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# Common fixed point theorems for a class of maps of $\Phi$-contraction in generalized metric spaces

Feng Gu[a,*], Yun Yin[b]

[a] *Institute of Applied Mathematics and Department of Mathematics, Hangzhou Normal University, Hangzhou, Zhejiang 310036, China.*

[b] *Zhanjiang No.2 Middle School, Zhanjiang, Guangdong 524022, China.*

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**Abstract**

In this paper, we prove a new common fixed point theorem for three pair of weakly compatible mappings satisfying $\phi$-contractive condition in the framework of generalized metric spaces. It is worth mentioning that our results do not rely on continuity of mappings involved therein. The main result of the paper generalizes several comparable results from the current literature. We also provide illustrative examples in support of our new results. 

*Keywords:* Generalized metric space, $\phi$-contraction, common fixed point, weakly compatible mappings.
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## 1. Introduction and preliminaries

In 2006, Mustafa in collaboration with Sims introduced a new notion of generalized metric space called $G$-metric space [17]. Then, based on the notion of generalized metric spaces, many authors obtained some fixed point and common fixed point results under some contractive conditions, see [1–10, 12–16, 18–23]. In the present work, we study some common fixed point results for six self-mappings in a $G$-metric space $X$ involving nonlinear contractions related to a function $\phi \in \Phi$, where $\Phi$ is the set of nondecreasing continuous functions $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying:

(a) $0 < \phi(t) < t$ for all $t > 0$;

*Corresponding author
*Email addresses:* gufeng_99@sohu.com (Feng Gu), 497308798@qq.com (Yun Yin)

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(b) the series $\sum_{n\geq 1}\phi^n(t)$ converge for all $t>0$.

From (b), we may have $\lim_{n\to\infty}\phi^n(t)=0$ for all $t>0$. Again from (a), we have $\phi(0)=0$.

Now, we present some necessary definitions and results in $G$-metric spaces, which will be needed in the sequel.

**Definition 1.1** ([17]). Let $X$ be a nonempty set, and let $G: X\times X\times X\longrightarrow R^+$ be a function satisfying the following axioms:

(G1) $G(x,y,z)$=0 if $x=y=z$;

(G2) $0<G(x,x,y)$ for all $x,y\in X$ with $x\neq y$;

(G3) $G(x,x,y)\leq G(x,y,z)$ for all $x,y,z\in X$ with $z\neq$ y;

(G4) $G(x,y,z)=G(x,z,y)=G(y,z,x)=\cdots$, (symmetry in all three variables);

(G5) $G(x,y,z)\leq G(x,a,a)+G(a,y,z)$ for all $x,y,z,a\in X$, (rectangle inequality).

Then the function $G$ is called a generalized metric, or, more specifically a $G$-metric on $X$ and the pair $(X,G)$ is called a $G$-metric space.

**Definition 1.2** ([17]). Let $(X,G)$ be a $G$-metric space, and let $\{x_n\}$ be a sequence of points in $X$, a point $x$ in $X$ is said to be the limit of the sequence $\{x_n\}$ if $\lim_{m,n\to\infty}G(x,x_n,x_m)=0$, and one says that sequence $\{x_n\}$ is $G$-convergent to $x$.

Thus, if $x_n\to x$ in a $G$-metric space $(X,G)$, then for any $\epsilon>0$, there exists $N\in\mathbb{N}$ such that $G(x,x_n,x_m)<\epsilon$ for all $n,m\geq N$.

**Proposition 1.3** ([17]). *Let $(X,G)$ be a $G$-metric space, then the following are equivalent:*

(1) *$\{x_n\}$ is $G$-convergent to $x$.*

(2) *$G(x_n,x_n,x)\to 0$ as $n\to\infty$.*

(3) *$G(x_n,x,x)\to 0$ as $n\to\infty$.*

(4) *$G(x_n,x_m,x)\to 0$ as $n,m\to\infty$.*

**Definition 1.4** ([17]). Let $(X,G)$ be a $G$-metric space. A sequence $\{x_n\}$ is called $G$-Cauchy sequence if for each $\epsilon>0$ there exists a positive integer $N\in\mathbb{N}$ such that $G(x_n,x_m,x_l)<\epsilon$ for all $n,m,l\geq N$; i.e., if $G(x_n,x_m,x_l)\to 0$ as $n,m,l\to\infty$.

**Definition 1.5** ([17]). A $G$-metric space $(X,G)$ is said to be $G$-complete, if every $G$-Cauchy sequence in $(X,G)$ is $G$-convergent in $X$.

**Proposition 1.6** ([17]). *Let $(X,G)$ be a $G$-metric space. Then the following are equivalent:*

(1) *The sequence $\{x_n\}$ is $G$-Cauchy.*

(2) *For every $\epsilon>0$, there exists $k\in\mathbb{N}$ such that $G(x_n,x_m,x_m)<\epsilon$, for all $n,m\geq k$.*

**Proposition 1.7** ([17]). *Let $(X,G)$ be a $G$-metric space. Then the function $G(x,y,z)$ is jointly continuous in all three of its variables.*

**Definition 1.8.** Let $f$ and $g$ be self-maps of a set $X$. If $w=fx=gx$ for some $x$ in $X$, then $x$ is called a coincidence point of $f$ and $g$, and $w$ is called point of coincidence of $f$ and $g$.

**Definition 1.9** ([11]). Two self-mappings $f$ and $g$ are said to be weakly compatible, if they commute at coincidence points.

**Proposition 1.10** ([17]). *Let $(X, G)$ be a $G$-metric space. Then for any $x, y, z, a$ in $X$ it follows that:*

(i) *if $G(x, y, z) = 0$, then $x = y = z$;*

(ii) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$;

(iii) $G(x, y, y) \leq 2G(y, x, x)$;

(iv) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$;

(v) $G(x, y, z) \leq \frac{2}{3}(G(x, y, a) + G(x, a, z) + G(a, y, z))$;

(vi) $G(x, y, z) \leq G(x, a, a) + G(y, a, a) + G(z, a, a)$.

## 2. Main results

From now on, unless otherwise stated, we mean by $\phi \in \Phi$.

**Theorem 2.1.** *Let $(X, G)$ be a $G$-metric space, $f, g, h, S, T$ and $R$ be six mappings of $X$ into itself such that*

$$\begin{aligned} G(fx, gy, hz) \leq \phi\Big(\max\Big\{ & G(Sx, Ty, Rz), \frac{1}{2}G(fx, Sx, Rz), \frac{1}{2}G(gy, Ty, Sx), \frac{1}{2}G(hz, Rz, Ty), \\ & \frac{1}{2}G(fx, Sx, gy), \frac{1}{2}G(gy, Ty, hz), \frac{1}{2}G(hz, Rz, fx)\Big\}\Big) \end{aligned} \tag{2.1}$$

*for all $x, y, z \in X$. If the following conditions are satisfied:*

(i) *$f(X) \subset T(X)$, $g(X) \subset R(X)$, $h(X) \subset S(X)$;*

(ii) *one of $S(X), T(X)$ and $R(X)$ is a $G$-complete subspace of $X$.*

*Then one of the pairs $(f, S), (g, T)$ and $(h, R)$ has a coincidence point in $X$. Moreover, if the pairs $(f, S)$, $(g, T)$ and $(h, R)$ are weakly compatible, then $f, g, h, S, T$ and $R$ have a unique common fixed point in $X$.*

*Proof.* Let $x_0 \in X$. From (i), there exist $x_1, x_2, x_3 \in X$ such that $y_0 = fx_0 = Tx_1$, $y_1 = gx_1 = Rx_2$ and $y_2 = hx_2 = Sx_3$. By the induction, there exist sequences $\{x_n\}$ and $\{y_n\}$ in $X$ such that

$$y_{3n} = fx_{3n} = Tx_{3n+1},\ y_{3n+1} = gx_{3n+1} = Rx_{3n+2},\ y_{3n+2} = hx_{3n+2} = Sx_{3n+3}$$

for all $n = 0, 1, 2, \cdots$.

If $y_{3n} = y_{3n+1}$, then $gx = Tx$, where $x = x_{3n+1}$. If $y_{3n+1} = y_{3n+2}$, then $hx = Rx$, where $x = x_{3n+2}$. If $y_{3n+2} = y_{3n+3}$, then $fx = Sx$, where $x = x_{3n+3}$. Assume that $y_n \neq y_{n+1}$ for all $n$. We claim that $\{y_n\}$ is a $G$-Cauchy sequence in $X$. For $n \in N$, we have

$$\begin{aligned} G(y_{3n}, y_{3n+1}, y_{3n+2}) &= G(fx_{3n}, gx_{3n+1}, hx_{3n+2}) \\ &\leq \phi\Big(\max\Big\{G(Sx_{3n}, Tx_{3n+1}, Rx_{3n+2}), \frac{1}{2}G(fx_{3n}, Sx_{3n}, Rx_{3n+2}), \\ &\qquad \frac{1}{2}G(gx_{3n+1}, Tx_{3n+1}, Sx_{3n}), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, Tx_{3n+1}), \\ &\qquad \frac{1}{2}G(fx_{3n}, Sx_{3n}, gx_{3n+1}), \frac{1}{2}G(gx_{3n+1}, Tx_{3n+1}, hx_{3n+2}), \end{aligned}$$

$$
\begin{aligned}
&\qquad \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, fx_{3n})\Big\}\Big) \\
&= \phi\Big(\max\Big\{G(y_{3n-1}, y_{3n}, y_{3n+1}), \frac{1}{2}G(y_{3n}, y_{3n-1}, y_{3n+1}), \frac{1}{2}G(y_{3n+1}, y_{3n}, y_{3n-1}), \\
&\qquad \frac{1}{2}G(y_{3n+2}, y_{3n+1}, y_{3n}), \frac{1}{2}G(y_{3n}, y_{3n-1}, y_{3n+1}), \frac{1}{2}G(y_{3n+1}, y_{3n}, y_{3n+2}), \\
&\qquad \frac{1}{2}G(y_{3n+2}, y_{3n+1}, y_{3n})\Big\}\Big) \\
&= \phi\left(\max\left\{G(y_{3n-1}, y_{3n}, y_{3n+1}), \frac{1}{2}G(y_{3n}, y_{3n+1}, y_{3n+2})\right\}\right).
\end{aligned}
$$

If

$$\max\left\{G(y_{3n-1}, y_{3n}, y_{3n+1}), \frac{1}{2}G(y_{3n}, y_{3n+1}, y_{3n+2})\right\} = \frac{1}{2}G(y_{3n}, y_{3n+1}, y_{3n+2}),$$

then, by using the fact that $\phi(t) < t, t \in (0, \infty)$, we have

$$G(y_{3n}, y_{3n+1}, y_{3n+2}) \le \phi\left(\frac{1}{2}G(y_{3n}, y_{3n+1}, y_{3n+2})\right) < \frac{1}{2}G(y_{3n}, y_{3n+1}, y_{3n+2}),$$

which is a contradiction, so

$$\max\left\{G(y_{3n-1}, y_{3n}, y_{3n+1}), \frac{1}{2}G(y_{3n}, y_{3n+1}, y_{3n+2})\right\} = G(y_{3n-1}, y_{3n}, y_{3n+1}).$$

For $\phi$ as a nondecreasing function, we get

$$G(y_{3n}, y_{3n+1}, y_{3n+2}) \le \phi(G(y_{3n-1}, y_{3n}, y_{3n+1})). \tag{2.2}$$

By the condition (2.1), we have

$$
\begin{aligned}
G(y_{3n+1}, y_{3n+2}, y_{3n+3}) &= G(gx_{3n+1}, hx_{3n+2}, fx_{3n+3}) = G(fx_{3n+3}, gx_{3n+1}, hx_{3n+2}) \\
&\le \phi\Big(\max\Big\{G(Sx_{3n+3}, Tx_{3n+1}, Rx_{3n+2}), \frac{1}{2}G(fx_{3n+3}, Sx_{3n+3}, Rx_{3n+2}), \\
&\qquad \frac{1}{2}G(gx_{3n+1}, Tx_{3n+1}, Sx_{3n+3}), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, Tx_{3n+1}), \\
&\qquad \frac{1}{2}G(fx_{3n+3}, Sx_{3n+3}, gx_{3n+1}), \frac{1}{2}G(gx_{3n+1}, Tx_{3n+1}, hx_{3n+2}), \\
&\qquad \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, fx_{3n+3})\Big\}\Big) \\
&= \phi\Big(\max\Big\{G(y_{3n+2}, y_{3n}, y_{3n+1}), \frac{1}{2}G(y_{3n+3}, y_{3n+2}, y_{3n+1}), \frac{1}{2}G(y_{3n+1}, y_{3n}, y_{3n+2}) \\
&\qquad \frac{1}{2}G(y_{3n+2}, y_{3n+1}, y_{3n}), \frac{1}{2}G(y_{3n+3}, y_{3n+2}, y_{3n+1}), \frac{1}{2}G(y_{3n+1}, y_{3n}, y_{3n+2}), \\
&\qquad \frac{1}{2}G(y_{3n+2}, y_{3n+1}, y_{3n+3})\Big\}\Big) \\
&= \phi\left(\max\left\{G(y_{3n}, y_{3n+1}, y_{3n+2}), \frac{1}{2}G(y_{3n+1}, y_{3n+2}, y_{3n+3})\right\}\right).
\end{aligned}
$$

If

$$\max\left\{G(y_{3n}, y_{3n+1}, y_{3n+2}), \frac{1}{2}G(y_{3n+1}, y_{3n+2}, y_{3n+3})\right\} = \frac{1}{2}G(y_{3n+1}, y_{3n+2}, y_{3n+3}),$$

then, by using the fact that $\phi(t) < t, t \in (0,\infty)$, we have

$$G(y_{3n+1}, y_{3n+2}, y_{3n+3}) \le \phi\left(\frac{1}{2}G(y_{3n+1}, y_{3n+2}, y_{3n+3})\right) < \frac{1}{2}G(y_{3n+1}, y_{3n+2}, y_{3n+3}),$$

which leads to a contradiction, this implies that

$$\max\left\{G(y_{3n}, y_{3n+1}, y_{3n+2}), \frac{1}{2}G(y_{3n+1}, y_{3n+2}, y_{3n+3})\right\} = G(y_{3n}, y_{3n+1}, y_{3n+2}).$$

For $\phi$ as a nondecreasing function, we have

$$G(y_{3n+1}, y_{3n+2}, y_{3n+3}) \le \phi(G(y_{3n}, y_{3n+1}, y_{3n+2})). \tag{2.3}$$

Again, by using (2.1), we can get

$$\begin{aligned}
&G(y_{3n+2}, y_{3n+3}, y_{3n+4})\\
&\quad= G(hx_{3n+2}, fx_{3n+3}, gx_{3n+4}) = G(fx_{3n+3}, gx_{3n+4}, hx_{3n+2})\\
&\quad\le \phi\Bigg(\max\Bigg\{G(Sx_{3n+3}, Tx_{3n+4}, Rx_{3n+2}), \frac{1}{2}G(fx_{3n+3}, Sx_{3n+3}, Rx_{3n+2}),\\
&\qquad\qquad \frac{1}{2}G(gx_{3n+4}, Tx_{3n+4}, Sx_{3n+3}), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, Tx_{3n+4}),\\
&\qquad\qquad \frac{1}{2}G(fx_{3n+3}, Sx_{3n+3}, gx_{3n+4}), \frac{1}{2}G(gx_{3n+4}, Tx_{3n+4}, hx_{3n+2}),\\
&\qquad\qquad \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, fx_{3n+3})\Bigg\}\Bigg)\\
&\quad= \phi\Bigg(\max\Bigg\{G(y_{3n+2}, y_{3n+3}, y_{3n+1}), \frac{1}{2}G(y_{3n+3}, y_{3n+2}, y_{3n+1}), \frac{1}{2}G(y_{3n+4}, y_{3n+3}, y_{3n+2}),\\
&\qquad\qquad \frac{1}{2}G(y_{3n+2}, y_{3n+1}, y_{3n+3}), \frac{1}{2}G(y_{3n+3}, y_{3n+2}, y_{3n+4}), \frac{1}{2}G(y_{3n+4}, y_{3n+3}, y_{3n+2}),\\
&\qquad\qquad \frac{1}{2}G(y_{3n+2}, y_{3n+1}, y_{3n+3})\Bigg\}\Bigg)\\
&\quad= \phi\left(\max\left\{G(y_{3n+1}, y_{3n+2}, y_{3n+3}), \frac{1}{2}G(y_{3n+2}, y_{3n+3}, y_{3n+4})\right\}\right).
\end{aligned}$$

If

$$\max\left\{G(y_{3n+1}, y_{3n+2}, y_{3n+3}), \frac{1}{2}G(y_{3n+2}, y_{3n+3}, y_{3n+4})\right\} = \frac{1}{2}G(y_{3n+2}, y_{3n+3}, y_{3n+4}),$$

then, since $\phi(t) < t, t \in (0,\infty)$, we have

$$G(y_{3n+2}, y_{3n+3}, y_{3n+4}) \le \phi\left(\frac{1}{2}G(y_{3n+2}, y_{3n+3}, y_{3n+4})\right) < \frac{1}{2}G(y_{3n+2}, y_{3n+3}, y_{3n+4}),$$

which is a contradiction, so we have

$$\max\left\{G(y_{3n+1}, y_{3n+2}, y_{3n+3}), \frac{1}{2}G(y_{3n+2}, y_{3n+3}, y_{3n+4})\right\} = G(y_{3n+1}, y_{3n+2}, y_{3n+3}).$$

For $\phi$ as a nondecreasing function, we have

$$G(y_{3n+2}, y_{3n+3}, y_{3n+4}) \le \phi(G(y_{3n+1}, y_{3n+2}, y_{3n+3})). \tag{2.4}$$

By combining (2.2), (2.3) and (2.4), for $n \in \mathbb{N}$, we have

$$G(y_n, y_{n+1}, y_{n+2}) \leq \phi(G(y_{n-1}, y_n, y_{n+1})) \leq \phi^2(G(y_{n-1}, y_{n-2}, y_n)) \leq \cdots \leq \phi^n(G(y_0, y_1, y_2)). \tag{2.5}$$

Therefore, for all $n, m \in \mathbb{N}$, by using conditions (G3), (G4), (G5) and (2.5), we have

$$\begin{aligned} G(y_n, y_{n+m}, y_{n+m}) &\leq G(y_n, y_{n+1}, y_{n+1}) + G(y_{n+1}, y_{n+2}, y_{n+2}) + \cdots + G(y_{n+m-1}, y_{n+m}, y_{n+m}) \\ &\leq \phi^n(G(y_0, y_1, y_2)) + \phi^{n+1}(G(y_0, y_1, y_2)) + \cdots + \phi^{n+m}(G(y_0, y_1, y_2)) \\ &\leq \sum_{k=n}^{n+m} \phi^k(G(y_0, y_1, y_2)) \leq \sum_{k=n}^{+\infty} \phi^k(G(y_0, y_1, y_2)). \end{aligned}$$

The property (b) yields that $\sum_{k=n}^{+\infty} \phi^k(G(y_0, y_1, y_2))$ tends to 0 as $n \to +\infty$. Therefore

$$\lim_{n \to +\infty} G(y_n, y_{n+m}, y_{n+m}) = 0, \quad \forall m \in \mathbb{N}.$$

That means the sequence $\{y_n\}$ is $G$-Cauchy.

Let us first assume that $S(X)$ is a $G$-complete subspace of $X$. Then there exist $p, t \in X$ such that $y_{3n+2} \to p = St$. Since $\{y_n\}$ is $G$-Cauchy, it follows that $y_{3n} \to p$ and $y_{3n+1} \to p$. By (2.1) we obtain

$$\begin{aligned} G(ft, y_{3n+1}, y_{3n+2}) &= G(ft, gx_{3n+1}, hx_{3n+2}) \\ &\leq \phi\Bigg(\max\Bigg\{G(St, Tx_{3n+1}, Rx_{3n+2}), \frac{1}{2}G(ft, St, Rx_{3n+2}), \\ &\qquad \frac{1}{2}G(gx_{3n+1}, Tx_{3n+1}, St), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, Tx_{3n+1}), \frac{1}{2}G(ft, St, gx_{3n+1}), \\ &\qquad \frac{1}{2}G(gx_{3n+1}, Tx_{3n+1}, hx_{3n+2}), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, ft)\Bigg\}\Bigg). \end{aligned}$$

By letting $n \to \infty$, we get

$$G(ft, p, p) \leq \phi\left(\frac{1}{2}G(ft, p, p)\right).$$

Hence $ft = p$. Thus $St = ft = p$.

Since $(f, S)$ is a weakly compatible pair, we have $fp = Sp$. From (2.1) we get

$$\begin{aligned} G(fp, y_{3n+1}, y_{3n+2}) &= G(fp, gx_{3n+1}, hx_{3n+2}) \\ &\leq \phi\Bigg(\max\Bigg\{G(Sp, Tx_{3n+1}, Rx_{3n+2}), \frac{1}{2}G(fp, Sp, Rx_{3n+2}), \\ &\qquad \frac{1}{2}G(gx_{3n+1}, Tx_{3n+1}, Sp), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, Tx_{3n+1}), \frac{1}{2}G(fp, Sp, gx_{3n+1}), \\ &\qquad \frac{1}{2}G(gx_{3n+1}, Tx_{3n+1}, hx_{3n+2}), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, fp)\Bigg\}\Bigg). \end{aligned}$$

By letting $n \to \infty$ and using inequality $G(fp, fp, p) \leq 2G(fp, p, p)$ we get

$$G(fp, p, p) \leq \phi\left(\max\left\{G(fp, p, p), \frac{1}{2}G(fp, fp, p)\right\}\right) \leq \phi(G(fp, p, p)).$$

If $fp \neq p$, then $G(fp, p, p) > 0$. From $\phi(t) < t,\ t \in (0, \infty)$, we obtain

$$G(fp, p, p) \leq \phi(G(fp, p, p)) < G(fp, p, p),$$

which is a contradiction. Thus $fp = p$, and so $fp = Sp = p$.

Since $f(X) \subset T(X)$ and $p = fp \in f(X)$, there exists $v \in X$ such that $p = fp = Tv$. From (2.1) we have

$$\begin{aligned}G(fp, gv, y_{3n+2}) &= G(fp, gv, hx_{3n+2})\\ &\le \phi\bigg(\max\bigg\{G(Sp, Tv, Rx_{3n+2}), \frac{1}{2}G(fp, Sp, Rx_{3n+2}),\\ &\qquad \frac{1}{2}G(gv, Tv, Sp), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, Tv), \frac{1}{2}G(fp, Sp, gv),\\ &\qquad \frac{1}{2}G(gv, Tv, hx_{3n+2}), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, fp)\bigg\}\bigg).\end{aligned}$$

By letting $n \to \infty$ we get, $G(p, gv, p) \le \phi(\frac{1}{2}G(gv, p, p))$. Since $\phi(t) < t,\ t \in (0, \infty)$, we have $G(p, gv, p) = 0$, so that $gv = p$. Thus $gv = Tv = p$. Because $(g, T)$ is a weakly compatible pair, we have $gp = Tp$. By (2.1) we have

$$\begin{aligned}G(fp, gp, y_{3n+2}) &= G(fp, gp, hx_{3n+2})\\ &\le \phi\bigg(\max\bigg\{G(Sp, Tp, Rx_{3n+2}), \frac{1}{2}G(fp, Sp, Rx_{3n+2}),\\ &\qquad \frac{1}{2}G(gp, Tp, Sp), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, Tp), \frac{1}{2}G(fp, Sp, gp),\\ &\qquad \frac{1}{2}G(gp, Tp, hx_{3n+2}), \frac{1}{2}G(hx_{3n+2}, Rx_{3n+2}, fp)\bigg\}\bigg).\end{aligned}$$

By letting $n \to \infty$ and using inequality $G(gp, gp, p) \le 2G(gp, p, p)$ we obtain

$$G(p, gp, p) \le \phi\left(\max\left\{G(p, gp, g), \frac{1}{2}G(gp, gp, p)\right\}\right) \le \phi(G(p, gp, p)).$$

Hence $gp = p$. Thus $gp = Tp = p$.

Since $g(X) \subset R(X)$ and $p = gp \in g(X)$, there is a point $w \in X$ such that $p = gp = Rw$. Again by using condition (2.1) we have

$$\begin{aligned}G(p, p, hw) &= G(fp, gp, hw)\\ &\le \phi\bigg(\max\bigg\{G(Sp, Tp, Rw), \frac{1}{2}G(fp, Sp, Rw), \frac{1}{2}G(gp, Tp, Sp),\\ &\qquad \frac{1}{2}G(hw, Rw, Tp), \frac{1}{2}G(fp, Sp, gp), \frac{1}{2}G(gp, Tp, hw), \frac{1}{2}G(hw, Rw, fp)\bigg\}\bigg)\\ &= \phi\left(\frac{1}{2}G(p, p, hw)\right).\end{aligned} \tag{2.6}$$

If $hw \ne p$, then $G(p, p, hw) > 0$ and so $\phi(\frac{1}{2}G(p, p, hw)) < \frac{1}{2}G(p, p, hw)$. Therefore, by (2.6) we have

$$G(p, p, hw) \le \phi\left(\frac{1}{2}G(p, p, hw)\right) < \frac{1}{2}G(p, p, hw),$$

which is a contradiction, and hence $hw = p = Rw$.

Since $(h, R)$ is a weakly compatible pair, we have $Rp = hp$. By using the condition (2.1) and

$$G(p, hp, hp) \le 2G(p, p, hp),$$

we have

$$\begin{aligned}G(p, p, hp) &= G(fp, gp, hp)\\ &\le \phi\bigg(\max\bigg\{G(Sp, Tp, Rp), \frac{1}{2}G(fp, Sp, Rp), \frac{1}{2}G(gp, Tp, Sp),\end{aligned}$$

$$\frac{1}{2}G(hp,Rp,Tp),\frac{1}{2}G(fp,Sp,gp),\frac{1}{2}G(gp,Tp,hp),\frac{1}{2}G(hp,Rp,fp)\Big\}\Big) \tag{2.7}$$
$$= \phi\left(\max\left\{G(p,p,hp),\frac{1}{2}G(p,hp,hp)\right\}\right)$$
$$\leq \phi(G(p,p,hp)).$$

If $hp \neq p$, then $G(p,p,hp) > 0$ and so $\phi(G(p,p,hp)) < G(p,p,hp)$. Therefore, by (2.7) we have

$$G(p,p,hp) \leq \phi(G(p,p,hp) < G(p,p,hp),$$

which is a contradiction, and hence $hp = Rp = p$.

From the above, it follows that $p$ is a common fixed point of $f,g,h,S,T$ and $R$.

Suppose $q$ is another common fixed point of $f,g,h,S,T$ and $R$. By (2.1) $G(p,q,q) \leq 2G(p,p,q)$ we have

$$\begin{aligned} G(p,p,q) &= G(fp,gp,hq) \\ &\leq \phi\Big(\max\Big\{G(Sp,Tp,Rq),\frac{1}{2}G(fp,Sp,Rq),\frac{1}{2}G(gp,Tp,Sp), \\ &\qquad \frac{1}{2}G(hq,Rq,Tp),\frac{1}{2}G(fp,Sp,gp),\frac{1}{2}G(gp,Tp,hq),\frac{1}{2}G(hq,Rq,fp)\Big\}\Big) \\ &\leq \phi\left(\max\left\{G(p,p,q),\frac{1}{2}G(p,q,q)\right\}\right) \\ &\leq \phi(G(p,p,q)), \end{aligned}$$

which implies that $q = p$. Thus $p$ is a unique common fixed point of $S,T,R,f,g$ and $h$.

If $T(X)$ or $R(X)$ is a $G$-complete subspace of $X$, then proof is similar. This completes the proof. □

*Remark* 2.2. The results obtained in this paper are innovative. The contractive conditions studied in this paper are new. As far as now, no author has investigated the problems.

*Remark* 2.3. Theorem 2.1 improves and extends the corresponding results of [7] and [10] in three aspects:

(1) Any two pairs of the three pairs of mappings do not need to satisfy common $(E.A)$ property.

(2) "One of $SX$, $TX$ and $RX$ is closed" is replaced by the "one of $SX$, $TX$ and $RX$ is complete".

(3) The contractive conditions studied in Theorem 2.1 with the literature [7] and [23] are quite different.

*Remark* 2.4. Theorem 2.1 extends and improves the main results of [8] in the following ways:

(1) The main results of [8] require, at least, one mapping is continuous. However, the Theorem 2.1 do not rely on continuity of any mappings.

(2) The mappings pair is weakly commuting is replaced by the more general weakly compatibility.

(3) "The $X$ be a complete $G$-metric spaces" is replaced by the "one of $SX$, $TX$ and $RX$ is $G$- complete subspace of $X$".

(4) The contractive conditions studied in Theorem 2.1 with the literature [7] and [23] are quite different.

*Remark* 2.5. Theorem 2.1 also extends and improves the main results of [9] and [21].

Now we introduce an example to support Theorem 2.1.

**Example 2.6.** Let $X = [0,1]$, and $(X,G)$ be a $G$-metric space defined by $G(x,y,z) = |x-y|+|y-z|+|z-x|$ for all $x,y,z$ in $X$. Let $f$, $g$, $h$, $S$, $T$ and $R$ be self-mappings defined by

$$fx = \begin{cases} \frac{7}{8}, & x \in [0,\frac{1}{2}], \\ \frac{8}{9}, & x \in (\frac{1}{2},1]. \end{cases}, \quad gx = \begin{cases} \frac{10}{11}, & x \in [0,\frac{1}{2}], \\ \frac{8}{9}, & x \in (\frac{1}{2},1]. \end{cases}, \quad hx = \begin{cases} \frac{9}{10}, & x \in [0,\frac{1}{2}], \\ \frac{8}{9}, & x \in (\frac{1}{2},1]. \end{cases}$$

$$Sx = \begin{cases} 1, & x \in [0,\frac{1}{2}], \\ \frac{8}{9}, & x \in (\frac{1}{2},1), \\ \frac{9}{10}, & x = 1. \end{cases}, \quad Tx = \begin{cases} 1, & x \in [0,\frac{1}{2}], \\ \frac{8}{9}, & x \in (\frac{1}{2},1), \\ \frac{7}{8}, & x = 1. \end{cases}, \quad Rx = \begin{cases} 1, & x \in [0,\frac{1}{2}], \\ \frac{8}{9}, & x \in (\frac{1}{2},1), \\ \frac{10}{11}, & x = 1. \end{cases}$$

Note that $f$, $g$, $h$, $S$, $T$ and $R$ are not $G$-continuous in $X$. Clearly we can get $f(X) \subset T(X)$, $g(X) \subset R(X)$, $h(X) \subset S(X)$.

By the definition of the mappings of $f$ and $S$, only for $x \in (\frac{1}{2},1)$, $fx = Sx = \frac{8}{9}$, at this time $fSx = f(\frac{8}{9}) = \frac{8}{9} = S(\frac{8}{9}) = Sfx$, so $fSx = Sfx$, thus we can obtain the pair $(f,S)$ is weakly compatible. Similarly we can show that the pair $(g,T)$ and $(h,R)$ are also weakly compatible.

Now we prove that the mappings $f$, $g$, $h$, $S$, $T$ and $R$ satisfy the condition (2.1) of Theorem 2.1 with $\phi(t) = \frac{6}{11}t$. Let

$$M(x,y,z) = \max\left\{ \begin{array}{c} G(Sx,Ty,Rz), \frac{1}{2}G(fx,Sx,Rz), \frac{1}{2}G(gy,Ty,Sx), \frac{1}{2}G(hz,Rz,Ty), \\ \frac{1}{2}G(fx,Sx,gy), \frac{1}{2}G(gy,Ty,hz), \frac{1}{2}G(hz,Rz,fx) \end{array} \right\}.$$

**Case 1.** If $x,y,z \in \left[0,\frac{1}{2}\right]$, then

$$G(fx,gy,hz) = G\left(\frac{7}{8},\frac{10}{11},\frac{9}{10}\right) = \frac{3}{44},$$

$$G(fx,Sx,Rz) = G\left(\frac{7}{8},1,1\right) = \frac{1}{4}.$$

Thus we have

$$G(fx,gy,hz) = \frac{3}{44} = \frac{6}{11}\cdot\frac{1}{2}\cdot\frac{1}{4} = \frac{6}{11}\cdot\frac{1}{2}\cdot G(fx,Sx,Rz) \le \frac{6}{11}\cdot M(x,y,z) = \phi(M(x,y,z)).$$

**Case 2.** If $x,y \in \left[0,\frac{1}{2}\right], z \in \left(\frac{1}{2},1\right]$, then

$$G(fx,gy,hz) = G\left(\frac{7}{8},\frac{10}{11},\frac{8}{9}\right) = \frac{3}{44},$$

$$G(fx,Sx,gy) = G\left(\frac{7}{8},1,\frac{10}{11}\right) = \frac{1}{4}.$$

Therefore we get

$$G(fx,gy,hz) = \frac{3}{44} = \frac{6}{11}\cdot\frac{1}{2}\cdot\frac{1}{4} = \frac{6}{11}\cdot\frac{1}{2}\cdot G(fx,Sx,gy) \le \frac{6}{11}\cdot M(x,y,z) = \phi(M(x,y,z)).$$

**Case 3.** If $x,z \in \left[0,\frac{1}{2}\right], y \in \left(\frac{1}{2},1\right]$, then

$$G(fx,gy,hz) = G\left(\frac{7}{8},\frac{8}{9},\frac{9}{10}\right) = \frac{1}{20},$$

$$G(hz,Rz,fx) = G\left(\frac{9}{10},1,\frac{7}{8}\right) = \frac{1}{4}.$$

Hence we have

$$G(fx,gy,hz) = \frac{1}{20} = \frac{2}{5}\cdot\frac{1}{2}\cdot\frac{1}{4} < \frac{6}{11}\cdot\frac{1}{2}\cdot\frac{1}{4} = \frac{6}{11}\cdot\frac{1}{2}\cdot G(hz,Rz,fx) \le \frac{6}{11}\cdot M(x,y,z) = \phi(M(x,y,z)).$$

**Case 4.** If $y, z \in \left[0, \frac{1}{2}\right], x \in \left(\frac{1}{2}, 1\right]$, then

$$G(fx, gy, hz) = G\left(\frac{8}{9}, \frac{10}{11}, \frac{9}{10}\right) = \frac{4}{99},$$

$$G(hz, Rz, fx) = G\left(\frac{9}{10}, 1, \frac{8}{9}\right) = \frac{2}{9}.$$

So we get

$$G(fx, gy, hz) = \frac{4}{99} = \frac{4}{11} \cdot \frac{1}{2} \cdot \frac{2}{9} < \frac{6}{11} \cdot \frac{1}{2} \cdot \frac{2}{9} = \frac{6}{11} \cdot \frac{1}{2} \cdot G(hz, Rz, fx) \leq \frac{6}{11} \cdot M(x, y, z) = \phi(M(x, y, z)).$$

**Case 5.** If $x \in \left[0, \frac{1}{2}\right], y, z \in \left(\frac{1}{2}, 1\right]$, then

$$G(fx, gy, hz) = G\left(\frac{7}{8}, \frac{8}{9}, \frac{8}{9}\right) = \frac{1}{36},$$

$$G(fx, Sx, gy) = G\left(\frac{7}{8}, 1, \frac{8}{9}\right) = \frac{1}{4}.$$

So we get

$$G(fx, gy, hz) = \frac{1}{36} = \frac{2}{9} \cdot \frac{1}{2} \cdot \frac{1}{4} < \frac{6}{11} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{6}{11} \cdot \frac{1}{2} \cdot G(fx, Sx, gy) \leq \frac{6}{11} \cdot M(x, y, z) = \phi(M(x, y, z)).$$

**Case 6.** If $y \in \left[0, \frac{1}{2}\right], x, z \in \left(\frac{1}{2}, 1\right]$, then

$$G(fx, gy, hz) = G\left(\frac{8}{9}, \frac{10}{11}, \frac{8}{9}\right) = \frac{4}{99},$$

$$G(gy, Ty, hz) = G\left(\frac{10}{11}, 1, \frac{8}{9}\right) = \frac{2}{9}.$$

Thus we have

$$G(fx, gy, hz) = \frac{4}{99} = \frac{4}{11} \cdot \frac{1}{2} \cdot \frac{2}{9} < \frac{6}{11} \cdot \frac{1}{2} \cdot \frac{2}{9} = \frac{6}{11} \cdot \frac{1}{2} \cdot G(gy, Ty, hz) \leq \frac{6}{11} \cdot M(x, y, z) = \phi(M(x, y, z)).$$

**Case 7.** If $z \in \left[0, \frac{1}{2}\right], x, y \in \left(\frac{1}{2}, 1\right]$, then

$$G(fx, gy, hz) = G\left(\frac{8}{9}, \frac{8}{9}, \frac{9}{10}\right) = \frac{1}{45},$$

$$G(hz, Rz, fx) = G\left(\frac{9}{10}, 1, \frac{7}{8}\right) = \frac{2}{9}.$$

Hence we have

$$G(fx, gy, hz) = \frac{1}{45} = \frac{1}{5} \cdot \frac{1}{2} \cdot \frac{2}{9} < \frac{6}{11} \cdot \frac{1}{2} \cdot \frac{2}{9} = \frac{6}{11} \cdot \frac{1}{2} \cdot G(hz, Rz, fx) \leq \frac{6}{11} \cdot M(x, y, z) = \phi(M(x, y, z)).$$

**Case 8.** If $x, y, z \in \left(\frac{1}{2}, 1\right]$, then

$$G(fx, gy, hz) = G\left(\frac{8}{9}, \frac{8}{9}, \frac{8}{9}\right) = 0 \leq \frac{6}{11} M(x, y, z) = \phi(M(x, y, z)).$$

Then in all the above cases, the mappings $f, g, h, S, T$ and $R$ satisfy the condition (2.1) of the Theorem 2.1 with $\phi(t) = \frac{6}{11} t$. So, all the conditions of Theorem 2.1 are satisfied. Moreover, $\frac{8}{9}$ is the unique common fixed point for all of the mappings $f, g, h, S, T$ and $R$.

*Remark* 2.7. In Theorem 2.1, if we take:

(1) $f = g = h$;

(2) $S = T = R$;

(3) $f = g = h$ and $S = T = R$;

(4) $g = h$ and $T = R$;

(5) $g = h$ and $T = R = I$, ($I$ is the identity mapping, the same below);

then several new results can be obtained.

In Theorem 2.1, if we take $S = T = R = I$, then we have the following corollary.

**Corollary 2.8.** *Let $(X, G)$ be a complete $G$-metric space, $f, g$ and $h$ be three mappings of $X$ into itself such that*

$$G(fx, gy, hz) \leq \phi\Bigg(\max\Bigg\{G(x, y, z), \frac{1}{2}G(fx, x, z), \frac{1}{2}G(gy, y, x), \frac{1}{2}G(hz, z, y), \frac{1}{2}G(fx, x, gy), \frac{1}{2}G(gy, y, hz), \frac{1}{2}G(hz, z, fx)\Bigg\}\Bigg)$$

*for all $x, y, z \in X$. Then $f, g$ and $h$ have a unique common fixed point in $X$.*

**Corollary 2.9.** *Let $(X, G)$ be a $G$-metric space, $f, g, h, S, T$ and $R$ be six mappings of $X$ into itself such that*

$$G(fx, gy, hz) \leq a_1G(Sx, Ty, Rz) + a_2G(fx, Sx, Rz) + a_3G(gy, Ty, Sx) + a_4G(hz, Rz, Ty) + a_5G(fx, Sx, gy) + a_6G(gy, Ty, hz) + a_7G(hz, Rz, fx)$$

*for all $x, y, z \in X$, where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are nonnegative real numbers such that $a_1 + 2a_2 + 2a_3 + 2a_4 + 2a_5 + 2a_6 + 2a_7 < 1$. If the following conditions are satisfied:*

(i) *$h(X) \subset S(X)$, $f(X) \subset T(X)$, $g(X) \subset R(X)$;*

(ii) *one of $S(X), T(X)$ and $R(X)$ is a $G$-complete subspace of $X$;*

*then one of the pairs $(f, S), (g, T)$ and $(h, R)$ has a coincidence point in $X$. Moreover, if the pairs $(f, S), (g, T)$ and $(h, R)$ are weakly compatible, then $S, T, R, f, g$ and $h$ have a unique common fixed point in $X$.*

*Proof.* For $x, y, z \in X$, let

$$M(x, y, z) = \max\left\{\begin{array}{c} G(Sx, Ty, Rz), \frac{1}{2}G(fx, Sx, Rz), \frac{1}{2}G(gy, Ty, Sx), \frac{1}{2}G(hz, Rz, Ty), \\ \frac{1}{2}G(fx, Sx, gy), \frac{1}{2}G(gy, Ty, hz), \frac{1}{2}G(hz, Rz, fx) \end{array}\right\}.$$

Then

$$a_1G(Sx, Ty, Rz) + a_2G(fx, Sx, Rz) + a_3G(gy, Ty, Sx) + a_4G(hz, Rz, Ty) + a_5G(fx, Sx, gy) + a_6G(gy, Ty, hz) + a_7G(hz, Rz, fx) \leq (a_1 + 2a_2 + 2a_3 + 2a_4 + 2a_5 + 2a_6 + 2a_7)M(x, y, z).$$

So, if

$$G(fx, gy, hz) \leq a_1G(Sx, Ty, Rz) + a_2G(fx, Sx, Rz) + a_3G(gy, Ty, Sx) + a_4G(hz, Rz, Ty) + a_5G(fx, Sx, gy) + a_6G(gy, Ty, hz) + a_7G(hz, Rz, fx),$$

then $G(fx, gy, hz) \leq (a_1 + 2a_2 + 2a_3 + 2a_4 + 2a_5 + 2a_6 + 2a_7)M(x, y, z)$. Define $\phi : [0, \infty) \to [0, \infty)$ by $\phi(t) = (a_1 + 2a_2 + 2a_3 + 2a_4 + 2a_5 + 2a_6 + 2a_7)t$. Then $\phi$ is a nondecreasing function. Also, if $a_1 + 2a_2 + 2a_3 + 2_4 + 2a_5 + 2a_6 + 2a_7 < 1$, then it is clear that $\phi \in \Phi$. Then the conclusion of Corollary 2.9 can be obtained from Theorem 2.1 immediately. This completes the proof. □

*Remark* 2.10. Corollary 2.9 generalizes and extends the corresponding results of Manro et al. [12, Theorem 2.1], Manro et al. [13, Theorem 1], Mustafa et al. [15, Theorem 2.1] and Mustafa et al. [16, Theorem 2.1].

*Remark* 2.11. In Corollary 2.9, if we take:

(1) $f = g = h$;

(2) $S = T = R$;

(3) $f = g = h$ and $S = T = R$;

(4) $g = h$ and $T = R$;

(5) $g = h$ and $T = R = I$;

then several new results can be obtained.

In Corollary 2.9, if we take $S = T = R = I$, then we have the following corollary.

**Corollary 2.12.** *Let $(X, G)$ be a $G$-metric space, $f, g$ and $h$ be three mappings of $X$ into itself such that*

$$\begin{aligned} G(fx, gy, hz) \leq\ & a_1G(x, y, z) + a_2G(fx, x, z) + a_3G(gy, y, x) + a_4G(hz, z, y) \\ & + a_5G(fx, x, gy) + a_6G(gy, y, hz) + a_7G(hz, z, fx) \end{aligned}$$

*for all $x, y, z \in X$, where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are nonnegative real numbers such that $a_1 + 2a_2 + 2a_3 + 2a_4 + 2a_5 + 2a_6 + 2a_7 < 1$. Then $f, g$ and $h$ have a unique common fixed point in $X$.*

**Corollary 2.13.** *Let $(X, G)$ be a $G$-metric space, $f, g, h, S, T$ and $R$ be six mappings of $X$ into itself such that*

$$\begin{aligned} G(fx, gy, hz) \leq k \max\Big\{ & G(Sx, Ty, Rz), \frac{1}{2}G(fx, Sx, Rz), \frac{1}{2}G(gy, Ty, Sx), \frac{1}{2}G(hz, Rz, Ty), \\ & \frac{1}{2}G(fx, Sx, gy), \frac{1}{2}G(gy, Ty, hz), \frac{1}{2}G(hz, Rz, fx)\Big\} \end{aligned}$$

*for all $x, y, z \in X$, where $0 \leq k < 1$. If the following conditions are satisfied:*

(i) *$h(X) \subset S(X)$, $f(X) \subset T(X)$, $g(X) \subset R(X)$;*

(ii) *one of $S(X), T(X)$ and $R(X)$ is a $G$-complete subspace of $X$;*

*then one of the pairs $(f, S), (g, T)$ and $(h, R)$ has a coincidence point in $X$. Moreover, if the pairs $(f, S), (g, T)$ and $(h, R)$ are weakly compatible, then $S, T, R, f, g$ and $h$ have a unique common fixed point in $X$.*

*Proof.* For all $x, y, z \in X$, we let

$$M(x, y, z) = \max\left\{ \begin{array}{c} G(Sx, Ty, Rz), \frac{1}{2}G(fx, Sx, Rz), \frac{1}{2}G(gy, Ty, Sx), \frac{1}{2}G(hz, Rz, Ty), \\ \frac{1}{2}G(fx, Sx, gy), \frac{1}{2}G(gy, Ty, hz), \frac{1}{2}G(hz, Rz, fx) \end{array} \right\}.$$

If

$$\begin{aligned} G(fx, gy, hz) \leq k \max\Big\{ & G(Sx, Ty, Rz), \frac{1}{2}G(fx, Sx, Rz), \frac{1}{2}G(gy, Ty, Sx), \frac{1}{2}G(hz, Rz, Ty), \\ & \frac{1}{2}G(fx, Sx, gy), \frac{1}{2}G(gy, Ty, hz), \frac{1}{2}G(hz, Rz, fx)\Big\}, \end{aligned}$$

then $G(fx, gy, hz) \leq kM(x, y, z)$. Define $\phi : [0, \infty) \to [0, \infty)$ by $\phi(t) = kt$, then it is clear that $\phi \in \Phi$. Then the conclusion of Corollary 2.13 can be obtained from Theorem 2.1 immediately. This completes the proof. □

*Remark* 2.14. In Corollary 2.13, if we take:

(1) $f = g = h$;

(2) $S = T = R$;

(3) $f = g = h$ and $S = T = R$;

(4) $g = h$ and $T = R$;

(5) $g = h$ and $T = R = I$;

then several new results can be obtained.

In Corollary 2.13, if we take $S = T = R = I$, then we have the following corollary.

**Corollary 2.15.** *Let $(X, G)$ be a $G$-metric space, $f, g$ and $h$ be three mappings of $X$ into itself such that*

$$\begin{aligned} G(fx, gy, hz) \leq & k \max \Big\{ G(x, y, z), \frac{1}{2}G(fx, x, z), \frac{1}{2}G(gy, y, x), \frac{1}{2}G(hz, z, y), \\ & \frac{1}{2}G(fx, x, gy), \frac{1}{2}G(gy, y, hz), \frac{1}{2}G(hz, z, fx) \Big\} \end{aligned}$$

*for all $x, y, z \in X$, where $0 \leq k < 1$. Then $f, g$ and $h$ have a unique common fixed point in $X$.*

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