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Common tripled fixed point theorem for W-compatible mappings in fuzzy metric spaces

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Abstract

In this paper we present a common tripled fixed point theorem for W-compatible mappings under ϕ-contractive conditions in fuzzy metric spaces. The result generalizes, extends and improves several classical and very recent related results in literature. For instance, we obtain an extension of Theorem 2.5 in [S. Sedghi, I. Altun, N. Shobe, Nonlinear Anal., **72** (2010), 1298–1304], an refinement of Theorem 4.1 in [X. Zhu, J. Xiao, Nonlinear Anal., **74** (2011), 5475–5479] and an improvement of Theorem 11 in [A. Roldán, J. Martínez-Moreno, C. Roldán, Fixed Point Theory Appl., **2013** (2013), 13 pages]. Finally, an example is given to illustrate the usability of our main result.

Keywords: Common tripled fixed point, tripled fixed point, fuzzy metric space, Hadžić type t-norm.
2010 MSC: 47H10.

1. Introduction

The notion of fuzzy metric space was introduced by Kramosil and Michalek [8]. To obtain a Hausdorff topology of fuzzy metric spaces, George and Veeramani [5] modified the concept of fuzzy metric space introduced by Kramosil and Michalek. Later, many fixed point results were established in this fuzzy metric space. For example, Sedghi, Altun and Shobe [15] and Zhu and Xiao [16] gave a coupled fixed point theorem for contractions in fuzzy metric spaces in the sense of George and Veeramani. Miheţ [9] presented

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two common fixed point theorems for a pair of weakly compatible maps in fuzzy metric spaces both in the sense of Kramosil and Michalek and in the sense of George and Veeramani.

The concept of tripled fixed point was introduced by Berinde and Borcut in [2]. Later, in 2012, Berinde and Borcut [3] presented the concept of tripled coincidence point for a pair of nonlinear contractive mappings $F : X \to X$ and $g : X \to X$ and obtained tripled coincidence point theorems which generalized the results of [2]. In 2013, Roldán, Martínez-Moreno and Roldán gave a slight modification of the concept of a tripled fixed point introduced by Berinde and Borcut [2] for nonlinear mappings, and established a common tripled fixed point theorem for contractive type mappings in fuzzy metric spaces.

The aim of this paper is to introduce the concepts of W-compatible mappings in fuzzy metric spaces. Based on this notion, a common tripled fixed point for mappings $F : X \times X \times X \to X$ and $g : X \to X$ are obtained. The presented theorem generalizes, extends and improves several well-known comparable results in literature. An example is also given in support of our results.

2. Preliminaries

The following definitions and results will be needed in the sequel.

Definition 2.1 ([7, 14]). A binary operation $* : [0,1]^2 \to [0,1]$ is called a continuous t-norm if the following properties are satisfied:
(i) $*$ is associative and commutative;
(ii) $a * 1 = a$ for all $a \in [0,1]$;
(iii) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0,1]$;
(iv) $*$ is continuous.

Definition 2.2 ([8]). A triple $(X, M, *)$ is called a fuzzy metric space (in the sense of Kramosil and Michalek; briefly, a FMS) if X is an arbitrary non-empty set, $*$ is a continuous t-norm and $M : X \times X \times [0,\infty) \to [0,1]$ is a fuzzy set satisfying the following conditions:
(KM-1) $M(x, y, 0) = 0$ for all $x, y \in X$;
(KM-2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;
(KM-3) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and all $t > 0$;
(KM-4) $M(x, y, \cdot) : [0,\infty) \to [0,1]$ is left-continuous for all $x, y \in X$;
(KM-5) $M(x, z, t + s) \ge M(x, y, t) * M(y, z, s)$ for all $x, y, z \in X$, and all $t, s \ge 0$.

To obtain a Hausdorff topology of fuzzy metric spaces, George and Veeramani [5] modified the concept of fuzzy metric space introduced by Kramosil and Michalek as follows.

Definition 2.3 ([5]). A triple $(X, M, *)$ is called a fuzzy metric space (in the sense of George and Veeramani) if X is an arbitrary non-empty set, $*$ is a continuous t-norm and $M : X \times X \times (0,\infty) \to [0,1]$ is a fuzzy set satisfying the following conditions:
(GV-1) $M(x, y, t) > 0$;
(GV-2) $M(x, y, t) = 1$ if and only if $x = y$;
(GV-3) $M(x, y, t) = M(y, x, t)$;
(GV-4) $M(x, y, \cdot) : (0,\infty) \to [0,1]$ is continuous;
(GV-5) $M(x, z, t + s) \ge M(x, y, t) * M(y, z, s)$ for $x, y, z \in X$, and $t, s > 0$.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with a center $x \in X$ and a radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}. \tag{2.1}$$

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let τ denote the family of all open subsets of X. Then τ is called the topology on X, induced by the fuzzy metric M. This topology is Hausdorff and first countable.

Definition 2.4 ([5]). Let $(X, M, *)$ be a fuzzy metric space.

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.

(ii) A sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exists a positive integer n_0 such that $M(x_n, x_m, t) > 1 - \varepsilon$ for each $n, m \geq n_0$.

(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Lemma 2.5 ([6]). *Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, \cdot)$ is nondecreasing for all $x, y \in X$.*

Lemma 2.6 ([11]). *M is a continuous function on $X^2 \times (0, \infty)$.*

Definition 2.7 ([7]). A t-norm $*$ is said to be Hadžić type t-norm if the family $\{*^p\}_{p\geq 0}$ of its iterates defined for each $s \in [0, 1]$ by

$$*^0(s) = 1, \quad *^{p+1}(s) = *(*^p(s), s),$$

is equi-continuous at $s = 1$, that is, given $\eta > 0$ there exists $\eta(\lambda) \in (0, 1)$ such that

$$\eta(\lambda) < s \leq 1 \Rightarrow *^{(p)}(s) > 1 - \lambda$$

for all $p \geq 0$.

Definition 2.8 ([15]). Let $(X, M, *)$ be a fuzzy metric space. M is said to satisfy the n-property on $X \times X \times (0, \infty)$ if

$$\lim_{n\to\infty} [M(x, y, l^n t)]^{n^p} = 1$$

whenever $x, y \in X, l > 1$ and $p > 0$.

Define $\Phi = \{\phi : \mathbb{R}^+ \to \mathbb{R}^+\}$, where $\mathbb{R}^+ = [0, +\infty)$ and each $\phi \in \Phi$ satisfies the following conditions:
$(\phi - 1)$ ϕ is non-decreasing;
$(\phi - 2)$ ϕ is upper semi-continuous from the right;
$(\phi - 3)$ $\sum_{n=0}^{\infty} = \phi^n(t) < +\infty$ for all $t > 0$, where $\phi^{n+1}(t) = \phi(\phi^n(t)), n \in \mathbb{N}$.
It is easy to prove that if $\phi \in \Phi$, then $\phi(t) < t$ for all $t > 0$.

Lemma 2.9 ([4]). *Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t-norm of H-type. If there exists $\phi \in \Phi$ such that*

$$M(x, y, \phi(t)) \geq M(x, y, t) \tag{2.2}$$

for all $t > 0$, then $x = y$.

Definition 2.10 ([13]). An element $(x, y, z) \in X \times X \times X$ is called a tripled fixed point of $F : X \times X \times X \to X$ if $F(x, y, z) = x, F(y, z, x) = y$, and $F(z, x, y) = z$.

Definition 2.11 ([1]). An element $(x, y, z) \in X \times X \times X$ is called a tripled coincidence point of the mappings $F : X \times X \times X \to X$ and $g : X \to X$ if

$$F(x, y, z) = g(x), \; F(y, z, x) = g(y), \; F(z, x, y) = g(z).$$

Moreover, (x, y, z) is called a common tripled fixed point of F and g if $F(x, y, z) = g(x) = x, F(y, z, x) = g(y) = y$, and $F(z, x, y) = g(z) = z$.

Definition 2.12 ([10]). Let X be a non-empty set, $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings. F is said to be commutative with g, if $gF(x, y, z) = F(g(x), g(y), g(z))$ for all $x, y, z \in X$.

In [1], Abbas *et al.* introduced the concept of W-compatible mappings. Here we give a similar concept in fuzzy metric spaces as follows.

Definition 2.13. Let $(X, M, *)$ be a fuzzy metric space, and let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings. F and g are said to be weakly compatible (or W-compatible) if they commute at their coupled coincidence points, i.e., if (x, y, z) is a tripled coincidence point of g and F, then $gF(x, y, z) = F(g(x), g(y), g(z))$.

3. Main results

In this section, the fuzzy metric space $(X, M, *)$ is in the sense of George and Veeramani and the fuzzy metric M is assumed to satisfy the condition:

(GV-6): $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$.

For simplicity, denote

$$[M(x, y, t)]^n = \underbrace{M(x, y, t) * M(x, y, t) * \cdots * M(x, y, t)}_{n}$$

for all $n \in \mathbb{N}$.

In [15], Sedghi, Altun and Shobe gave the following results.

Theorem 3.1 ([15])**.** *Let $a * b > ab$ for all $a, b \in [0, 1]$ and let $(X, M, *)$ be a complete fuzzy metric space such that M has an n-property. Let $F : X \times X \to X$ and $g : X \to X$ be two functions such that*

$$M(F(x, y), F(u, v), kt) \geq M(g(x), g(u), t) * M(g(y), g(v), t)$$

for all $x, y, u, v \in X$, where $0 < k < 1$,

Suppose that $F(X \times X) \subseteq g(X)$ and $g(X)$ is continuous and commuting with F. Then F and g have a unique common coupled fixed point in X.

Remark 3.2. It is easy to prove that if M satisfies n-property, then M satisfy the condition (GV-6).

In [16], Zhu and Xiao presented the following result.

Theorem 3.3 ([16])**.** *Let $*$ be a t-norm of H-type such that $s * t \geq st$ for all $s, t \in [0, 1]$. Let $(X, M, *)$ be a complete FMS and let $F : X \times X \to X$ and $g : X \to X$ be two mappings such that*

$$M(F(x, y), F(u, v), kt) \geq [M(g(x), g(u), t)]^{\frac{1}{2}} * [M(g(y), g(v), t)]^{\frac{1}{2}} \tag{3.1}$$

for all $x, y, u, v \in X$, where $0 < k < 1$, $t > 0$.

Suppose that $F(X \times X) \subseteq g(X)$ and $g(X)$ is continuous and commuting with F. Then F and g have a unique common coupled fixed point in X.

A. Roldán, J. Martínez-Moreno and C. Roldán [12] proved the following result.

Theorem 3.4 ([12])**.** *Let $*$ be a t-norm of H-type such that $s * t \geq st$ for all $s, t \in [0, 1]$. Let $k \in (0, 1)$ and $a, b, c \in [0, 1]$ be real numbers such that $a + b + c \leq 1$. Let $(X, M, *)$ be a complete FMS and let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings such that*

$$M(F(x, y, z), F(u, v, w), kt) \geq [M(g(x), g(u), t)]^a * [M(g(y), g(v), t)]^b * [M(g(z), g(w), t)]^c \tag{3.2}$$

for all $x, y, z, u, v, w \in X, t > 0$.

Suppose that $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is continuous and commuting with F. Then F and g have a unique common tripled fixed point in X.

Now we give our main result.

Theorem 3.5. *Let $(X, M, *)$ be a FMS such that $*$ is a t-norm of H-type. Let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings, and there exists $\phi \in \Phi$ satisfying*

$$M(F(x, y, z), F(u, v, w), \phi(t)) \geq M(g(x), g(u), t) * M(g(y), g(v), t) * M(g(z), g(w), t) \tag{3.3}$$

for all $x, y, z, u, v, w \in X, t > 0$.

Suppose that $F(X \times X \times X) \subseteq g(X)$, $g(X)$ is complete, F and g are weakly compatible. Then F and g have a unique common tripled fixed point in X.

Proof. Let $x_0, y_0, z_0 \in X$ be three arbitrary points in X. Since $F(X \times X \times X) \subseteq g(X)$, we can choose $x_1, y_1, z_1 \in X$ such that $g(x_1) = F(x_0, y_0, z_0)$, $g(y_1) = F(y_0, z_0, x_0)$ and $g(z_1) = F(z_0, x_0, y_0)$. Continuing this process, we can construct three sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in X such that

$$g(x_{n+1}) = F(x_n, y_n, z_n),\ \ g(y_{n+1}) = F(y_n, z_n, x_n),\ \ g(z_{n+1}) = F(z_n, x_n, y_n) \tag{3.4}$$

for all $n \geq 0$.

We shall do the proof in four steps.

Step I. We shall show that $\{g(x_n)\}$, $\{g(y_n)\}$ and $\{g(z_n)\}$ are Cauchy sequences.
Since $*$ is a t-norm of H-type, for any $\lambda > 0$, there exists an $\mu > 0$ such that

$$\underbrace{(1-\mu) * (1-\mu) * \cdots * (1-\mu)}_{k} \geq 1 - \lambda$$

for all $k \in \mathbb{N}$.

Since $M(x, y, \cdot)$ is continuous and $\lim\limits_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$, so there exists $t_0 > 0$ such that

$$\begin{aligned} &M(g(x_0), g(x_1), t_0) \geq 1 - \mu, \\ &M(g(y_0), g(y_1), t_0) \geq 1 - \mu, \\ &M(g(z_0), g(z_1), t_0) \geq 1 - \mu. \end{aligned} \tag{3.5}$$

On the other hand, since $\phi \in \Phi$, by condition $(\phi - 3)$, we have $\sum\limits_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$t > \sum_{k=n_0}^{\infty} \phi^k(t_0). \tag{3.6}$$

By use of condition (3.3), we get

$$\begin{aligned} M(g(x_1), g(x_2), \phi(t_0)) &= M(F(x_0, y_0, z_0), F(x_1, y_1, z_1), \phi(t_0)) \\ &\geq M(g(x_0), g(x_1), t_0) * M(g(y_0), g(y_1), t_0) * M(g(z_0), g(z_1), t_0), \end{aligned}$$

$$\begin{aligned} M(g(y_1), g(y_2), \phi(t_0)) &= M(F(y_0, z_0, x_0), F(y_1, z_1, x_1), \phi(t_0)) \\ &\geq M(g(y_0), g(y_1), t_0) * M(g(z_0), g(z_1), t_0) * M(g(x_0), g(x_1), t_0) \end{aligned}$$

and

$$\begin{aligned} M(g(z_1), g(z_2), \phi(t_0)) &= M(F(z_0, x_0, y_0), F(z_1, x_1, y_1), \phi(t_0)) \\ &\geq M(g(z_0), g(z_1), t_0) * M(g(x_0), g(x_1), t_0) * M(g(y_0), g(y_1), t_0). \end{aligned}$$

Similarly, we have

$$\begin{aligned} M(g(x_2), g(x_3), \phi^2(t_0)) &= M(F(x_1, y_1, z_1), F(x_2, y_2, z_2), \phi^2(t_0)) \\ &\geq M(g(x_1), g(x_2), \phi(t_0)) * M(g(y_1), g(y_2), \phi(t_0)) * M(g(z_1), g(z_2), \phi(t_0)) \\ &\geq [M(g(x_0), g(x_1), t_0)]^3 * [M(g(y_0), g(y_1), t_0)]^3 * [M(g(z_0), g(z_1), t_0)]^3, \end{aligned}$$

$$\begin{aligned} M(g(y_2), g(y_3), \phi^2(t_0)) &= M(F(y_1, z_1, x_1), F(y_2, z_2, x_2), \phi^2(t_0)) \\ &\geq [M(g(y_0), g(y_1), t_0)]^3 * [M(g(z_0), g(z_1), t_0)]^3 * [M(g(x_0), g(x_1), t_0)]^3 \end{aligned}$$

and

$$\begin{aligned} M(g(z_2), g(z_3), \phi^2(t_0)) &= M(F(z_1, x_1, y_1), F(z_2, x_2, y_2), \phi^2(t_0)) \\ &\geq [M(g(z_0), g(z_1), t_0)]^3 * [M(g(x_0), g(x_1), t_0)]^3 * [M(g(y_0), g(y_1), t_0)]^3. \end{aligned}$$

From the inequalities above and by induction, it is easy to find that

$$\begin{aligned} M(g(x_n), g(x_{n+1}), \phi^n(t_0)) \geq {} & [M(g(x_0), g(x_1), t_0)]^{3^{n-1}} * [M(g(y_0), g(y_1), t_0)]^{3^{n-1}} \\ & * [M(g(z_0), g(z_1), t_0)]^{3^{n-1}}, \end{aligned} \tag{3.7}$$

$$\begin{aligned} M(g(y_n), g(y_{n+1}), \phi^n(t_0)) \geq {} & [M(g(y_0), g(y_1), t_0)]^{3^{n-1}} * [M(g(x_0), g(x_1), t_0)]^{3^{n-1}} \\ & * [M(g(z_0), g(z_1), t_0)]^{3^{n-1}} \end{aligned} \tag{3.8}$$

and

$$\begin{aligned} M(g(z_n), g(z_{n+1}), \phi^n(t_0)) \geq {} & [M(g(z_0), g(z_1), t_0)]^{3^{n-1}} * [M(g(x_0), g(x_1), t_0)]^{3^{n-1}} \\ & * [M(g(y_0), g(y_1), t_0)]^{3^{n-1}}. \end{aligned} \tag{3.9}$$

So, from (3.5), (3.6), (GV-5) and (3.7), for $m > n \geq n_0$, we have

$$\begin{aligned} M(g(x_n), g(x_m), t) &\geq M\Big(g(x_n), g(x_m), \sum_{k=n_0}^{\infty} \phi^k(t_0)\Big) \\ &\geq M\Big(g(x_n), g(x_m), \sum_{k=n}^{m-1} \phi^k(t_0)\Big) \\ &\geq M(g(x_n), g(x_{n+1}), \phi^n(t_0)) * M(g(x_{n+1}), g(x_{n+2}), \phi^{n+1}(t_0)) * \cdots \\ &\quad * M(g(x_{m-1}), g(x_m), \phi^{m-1}(t_0)) \\ &\geq [M(g(x_0), g(x_1), t_0)]^{3^{n-1}} * [M(g(y_0), g(y_1), t_0)]^{3^{n-1}} * [M(g(z_0), g(z_1), t_0)]^{3^{n-1}} \\ &\quad * [M(g(x_0), g(x_1), t_0)]^{3^{n}} * [M(g(y_0), g(y_1), t_0)]^{3^{n}} * [M(g(z_0), g(z_1), t_0)]^{3^{n}} \\ &\quad * \cdots * [M(g(x_0), g(x_1), t_0)]^{3^{m-2}} \\ &\quad * [M(g(y_0), g(y_1), t_0)]^{3^{m-2}} * [M(g(z_0), g(z_1), t_0)]^{3^{m-2}} \\ &= [M(g(x_0), g(x_1), t_0)]^{\frac{3^{m-1}-2^{n-1}}{2}} * [M(g(y_0), g(y_1), t_0)]^{\frac{3^{m-1}-2^{n-1}}{2}} \\ &\quad * [M(g(z_0), g(z_1), t_0)]^{\frac{3^{m-1}-2^{n-1}}{2}} \\ &\geq \underbrace{(1-\mu) * (1-\mu) * \cdots * (1-\mu)}_{\frac{3(3^{m-1}-2^{n-1})}{2}} \geq 1 - \lambda, \end{aligned}$$

which implies that

$$M(g(x_n), g(x_m), t) > 1 - \lambda \tag{3.10}$$

for all $m, n \in \mathbb{N}$ with $m > n \geq n_0$ and $t > 0$. So $\{g(x_n)\}$ is a Cauchy sequence. Similarly, we can prove that $\{g(y_n)\}$ and $\{g(z_n)\}$ are also Cauchy sequences.

Step II. We shall show that g and F have a tripled coincidence point. Since $g(X)$ is complete, there exist $\hat{x}, \hat{y}, \hat{z} \in g(X)$, and exist $a, b, c \in X$ such that

$$\begin{aligned} \lim_{n\to\infty} g(x_n) &= \lim_{n\to\infty} F(x_n, y_n, z_n) = g(a) = \hat{x}, \\ \lim_{n\to\infty} g(y_n) &= \lim_{n\to\infty} F(y_n, z_n, x_n) = g(b) = \hat{y}, \\ \lim_{n\to\infty} g(z_n) &= \lim_{n\to\infty} F(z_n, x_n, y_n) = g(c) = \hat{z}. \end{aligned} \tag{3.11}$$

By use of condition (3.3), we get

$$M(F(x_n, y_n, z_n), F(a, b, c), \phi(t)) \geq M(g(x_n), g(a), t) * M(g(y_n), g(b), t) * M(g(z_n), g(c), t).$$

Taking the limit as $n \to \infty$ in the above inequality, by continuity of M and using (3.11), we have

$$M(g(a), F(a, b, c), \phi(t)) = 1,$$

which implies that $F(a,b,c) = g(a) = \hat{x}$. In a similar way, we can prove that $F(b,c,a) = g(b) = \hat{y}$ and $F(c,a,b) = g(c) = \hat{z}$. Since F and g are weakly compatible, we get that $gF(a,b,c) = F(g(a),g(b),g(c))$, $gF(b,c,a) = F(g(b),g(c),g(a))$ and $gF(c,a,b) = F(g(c),g(a),g(b))$, which implies that $g(\hat{x}) = F(\hat{x},\hat{y},\hat{z})$, $g(\hat{y}) = F(\hat{y},\hat{z},\hat{x})$ and $g(\hat{z}) = F(\hat{z},\hat{x},\hat{y})$.

Step III. We shall show that $g(\hat{x}) = \hat{y}$, $g(\hat{y}) = \hat{z}$ and $g(\hat{z}) = \hat{x}$.
Since $*$ is a t-norm of H-type, for any $\lambda > 0$, there exists an $\mu > 0$ such that

$$\underbrace{(1-\mu) * (1-\mu) * \cdots * (1-\mu)}_{k} \geq 1 - \lambda$$

for all $k \in \mathbb{N}$.

Since $M(x,y,\cdot)$ is continuous and $\lim_{t\to\infty} M(x,y,t) = 1$ for all $x,y \in X$, there exists $t_0 > 0$ such that $M(g(\hat{x}),\hat{y},t_0) \geq 1-\mu$, $M(g(\hat{y}),\hat{z},t_0) \geq 1-\mu$ and $M(g(\hat{z}),\hat{x},t_0) \geq 1-\mu$.

On the other hand, since $\phi \in \Phi$, by condition $(\phi - 3)$, we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Thus, for any $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$. Since

$$\begin{aligned} M(g(\hat{x}), g(y_{n+1}), \phi(t_0)) &= M(F(\hat{x},\hat{y},\hat{z}), F(y_n,z_n,x_n), \phi(t_0)) \\ &\geq M(g(\hat{x}), g(y_n), t_0) * M(g(\hat{y}), g(z_n), t_0) * M(g(\hat{z}), g(x_n), t_0), \end{aligned}$$

$$\begin{aligned} M(g(\hat{y}), g(z_{n+1}), \phi(t_0)) &= M(F(\hat{y},\hat{z},\hat{x}), F(z_n,x_n,y_n), \phi(t_0)) \\ &\geq M(g(\hat{y}), g(z_n), t_0) * M(g(\hat{z}), g(x_n), t_0) * M(g(\hat{x}), g(y_n), t_0) \end{aligned}$$

and

$$\begin{aligned} M(g(\hat{z}), g(x_{n+1}), \phi(t_0)) &= M(F(\hat{z},\hat{x},\hat{y}), F(x_n,y_n,z_n), \phi(t_0)) \\ &\geq M(g(\hat{z}), g(x_n), t_0) * M(g(\hat{x}), g(y_n), t_0) * M(g(\hat{y}), g(z_n), t_0). \end{aligned}$$

Letting $n \to \infty$ in the above inequalities, we obtain

$$M(g(\hat{x}),\hat{y},\phi(t_0)) \geq M(g(\hat{x}),\hat{y},t_0) * M(g(\hat{y}),\hat{z},t_0) * M(g(\hat{z}),\hat{x},t_0), \tag{3.12}$$

$$M(g(\hat{y}),\hat{z},\phi(t_0)) \geq M(g(\hat{y}),\hat{z},t_0) * M(g(\hat{z}),\hat{x},t_0) * M(g(\hat{x}),\hat{y},t_0) \tag{3.13}$$

and

$$M(g(\hat{z}),\hat{x},\phi(t_0)) \geq M(g(\hat{z}),\hat{x},t_0) * M(g(\hat{x}),\hat{y},t_0) * M(g(\hat{y}),\hat{z},t_0). \tag{3.14}$$

According to (3.12), (3.13) and (3.14), we get that

$$\begin{aligned} &M(g(\hat{x}),\hat{y},\phi(t_0)) * M(g(\hat{y}),\hat{z},\phi(t_0)) * M(g(\hat{z}),\hat{x},\phi(t_0)) \\ &\qquad \geq [M(g(\hat{x}),\hat{y},t_0)]^3 * [M(g(\hat{y}),\hat{z},t_0)]^3 * [M(g(\hat{z}),\hat{x},t_0)]^3. \end{aligned}$$

Therefore, from this inequality and by induction, we obtain that

$$\begin{aligned} &M(g(\hat{x}),\hat{y},\phi^n(t_0)) * M(g(\hat{y}),\hat{z},\phi^n(t_0)) * M(g(\hat{z}),\hat{x},\phi^n(t_0)) \\ &\quad \geq [M(g(\hat{x}),\hat{y},\phi^{n-1}(t_0))]^3 * [M(g(\hat{y}),\hat{z},\phi^{n-1}(t_0))]^3 * [M(g(\hat{z}),\hat{x},\phi^{n-1}(t_0))]^3 \\ &\quad \geq [M(g(\hat{x}),\hat{y},t_0)]^{3^n} * [M(g(\hat{y}),\hat{z},t_0)]^{3^n} * [M(g(\hat{z}),\hat{x},t_0)]^{3^n} \end{aligned}$$

for all $n \in \mathbb{N}$. Since $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$, we get

$$M(g(\hat{x}),\hat{y},t) * M(g(\hat{y}),\hat{z},t) * M(g(\hat{z}),\hat{x},t)$$

$$\begin{aligned}
&\geq M\Big(g(\hat{x}),\hat{y},\sum_{k=n_0}^{\infty}\phi^k(t_0)\Big)*M\Big(g(\hat{y}),\hat{z},\sum_{k=n_0}^{\infty}\phi^k(t_0)\Big)*M\Big(g(\hat{z}),\hat{x},\sum_{k=n_0}^{\infty}\phi^k(t_0)\Big)\\
&\geq M(g(\hat{x}),\hat{y},\phi^{n_0}(t_0))*M(g(\hat{y}),\hat{z},\phi^{n_0}(t_0))*M(g(\hat{z}),\hat{x},\phi^{n_0}(t_0))\\
&\geq [M(g(\hat{x}),\hat{y},t_0)]^{3^{n_0}}*[M(g(\hat{y}),\hat{z},t_0)]^{3^{n_0}}*[M(g(\hat{z}),\hat{x},t_0)]^{3^{n_0}}\\
&\geq \underbrace{(1-\mu)*(1-\mu)*\cdots*(1-\mu)}_{3^{n_0+1}}\geq 1-\lambda.
\end{aligned}$$

Thus, for any $\lambda>0$, we have

$$M(g(\hat{x}),\hat{y},t)*M(g(\hat{y}),\hat{z},t)*M(g(\hat{z}),\hat{x},t)\geq 1-\lambda \tag{3.15}$$

for all $t>0$, which implies that $g(\hat{x})=\hat{y}$, $g(\hat{y})=\hat{z}$ and $g(\hat{z})=\hat{x}$.

Step IV. We shall prove that $\hat{x}=\hat{y}=\hat{z}$.

Since $*$ is a t-norm of H-type, for any $\lambda>0$, there exists an $\mu>0$ such that

$$\underbrace{(1-\mu)*(1-\mu)*\cdots*(1-\mu)}_{k}\geq 1-\lambda$$

for all $k\in\mathbb{N}$.

Since $M(x,y,\cdot)$ is continuous and $\lim\limits_{t\to+\infty}M(x,y,t)=1$ for all $x,y\in X$, there exists $t_0>0$ such that $M(\hat{x},\hat{y},t_0)\geq 1-\mu$, $M(\hat{y},\hat{z},t_0)\geq 1-\mu$ and $M(\hat{z},\hat{x},t_0)\geq 1-\mu$.

On the other hand, since $\phi\in\Phi$, by condition $(\phi-3)$, we have $\sum\limits_{n=1}^{\infty}\phi^n(t_0)<\infty$. Then, for any $t>0$, there exists $n_0\in\mathbb{N}$ such that $t>\sum\limits_{k=n_0}^{\infty}\phi^k(t_0)$.

By use of condition (3.3), we have

$$\begin{aligned}
M(g(x_{n+1}),g(y_{n+1}),\phi(t_0))&=M(F(x_n,y_n,z_n),F(y_n,z_n,x_n),\phi(t_0))\\
&\geq M(g(x_n),g(y_n),t_0)*M(g(y_n),g(z_n),t_0)*M(g(z_n),g(x_n),t_0),
\end{aligned}$$

$$\begin{aligned}
M(g(y_{n+1}),g(z_{n+1}),\phi(t_0))&=M(F(y_n,z_n,x_n),F(z_n,x_n,y_n),\phi(t_0))\\
&\geq M(g(y_n),g(z_n),t_0)*M(g(z_n),g(x_n),t_0)*M(g(x_n),g(y_n),t_0)
\end{aligned}$$

and

$$\begin{aligned}
M(g(z_{n+1}),g(x_{n+1}),\phi(t_0))&=M(F(z_n,x_n,y_n),F(x_n,y_n,z_n),\phi(t_0))\\
&\geq M(g(z_n),g(x_n),t_0)*M(g(x_n),g(y_n),t_0)*M(g(y_n),g(z_n),t_0).
\end{aligned}$$

Letting $n\to\infty$ in the above inequalities, we have

$$M(\hat{x},\hat{y},\phi(t_0))\geq M(\hat{x},\hat{y},t_0)*M(\hat{y},\hat{z},t_0)*M(\hat{z},\hat{x},t_0),$$

$$M(\hat{y},\hat{z},\phi(t_0))\geq M(\hat{y},\hat{z},t_0)*M(\hat{z},\hat{x},t_0)*M(\hat{x},\hat{y},t_0)$$

and

$$M(\hat{z},\hat{x},\phi(t_0))\geq M(\hat{z},\hat{x},t_0)*M(\hat{x},\hat{y},t_0)*M(\hat{y},\hat{z},t_0).$$

Therefore, we obtain that

$$M(\hat{x},\hat{y},\phi(t_0))*M(\hat{y},\hat{z},\phi(t_0))*M(\hat{z},\hat{x},\phi(t_0))\geq[M(\hat{x},\hat{y},t_0)]^3*[M(\hat{y},\hat{z},t_0)]^3*[M(\hat{z},\hat{x},t_0)]^3.$$

From this inequality and by induction, we can obtain

$$\begin{aligned}
&M(\hat{x},\hat{y},\phi^n(t_0))*M(\hat{y},\hat{z},\phi^n(t_0))*M(\hat{z},\hat{x},\phi^n(t_0))\\
&\quad\geq[M(\hat{x},\hat{y},\phi^{n-1}(t_0))]^3*[M(\hat{y},\hat{z},\phi^{n-1}(t_0))]^3*[M(\hat{z},\hat{x},\phi^{n-1}(t_0))]^3\\
&\quad\geq[M(\hat{x},\hat{y},t_0)]^{3^n}*[M(\hat{y},\hat{z},t_0)]^{3^n}*[M(\hat{z},\hat{x},t_0)]^{3^n}
\end{aligned}$$

for all $n \in \mathbb{N}$. Since $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$, we have

$$\begin{aligned}
&M(\hat{x}, \hat{y}, t) * M(\hat{y}, \hat{z}, t) * M(\hat{z}, \hat{x}, t) \\
&\quad \geq M\Big(\hat{x}, \hat{y}, \sum_{k=n_0}^{\infty} \phi^k(t_0)\Big) * M\Big(\hat{y}, \hat{z}, \sum_{k=n_0}^{\infty} \phi^k(t_0)\Big) * M\Big(\hat{z}, \hat{x}, \sum_{k=n_0}^{\infty} \phi^k(t_0)\Big) \\
&\quad \geq M(\hat{x}, \hat{y}, \phi^{n_0}(t_0)) * M(\hat{y}, \hat{z}, \phi^{n_0}(t_0)) * M(\hat{z}, \hat{x}, \phi^{n_0}(t_0)) \\
&\quad \geq [M(\hat{x}, \hat{y}, t_0)]^{3^{n_0}} * [M(\hat{y}, \hat{z}, t_0)]^{3^{n_0}} * [M(\hat{z}, \hat{x}, t_0)]^{3^{n_0}} \\
&\quad \geq \underbrace{(1-\mu) * (1-\mu) * \cdots * (1-\mu)}_{3^{n_0+1}} \geq 1 - \lambda,
\end{aligned}$$

which implies that $\hat{x} = \hat{y} = \hat{z}$.

Thus, we proved that F and g have a common tripled fixed point in X.

The uniqueness of the fixed point can be easily proved in the same way as above. This completes the proof of Theorem 3.5. □

Corollary 3.6. *Let $a, b, c \in [0, 1]$ be real numbers such that $a + b + c \leq 1$. Let $(X, M, *)$ be a FMS with a Hadžić type t-norm. Let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings, and there exists $\phi \in \Phi$ satisfying*

$$M(F(x, y, z), F(u, v, w), \phi(t)) \geq [M(g(x), g(u), t)]^a * [M(g(y), g(v), t)]^b * [M(g(z), g(w), t)]^c \tag{3.16}$$

for all $x, y, z, u, v, w \in X, t > 0$.

Suppose that $F(X \times X \times X) \subseteq g(X)$, $g(X)$ is complete, F and g are weakly compatible. Then F and g have a unique common tripled fixed point in X.

Proof. Due to

$$\begin{aligned}
M(F(x, y, z), F(u, v, w), \phi(t)) &\geq [M(g(x), g(u), t)]^a * [M(g(y), g(v), t)]^b * [M(g(z), g(w), t)]^c \\
&\geq M(g(x), g(u), t) * M(g(y), g(v), t) * M(g(z), g(w), t),
\end{aligned} \tag{3.17}$$

we conclude from Theorem 3.5 that the mappings F and g have a unique tripled common fixed point in X. □

Taking $\phi(t) = kt$, $k \in (0, 1)$ in Theorem 3.5 and Corollary 3.6 respectively, we get the following results.

Corollary 3.7. *Let $(X, M, *)$ be a FMS with a Hadžić type t-norm. Let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings such that*

$$M(F(x, y, z), F(u, v, w), kt) \geq M(g(x), g(u), t) * M(g(y), g(v), t) * M(g(z), g(w), t) \tag{3.18}$$

for all $x, y, z, u, v, w \in X, t > 0, 0 < k < 1$.

Suppose that $F(X \times X \times X) \subseteq g(X)$, $g(X)$ is complete, F and g are weakly compatible. Then F and g have a unique common tripled fixed point in X.

Corollary 3.8. *Let $k \in (0, 1)$ and $a, b, c \in [0, 1]$ be real numbers such that $a + b + c \leq 1$. Let $(X, M, *)$ be a FMS with a Hadžić type t-norm. Let $F : X \times X \times X \to X$ and $g : X \to X$ be two mappings such that*

$$M(F(x, y, z), F(u, v, w), kt) \geq [M(g(x), g(u), t)]^a * [M(g(y), g(v), t)]^b * [M(g(z), g(w), t)]^c \tag{3.19}$$

for all $x, y, z, u, v, w \in X, t > 0$.

Suppose that $F(X \times X \times X) \subseteq g(X)$, $g(X)$ is complete, F and g are weakly compatible. Then F and g have a unique common tripled fixed point in X.

Taking $g = I$ (the identity mapping) in Theorem 3.5 and Corollary 3.6 respectively, we get the following consequences.

Corollary 3.9. *Let $(X, M, *)$ be a fuzzy metric space with a Hadžić type t-norm. Let $F : X \times X \times X \to X$ is complete, and there exists $\phi \in \Phi$ such that*

$$M(F(x, y, z), F(u, v, w), \phi(t)) \geq M(x, u, t) * M(y, v, t) * M(z, w, t) \tag{3.20}$$

for all $x, y, z, u, v, w \in X, t > 0$.

Then there exists $x \in X$ such that $x = F(x, x, x)$; that is, F admits a unique fixed point in X.

Corollary 3.10. *Let $a, b, c \in [0, 1]$ be real numbers such that $a + b + c \leq 1$. Let $(X, M, *)$ be a fuzzy metric space with a Hadžić type t-norm. Let $F : X \times X \times X \to X$ is complete, and there exists $\phi \in \Phi$ such that*

$$M(F(x, y, z), F(u, v, w), \phi(t)) \geq [M(x, u, t)]^a * [M(y, v, t)]^b * [M(z, w, t)]^c \tag{3.21}$$

for all $x, y, z, u, v, w \in X, t > 0$.

Then there exist $x \in X$ such that $x = F(x, x, x)$; that is, F admits a unique fixed point in X.

Remark 3.11. Our results improve Theorem 3.4 ([12] Theorem 11) as follows:

(1) from kt to $\varphi(t)$;
(2) in our results, g is not required to be continuous, but the function g in Theorem 3.4 ([12] Theorem 11) is required to be continuous.
(3) we assume that F and g are weakly compatible, which is weaker than the condition in Theorem 3.4 ([12] Theorem 11), where Theorem 3.4 ([12] Theorem 11) assumes commutation for F and g.

By the same methods as Theorem 3.5, we can obtain the following result.

Corollary 3.12. *Let $k \in (0, 1)$ and $a, b \in [0, 1]$ be real numbers such that $a + b \leq 1$. Let $(X, M, *)$ be a FMS with a Hadžić type t-norm. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings, and there exists $\phi \in \Phi$ satisfying*

$$M(F(x, y), F(u, v), \phi(t)) \geq [M(g(x), g(u), t)]^a * [M(g(y), g(v), t)]^b \tag{3.22}$$

for all $x, y, u, v \in X, t > 0$.

Suppose that $F(X \times X) \subseteq g(X)$, $g(X)$ is complete, F and g are weakly compatible. Then F and g have a unique common coupled fixed point in X.

Now, we illustrate Theorem 3.5 by the following example.

Example 3.13. Let $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\}$, $* = \min$, and let $M(x, y, t) = \frac{t}{|x-y|+t}$, for all $x, y \in X, t > 0$. Then $(X, M, *)$ is a fuzzy metric space.

Let $\phi(t) = \frac{t}{2}$. Define the mappings $F : X \times X \times X \to X$ and $g : X \to X$ by

$$F(x, y, z) = \begin{cases} \frac{1}{(2n+1)^4}, & (x, y, z) = (\frac{1}{2n}, \frac{1}{2n}, \frac{1}{2n}), \\ 0, & \text{others} \end{cases}$$

and

$$g(x) = \begin{cases} 0, & x = 0, \\ 1, & x = \frac{1}{2n+1}, \\ \frac{1}{2n+1}, & x = \frac{1}{2n}. \end{cases}$$

Since

$$gF\left(\frac{1}{2n}, \frac{1}{2n}, \frac{1}{2n}\right) = 1,$$

$$F\left(g\left(\frac{1}{2n}\right), g\left(\frac{1}{2n}\right), g\left(\frac{1}{2n}\right)\right) = F\left(\frac{1}{2n+1}, \frac{1}{2n+1}, \frac{1}{2n+1}\right) = 0,$$

that is

$$gF\left(\frac{1}{2n}, \frac{1}{2n}, \frac{1}{2n}\right) \neq F\left(g\left(\frac{1}{2n}\right), g\left(\frac{1}{2n}\right), g\left(\frac{1}{2n}\right)\right),$$

which implies that g and F are not commutative. From $F(x, y, z) = g(x)$, $F(y, z, x) = g(y)$ and $F(z, x, y) = g(z)$, we can obtain $(x, y, z) = (0, 0, 0)$, and we have $gF(0, 0, 0) = F(g(0), g(0), g(0))$, which implies that F and g are weakly compatible. It is easy to prove that

$$\frac{t}{P+t} \geq \min\left\{\frac{t}{X+t}, \frac{t}{Y+t}, \frac{t}{Z+t}\right\} \Leftrightarrow P \leq \max\{X, Y, Z\} \tag{3.23}$$

for any $P, X, Y, Z \geq 0$ and $t > 0$. From inequality (3.23) and the definition of M and ϕ, we obtain that inequality (3.3) is equivalent to the following

$$2|F(x, y, z) - F(u, v, w)| \leq \max\{|g(x) - g(u)|, |g(y) - g(v)|, |g(z) - g(w)|\}. \tag{3.24}$$

Next, we shall prove that inequality (3.24) is valid. Let $A = \{\frac{1}{2n}, n \in \mathbb{N}\}, B = X - A$. By the symmetry and without loss of generality, (x, y, z), (u, v, w) have the following 10 possibilities.

Case 1: $(x, y, z) \in B \times B \times B, (u, v, w) \in B \times B \times B$. It is obvious that (3.24) holds.

Case 2: $(x, y, z) \in B \times B \times B, (u, v, w) \in B \times B \times A$. It is obvious that (3.24) holds.

Case 3: $(x, y, z) \in B \times B \times B, (u, v, w) \in B \times A \times A$. It is obvious that (3.24) holds.

Case 4: $(x, y, z) \in B \times B \times B, (u, v, w) \in A \times A \times A$. (i). If $u = v = w$, let $u = v = w = \frac{1}{2n}$, then

$$2|F(x, y, z) - F(u, v, w)| = \frac{2}{(2n+1)^4},$$
$$\max\{|g(x) - g(u)|, |g(y) - g(v)|, |g(z) - g(w)|\} = \begin{cases} \frac{1}{2n+1} & \text{if } x = y = z = 0, \\ \frac{2n}{2n+1} & \text{otherwise,} \end{cases}$$

which implies that (3.24) holds.

(ii). If $u \neq v$ or $v \neq w$ or $u \neq w$, then inequality (3.24) holds obviously.

Case 5: $(x, y, z) \in B \times B \times A, (u, v, w) \in B \times B \times A$. It is obvious that (3.24) holds.

Case 6: $(x, y, z) \in B \times B \times A, (u, v, w) \in B \times A \times A$. It is obvious that (3.24) holds.

Case 7: $(x, y, z) \in B \times B \times A, (u, v, w) \in A \times A \times A$.

(i). If $u = v = w$, let $u = v = w = \frac{1}{2n}$, $z = \frac{1}{2m}$, then

$$2|F(x, y, z) - F(u, v, w)| = \frac{2}{(2n+1)^4},$$
$$\max\{|g(x) - g(u)|, |g(y) - g(v)|, |g(z) - g(w)|\} = \begin{cases} \max\{\frac{1}{2n+1}, |\frac{1}{2m+1} - \frac{1}{2n+1}|\} & \text{if } x = y = 0 \\ \max\{\frac{2n}{2n+1}, |\frac{1}{2m+1} - \frac{1}{2n+1}|\} & \text{otherwise} \end{cases},$$

which implies that (3.24) holds.

(ii). If $u \neq v$ or $v \neq w$ or $u \neq w$, then inequality (3.24) holds obviously.

Case 8: $(x, y, z) \in B \times A \times A, (u, v, w) \in B \times A \times A$. It is obvious that (3.24) holds.

Case 9: $(x, y, z) \in B \times A \times A, (u, v, w) \in A \times A \times A$. (i). If $u = v = w$, let $u = v = w = \frac{1}{2n}$, $y = \frac{1}{2i}$, $z = \frac{1}{2j}$, then

$$2|F(x, y, z) - F(u, v, w)| = \frac{2}{(2n+1)^4},$$
$$\max\{|g(x) - g(u)|, |g(y) - g(v)|, |g(z) - g(w)|\} = \begin{cases} \max\{\frac{1}{2n+1}, |\frac{1}{2i+1} - \frac{1}{2n+1}|, |\frac{1}{2j} - \frac{1}{2n+1}|\} & \text{if } x = 0 \\ \max\{\frac{2n}{2n+1}, |\frac{1}{2i+1} - \frac{1}{2n+1}|, |\frac{1}{2j+1} - \frac{1}{2n+1}|\} & \text{otherwise} \end{cases},$$

which implies that (3.24) holds.

(ii). If $u \neq v$ or $v \neq w$ or $u \neq w$, then inequality (3.24) holds obviously.

Case 10: $(x, y, z) \in A \times A \times A, (u, v, w) \in A \times A \times A$.

(i). If $u = v = w$, $x = y = z$, let $u = v = w = \frac{1}{2n}$, $x = y = z = \frac{1}{2m}$, then

$$2|F(x,y,z) - F(u,v,w)| = \left|\frac{2}{(2m+1)^4} - \frac{2}{(2n+1)^4}\right|,$$
$$\max\{|g(x) - g(u)|, |g(y) - g(v)|, |g(z) - g(w)|\} = \left|\frac{1}{2m+1} - \frac{1}{2n+1}\right|,$$

which implies that (3.24) holds.

(ii). If $u = v = w$, $x \neq y$ or $y \neq z$ or $x \neq z$, without loss of generality, let $u = v = w = \frac{1}{2n}$, $x = \frac{1}{2i}$, $y = \frac{1}{2j} = z$, $i \neq j$, then

$$2|F(x,y,z) - F(u,v,w)| = \frac{2}{(2n+1)^4},$$
$$\max\{|g(x) - g(u)|, |g(y) - g(v)|, |g(z) - g(w)|\} = \max\left\{\left|\frac{1}{2i+1} - \frac{1}{2n+1}\right|, \left|\frac{1}{2j+1} - \frac{1}{2n+1}\right|\right\},$$

which implies that (3.24) holds.

(iii). If $u \neq v$ or $v \neq w$ or $u \neq w$, $x \neq y$ or $y \neq z$ or $x \neq z$, then inequality (3.24) holds obviously.

Thus all the conditions in Theorem 3.5 are satisfied, and 0 is the unique common fixed point of g and F.

Note that g and F are not commutative, and therefore the unique tripled common fixed point of g and F cannot be obtained by Theorem 3.4. It means that Theorem 3.5 is a proper improvement of Theorem 3.4.

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