{2n}, Sx_{2n+1}) \leq cd(x_{2n}, x_{2n+1}) + \frac{|c|}{1-\lambda} d(x_{2n+1}, x_{2n+2}), \tag{2.8}$$

$$cd(x_{2n}, Sx_{2n+1}) \leq |c| d(x_{2n}, x_{2n+1}) + \frac{c}{1-\lambda} d(x_{2n+1}, x_{2n+2}). \tag{2.9}$$

Case 1 : by (2.2)-(2.5), (2.6) and (2.8), we have

$$\begin{aligned}\frac{k}{1-\lambda}d(x_{2n+1},x_{2n+2})-\frac{|k|\lambda}{1-\lambda}d(x_{2n},x_{2n+1}) \leq & ad(x_{2n},x_{2n+1})+\frac{b}{1-\lambda}d(x_{2n},x_{2n+1})\\ &+\frac{b}{1-\lambda}d(x_{2n+1},x_{2n+2})+\frac{c\lambda}{1-\lambda}d(x_{2n},x_{2n+1})\\ &+cd(x_{2n},x_{2n+1})+\frac{|c|}{1-\lambda}d(x_{2n+1},x_{2n+2}).\end{aligned}$$

This implies that

$$\frac{k-b-|c|}{1-\lambda}d(x_{2n+1},x_{2n+2}) \leq \frac{a+b+c+(|k|-a)\lambda}{1-\lambda}d(x_{2n},x_{2n+1}).$$

Case 2 : as Case 1, by (2.2)-(2.5), (2.6) and (2.9), we have

$$\frac{k-b-c}{1-\lambda}d(x_{2n+1},x_{2n+2}) \leq \frac{a+b+|c|+(|k|+c-a-|c|)\lambda}{1-\lambda}d(x_{2n},x_{2n+1}).$$

Case 3 : as Case 1, by (2.2)-(2.5), (2.7) and (2.8), we have

$$\frac{-|k|-b-c}{1-\lambda}d(x_{2n+1},x_{2n+2}) \leq \frac{a+b+c-(k+a)\lambda}{1-\lambda}d(x_{2n},x_{2n+1}).$$

Case 4 : as Case 1, by (2.2)-(2.5), (2.7) and (2.9), we have

$$\frac{-|k|-b-c}{1-\lambda}d(x_{2n+1},x_{2n+2}) \leq \frac{a+b+|c|-(k+a+|c|-c)\lambda}{1-\lambda}d(x_{2n},x_{2n+1}).$$

The four above cases and $(k,a,b,c) \in \Gamma_\rho$ imply that Claim I holds.

Claim II. There exists $\theta \in [0,1)$ such that for all $n \geq 1$,

$$d(x_{2n},x_{2n+1}) \leq \theta d(x_{2n-1},x_{2n}).$$

From Lemma 2.1 and (2.1), we also have

$$d(x_{2n+1},x_{2n}) = d(W(x_{2n},Tx_{2n};\lambda),x_{2n}) = (1-\lambda)d(x_{2n},Tx_{2n}), \tag{2.10}$$

$$d(x_{2n-1},x_{2n}) = d(x_{2n-1},W(x_{2n-1},Sx_{2n-1};\lambda)) = (1-\lambda)d(x_{2n-1},Sx_{2n-1}), \tag{2.11}$$

for all $n \geq 1$. Now, substituting x with x_{2n} and y with x_{2n-1} in (1.1), we obtain

$$kd(Tx_{2n},Sx_{2n-1}) \leq ad(x_{2n},x_{2n-1})+b[d(x_{2n},Tx_{2n})+d(x_{2n-1},Sx_{2n-1})]$$

$$+c[d(x_{2n},Sx_{2n-1})+d(x_{2n-1},Tx_{2n})]$$

By the same method as Claim I, we can prove Claim II.

Claim III. $\{x_n\}$ is a Cauchy sequence in K.

From Claim I and Claim II, for any positive integer $n \geq 1$, we have

$$d(x_{n+1},x_n) \leq \theta d(x_n,x_{n-1}),$$

where $\theta \in [0,1)$. Thus, $d(x_{n+1},x_n) \leq \theta^n d(x_1,x_0)$, where $x_1 = W(x_0,Tx_0;\lambda)$. For any $m > n$, we have

$$d(x_m,x_n) \leq [\theta^n+\theta^{n+1}+\cdots+\theta^{m-1}]d(x_1,x_0) \leq \frac{\theta^n}{1-\theta}d(x_1,x_0).$$

Hence, $\{x_n\}$ is a Cauchy sequence in K. Since K is a closed convex subset in X, there exists $x^* \in K$ such that $\lim_{n\to\infty} x_n = x^*$.

Claim V. x^* is a common fixed point for a pair of generalized nonlinear mappings (1.1).

Letting $n \to \infty$ in (2.2) and (2.11), we get that $\lim_{n\to\infty} Sx_n = x^*$. By substituting x with x^* and y with x_n in (1.1), we have

$$kd(Tx^*, Sx_n) \le ad(x^*, x_n) + b[d(x^*, Tx^*) + d(x_n, Sx_n)] + c[d(x^*, Sx_n) + d(x_n, Tx^*)]$$

for all $n \in \mathbb{N}$. Letting $n \to \infty$ in the above inequality, we have

$$kd(Tx^*, x^*) \le (b + c)d(Tx^*, x^*).$$

Since $(k, a, b, c) \in \Gamma_\rho$, it follows that $d(Tx^*, x^*) = 0$. Hence, $x^* = Tx^*$. On the other hand, letting $n \to \infty$ in (2.3) and (2.10), we get that $\lim_{n\to\infty} Tx_n = x^*$. By substituting x with x^* and y with x_n in (1.1), we have

$$kd(Tx_n, Sx^*) \le ad(x_n, x^*) + b[d(x_n, Tx_n) + d(x^*, Sx^*)] + c[d(x_n, Sx^*) + d(x^*, Tx_n)]$$

for all $n \in \mathbb{N}$. Letting $n \to \infty$ in the above inequality, we have

$$kd(x^*, Sx^*) \le (b + c)d(x^*, Sx^*).$$

Since $(k, a, b, c) \in \Gamma_\rho$, it follows that $x^* = Sx^*$. Therefore, Claim V holds.

Claim VI. If $(k, a, b, c) \in \Gamma_\rho$ and $a + 2c < k$, then x^* is the unique common fixed point of T and S.

Assume that there exist $x_1, x_2 \in K$ such that $Tx_1 = Sx_1 = x_1$, $Tx_2 = Sx_2 = x_2$. Substituting x with x_1 and y with x_2 in (1.1), we have

$$kd(Tx_1, Sx_2) \le ad(x_1, x_2) + b[d(x_1, Tx_1) + d(x_2, Sx_2)] + c[d(x_1, Sx_2) + d(x_2, Tx_1)].$$

That is

$$kd(x_1, x_2) \le (a + 2c)d(x_1, x_2).$$

This implies that $x_1 = x_2$. We conclude that Claim VI holds. □

Remark 2.3. (i) Instead of Picard iterations considered in [1, 2, 3, 4, 5, 8, 13, 14], we use Krasnoselskii type iterations (2.1) to converge common fixed points for a pair of generalized nonlinear mappings (1.1) in convex metric space.

(ii) From Theorem 2.2, we can also study some sufficient conditions for existence of common fixed points for the following two mappings in convex metric space:

$$kd(Tx, Sy) \le a_1d(x, y) + a_2d(x, Tx) + a_3d(y, Sy) + a_4d(x, Sy) + a_5d(y, Tx).$$

Now, we give a special example to illustrate the results of Theorem 2.2.

Example 2.4. Let $X = \mathbb{R}$ (the set of real numbers) with the usual metric, $K = [0, \infty)$ and $T, S : K \to K$ be given by

$$Tx = \begin{cases} \frac{x}{4}, & x \in U = [0, \frac{1}{2}); \\ \frac{x}{8}, & x \in V = [\frac{1}{2}, \infty). \end{cases} \qquad Sx = \begin{cases} \frac{x}{8}, & x \in U = [0, \frac{1}{2}); \\ \frac{x}{4}, & x \in V = [\frac{1}{2}, \infty). \end{cases}$$

Then we have the following statements:

(i) T and S has a unique common fixed point 0;

(ii) T and S are a pair of generalized nonlinear mappings (1.1) with $k = 1, a = \frac{1}{4}, b = \frac{1}{3}, c = 0$ and $\lambda \in [0, \frac{1}{9})$ (which satisfy $(k, a, b, c) \in \Gamma_2$ and $a + 2c < 1$). In fact, let $M(x, y) = ad(x, y) + b[d(x, Tx) + d(y, Ty)] + c[d(x, Ty) + d(y, Tx)]$, we know that

Case 1: For $x, y \in U$,

$$d(Tx, Sy) = |\frac{x}{4} - \frac{y}{8}| \leq \frac{x}{4} + \frac{y}{8} \leq \frac{1}{4}|x - y| + \frac{1}{3}[\frac{3}{4}x + \frac{7}{8}y] + 0[|x - \frac{y}{8}| + |y - \frac{x}{4}|] = M(x, y),$$

Case 2: For $x, y \in V$,

$$d(Tx, Sy) = |\frac{x}{8} - \frac{y}{4}| \leq \frac{x}{8} + \frac{y}{4} \leq \frac{1}{4}|x - y| + \frac{1}{3}[\frac{7}{8}x + \frac{3}{4}y] + 0[|x - \frac{y}{4}| + |y - \frac{x}{8}|] = M(x, y),$$

Case 3: For $x \in U, y \in V$,

$$d(Tx, Sy) = |\frac{x}{4} - \frac{y}{4}| \leq \frac{1}{4}|x - y| + \frac{1}{3}[|x - \frac{x}{4}| + |y - \frac{y}{4}|] + 0[|x - \frac{y}{4}| + |y - \frac{x}{4}|] = M(x, y),$$

Case 4: For $x \in V, y \in U$,

$$d(Tx, Sy) = |\frac{x}{8} - \frac{y}{8}| \leq \frac{1}{4}|x - y| + \frac{1}{3}[|x - \frac{x}{8}| + |y - \frac{y}{8}|] + 0[|x - \frac{y}{8}| + |y - \frac{x}{8}|] = M(x, y);$$

(iii) Let $k = 1, a = \frac{1}{4}, b = \frac{1}{3}, c = 0$, for $\lambda \in [0, \frac{1}{9})$, a family iterations $\{x_n\}$ defined by

$$\begin{cases} x_{2n+1} = W(x_{2n}, Tx_{2n}; \lambda), \\ x_{2n+2} = W(x_{2n+1}, Sx_{2n+1}; \lambda), \end{cases}$$

converge to the unique fixed point 0.

Let $k = 1$ and $\lambda = 0$ in Theorem 2.2, we get the following result for a pair of generalized contractive type mappings.

Corollary 2.5. *Let (X, d, W) be a complete convex metric space, and K be a nonempty, closed and convex subset of X. Suppose $T, S : K \to K$ such that*

$$d(Tx, Sy) \leq ad(x, y) + b[d(x, Tx) + d(y, Sy)] + c[d(x, Sy) + d(y, Tx)] \tag{2.12}$$

where $b + c \leq a + 2b + c + |c| < 1$. Then, T and S have a common fixed point in K. Furthermore, if $a + 2c < 1$, then T and S have a unique common fixed point in K.

Proof. It follows from $k = 1$ and $\lambda = 0$ that

$$\Gamma_3 \subseteq \Gamma_4 \subseteq \Gamma_2, \quad \Gamma_3 \subseteq \Gamma_1 \subseteq \Gamma_2,$$

where $\Gamma_2 = \{(1, a, b, c)|\ b + c \leq a + 2b + c + |c| < 1\}$. Therefore, by Theorem 2.2, the result follows. □

Remark 2.6. Corollary 2.5 extends the results in [1, 2, 4, 5, 8, 14] to convex metric spaces and generalized the corresponding results in [2, 4, 6, 8, 10] as a pair of generalized contractive type mappings (2.12).

Let $k = -1$ and $\lambda = 0$ in Theorem 2.2, we get the following result for generalized expansive type mappings.

Corollary 2.7. *Let (X, d, W) be a complete convex metric space, and K be a nonempty, closed and convex subset of X. Suppose $T, S : K \to K$ such that*

$$d(Tx, Sy) \geq a'd(x, y) + b'[d(x, Tx) + d(y, Sy)] + c'[d(x, Sy) + d(y, Tx)] \tag{2.13}$$

where $1 < a' + 2b' + c' - |c'| \leq b' + c'$. Then, T and S have a common fixed point in K. Furthermore, if $a' + 2c' > 1$, then T and S have a unique common fixed point in K.

Proof. It follows from $k = -1$ and $\lambda = 0$ that

$$\Gamma_3 = \Gamma_1 \subseteq \Gamma_2 = \Gamma_4,$$

where $\Gamma_2 = \{(-1, a, b, c) |\ b + c \leq a + 2b + c + |c|\ < -1\}$. Let $a' = -a, b' = -b, c' = -c$, from Theorem 2.2, the result follows. □

Remark 2.8. If the domain is a closed convex subset of convex metric spaces, we give a sufficient condition for existence of common fixed points for a pair of generalized expansive type mappings (2.13). Hence Corollary 2.7 improves the corresponding results in [3, 13].

Theorem 2.9. *Let (X, d, W) be a complete convex metric space, and K be a nonempty, closed and convex subset of X. Suppose $T, S : K \to K$, there exist positive integer p, q such that*

$$kd(T^p x, S^q y) \leq ad(x, y) + b[d(x, T^p x) + d(y, S^q y)] + c[d(x, S^q y) + d(y, T^p x)]$$

where $(k, a, b, c) \in \Gamma_\rho$, for any $\rho \in \{1, 2, 3, 4\}$. If $a + 2c < k$, then T and S have a unique common fixed point in K.

Proof. Let $T' = T^p$ and $S' = S^q$, from Theorem 2.2, we know that T' and S' have a unique common fixed point $u \in K$. Since

$$T^p(Tu) = T(T^p u) = Tu, \qquad S^q(Su) = S(S^q u) = Su,$$

it follows that $Tu = Su = u$. Hence T and S have a unique common fixed point $u \in K$. □

Remark 2.10. As Corollary 2.5 and Corollary 2.7, we can also get some corollaries for Theorem 2.9.

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